

RESEARCH CENTRE

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2020

ACTIVITY REPORT

Project-Team

REALOPT

Reformulations based algorithms for Combinatorial Optimization

IN COLLABORATION WITH: Institut de Mathématiques de Bordeaux
(IMB)

DOMAIN

**Applied Mathematics, Computation and
Simulation**

THEME

**Optimization, machine learning and
statistical methods**

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Project-Team REALOPT

Creation of the Project-Team: 2009 January 01

Keywords

Computer sciences and digital sciences

- A6.2.6. – Optimization
- A7.1.3. – Graph algorithms
- A8.1. – Discrete mathematics, combinatorics
- A8.2. – Optimization
- A8.2.1. – Operations research
- A8.7. – Graph theory
- A9.7. – AI algorithmics

Other research topics and application domains

- B3.1. – Sustainable development
- B3.1.1. – Resource management
- B4.2. – Nuclear Energy Production
- B4.4. – Energy delivery
- B6.5. – Information systems
- B7. – Transport and logistics
- B9.5.2. – Mathematics

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2 Overall objectives

Reformulation techniques in Mixed Integer Programming (MIP), Polyhedral approaches (cut generation), Robust Optimization, Approximation Algorithms, Extended formulations, Lagrangian Relaxation (Column Generation) based algorithms, Dantzig and Benders Decomposition, Primal Heuristics, Graph Theory, Constraint Programming.

Quantitative modeling is routinely used in both industry and administration to design and operate transportation, distribution, or production systems. Optimization concerns every stage of the decision-making process: long term investment budgeting and activity planning, tactical management of scarce resources, or the control of day-to-day operations. In many optimization problems that arise in decision support applications the most important decisions (control variables) are discrete in nature: such as on/off decision to buy, to invest, to hire, to send a vehicle, to allocate resources, to decide on precedence in operation planning, or to install a connection in network design. Such *combinatorial optimization* problems can be modeled as linear or nonlinear programs with integer decision variables and extra variables to deal with continuous adjustments. The most widely used modeling tool consists in defining the feasible decision set using linear inequalities with a mix of integer and continuous variables, so-called Mixed Integer Programs (MIP), which already allow a fair description of reality and are also well-suited for global optimization. The solution of such models is essentially based on enumeration techniques and is notoriously difficult given the huge size of the solution space.

Commercial solvers have made significant progress but remain quickly overwhelmed beyond a certain problem size. A key to further progress is the development of better problem formulations that provide strong continuous approximations and hence help to prune the enumerative solution scheme. Effective solution schemes are a complex blend of techniques: cutting planes to better approximate the convex hull of feasible (integer) solutions, extended reformulations (combinatorial relations can be formulated better with extra variables), constraint programming to actively reduce the solution domain through logical implications along variable fixing based on reduced cost, Lagrangian decomposition methods to produce powerful relaxations, and Bender's decomposition to project the formulation, reducing the problem to the important decision variables, and to implement multi-level programming that models a hierarchy of decision levels or recourse decision in the case of data adjustment, primal heuristics and meta-heuristics (greedy, local improvement, or randomized partial search procedures) to produce good candidates at all stage of the solution process, and branch-and-bound or dynamic programming enumeration schemes to find a global optimum, with specific strong strategies for the selection on the sequence of fixings. The real challenge is to integrate the most efficient methods in one global system so as to prune what is essentially an enumeration based solution technique. The progress are measured in terms of the large scale of input data that can now be solved, the integration of many decision levels into planning models, and not least, the account taken for random (or dynamically adjusted)

data by way of modeling expectation (stochastic approaches) or worst-case behavior (robust approaches).

Building on complementary expertise, our team's overall goals are threefold:

- (i) **Methodologies:** To design tight formulations for specific combinatorial optimization problems and generic models, relying on delayed cut and column generation, decomposition, extended formulations and projection tools for linear and nonlinear mixed integer programming models. To develop generic methods based on such strong formulations by handling their large scale dynamically. To generalize algorithmic features that have proven efficient in enhancing performance of exact optimization approaches. To develop approximation schemes with proven optimality gap and low computational complexity. More broadly, to contribute to theoretical and methodological developments of exact and approximate approaches in combinatorial optimization, while extending the scope of applications and their scale.
- (ii) **Problem solving:** To demonstrate the strength of cooperation between complementary exact mathematical optimization techniques, dynamic programming, robust and stochastic optimization, constraint programming, combinatorial algorithms and graph theory, by developing "efficient" algorithms for specific mathematical models. To tackle large-scale real-life applications, providing provably good approximate solutions by combining exact, approximate, and heuristic methods.
- (iii) **Software platform & Transfer:** To provide prototypes of modelers and solvers based on generic software tools that build on our research developments, writing code that serves as the proof-of-concept of the genericity and efficiency of our approaches, while transferring our research findings to internal and external users.

3 Research program

3.1 Introduction

Keywords: integer programming, graph theory, decomposition approaches, polyhedral approaches, quadratic programming approaches, constraint programming..

Combinatorial optimization is the field of discrete optimization problems. In many applications, the most important decisions (control variables) are binary (on/off decisions) or integer (indivisible quantities). Extra variables can represent continuous adjustments or amounts. This results in models known as *mixed integer programs* (MIP), where the relationships between variables and input parameters are expressed as linear constraints and the goal is defined as a linear objective function. MIPs are notoriously difficult to solve: good quality estimations of the optimal value (bounds) are required to prune enumeration-based global-optimization algorithms whose complexity is exponential. In the standard approach to solving an MIP is so-called *branch-and-bound algorithm*: (i) one solves the linear programming (LP) relaxation using the simplex method; (ii) if the LP solution is not integer, one adds a disjunctive constraint on a fractional component (rounding it up or down) that defines two sub-problems; (iii) one applies this procedure recursively, thus defining a binary enumeration tree that can be pruned by comparing the local LP bound to the best known integer solution. Commercial MIP solvers are essentially based on branch-and-bound (such IBM-CPLEX, FICO-Xpress-mp, or GUROBI). They have made tremendous progress over the last decade (with a speedup by a factor of 60). But extending their capabilities remains a continuous challenge; given the combinatorial explosion inherent to enumerative solution techniques, they remain quickly overwhelmed beyond a certain problem size or complexity.

Progress can be expected from the development of tighter formulations. Central to our field is the characterization of polyhedra defining or approximating the solution set and combinatorial algorithms to identify "efficiently" a minimum cost solution or separate an unfeasible point. With properly chosen formulations, exact optimization tools can be competitive with other methods (such as meta-heuristics) in constructing good approximate solutions within limited computational time, and of course has the important advantage of being able to provide a performance guarantee through the relaxation

bounds. Decomposition techniques are implicitly leading to better problem formulation as well, while constraint propagation are tools from artificial intelligence to further improve formulation through intensive preprocessing. A new trend is robust optimization where recent progress have been made: the aim is to produce optimized solutions that remain of good quality even if the problem data has stochastic variations. In all cases, the study of specific models and challenging industrial applications is quite relevant because developments made into a specific context can become generic tools over time and see their way into commercial software.

Our project brings together researchers with expertise in mathematical programming (polyhedral approaches, decomposition and reformulation techniques in mixed integer programming, robust and stochastic programming, and dynamic programming), graph theory (characterization of graph properties, combinatorial algorithms) and constraint programming in the aim of producing better quality formulations and developing new methods to exploit these formulations. These new results are then applied to find high quality solutions for practical combinatorial problems such as routing, network design, planning, scheduling, cutting and packing problems, High Performance and Cloud Computing.

3.2 Polyhedral approaches for MIP

Adding valid inequalities to the polyhedral description of an MIP allows one to improve the resulting LP bound and hence to better prune the enumeration tree. In a cutting plane procedure, one attempt to identify valid inequalities that are violated by the LP solution of the current formulation and adds them to the formulation. This can be done at each node of the branch-and-bound tree giving rise to a so-called *branch-and-cut algorithm* [49]. The goal is to reduce the resolution of an integer program to that of a linear program by deriving a linear description of the convex hull of the feasible solutions. Polyhedral theory tells us that if X is a mixed integer program: $X = P \cap \mathbb{Z}^n \times \mathbb{R}^p$ where $P = \{x \in \mathbb{R}^{n+p} : Ax \leq b\}$ with matrix $(A, b) \in \mathbb{Q}^{m \times (n+p+1)}$, then $\text{conv}(X)$ is a polyhedron that can be described in terms of linear constraints, i.e. it writes as $\text{conv}(X) = \{x \in \mathbb{R}^{n+p} : Cx \leq d\}$ for some matrix $(C, d) \in \mathbb{Q}^{m' \times (n+p+1)}$ although the dimension m' is typically quite large. A fundamental result in this field is the equivalence of complexity between solving the combinatorial optimization problem $\min\{cx : x \in X\}$ and solving the *separation problem* over the associated polyhedron $\text{conv}(X)$: if $\tilde{x} \notin \text{conv}(X)$, find a linear inequality $\pi x \geq \pi_0$ satisfied by all points in $\text{conv}(X)$ but violated by \tilde{x} . Hence, for NP-hard problems, one can not hope to get a compact description of $\text{conv}(X)$ nor a polynomial time exact separation routine. Polyhedral studies focus on identifying some of the inequalities that are involved in the polyhedral description of $\text{conv}(X)$ and derive efficient *separation procedures* (cutting plane generation). Only a subset of the inequalities $Cx \leq d$ can offer a good approximation, that combined with a branch-and-bound enumeration techniques permits to solve the problem. Using *cutting plane algorithm* at each node of the branch-and-bound tree, gives rise to the algorithm called *branch-and-cut*.

3.3 Decomposition-and-reformulation-approaches

An hierarchical approach to tackle complex combinatorial problems consists in considering separately different substructures (subproblems). If one is able to implement relatively efficient optimization on the substructures, this can be exploited to reformulate the global problem as a selection of specific subproblem solutions that together form a global solution. If the subproblems correspond to subset of constraints in the MIP formulation, this leads to Dantzig-Wolfe decomposition. If it corresponds to isolating a subset of decision variables, this leads to Bender's decomposition. Both lead to extended formulations of the problem with either a huge number of variables or constraints. Dantzig-Wolfe approach requires specific algorithmic approaches to generate subproblem solutions and associated global decision variables dynamically in the course of the optimization. This procedure is known as *column generation*, while its combination with branch-and-bound enumeration is called *branch-and-price*. Alternatively, in Bender's approach, when dealing with exponentially many constraints in the reformulation, the *cutting plane procedures* that we defined in the previous section are well-suited tools. When optimization on a substructure is (relatively) easy, there often exists a tight reformulation of this substructure typically in an extended variable space. This gives rise powerful reformulation of the global problem, although it might be impractical given its size (typically pseudo-polynomial). It can be possible to project (part of) the extended formulation in a smaller dimensional space if not the original variable

space to bring polyhedral insight (cuts derived through polyhedral studies can often be recovered through such projections).

3.4 Integration of Artificial Intelligence Techniques in Integer Programming

When one deals with combinatorial problems with a large number of integer variables, or tightly constrained problems, mixed integer programming (MIP) alone may not be able to find solutions in a reasonable amount of time. In this case, techniques from artificial intelligence can be used to improve these methods. In particular, we use variable fixing techniques, primal heuristics and constraint programming.

Primal heuristics are useful to find feasible solutions in a small amount of time. We focus on heuristics that are either based on integer programming (rounding, diving, relaxation induced neighborhood search, feasibility pump), or that are used inside our exact methods (heuristics for separation or pricing subproblem, heuristic constraint propagation, ...). Such methods are likely to produce good quality solutions only if the integer programming formulation is of top quality, i.e., if its LP relaxation provides a good approximation of the IP solution.

In the same line, variable fixing techniques, that are essential in reducing the size of large scale problems, rely on good quality approximations: either tight formulations or tight relaxation solvers (as a dynamic program combined with state space relaxation). Then if the dual bound derives when the variable is fixed to one exceeds the incumbent solution value, the variable can be fixed to zero and hence removed from the problem. The process can be apply sequentially by refining the degree of relaxation.

Constraint Programming (CP) focuses on iteratively reducing the variable domains (sets of feasible values) by applying logical and problem-specific operators. The latter propagates on selected variables the restrictions that are implied by the other variable domains through the relations between variables that are defined by the constraints of the problem. Combined with enumeration, it gives rise to exact optimization algorithms. A CP approach is particularly effective for tightly constrained problems, feasibility problems and min-max problems. Mixed Integer Programming (MIP), on the other hand, is known to be effective for loosely constrained problems and for problems with an objective function defined as the weighted sum of variables. Many problems belong to the intersection of these two classes. For such problems, it is reasonable to use algorithms that exploit complementary strengths of Constraint Programming and Mixed Integer Programming.

3.5 Robust Optimization

Decision makers are usually facing several sources of uncertainty, such as the variability in time or estimation errors. A simplistic way to handle these uncertainties is to overestimate the unknown parameters. However, this results in over-conservatism and a significant waste in resource consumption. A better approach is to account for the uncertainty directly into the decision aid model by considering mixed integer programs that involve uncertain parameters. Stochastic optimization account for the expected realization of random data and optimize an expected value representing the average situation. Robust optimization on the other hand entails protecting against the worst-case behavior of unknown data. There is an analogy to game theory where one considers an oblivious adversary choosing the realization that harms the solution the most. A full worst case protection against uncertainty is too conservative and induces very high over-cost. Instead, the realization of random data are bound to belong to a restricted feasibility set, the so-called uncertainty set. Stochastic and robust optimization rely on very large scale programs where probabilistic scenarios are enumerated. There is hope of a tractable solution for realistic size problems, provided one develops very efficient ad-hoc algorithms. The techniques for dynamically handling variables and constraints (column-and-row generation and Bender's projection tools) that are at the core of our team methodological work are specially well-suited to this context.

3.6 Polyhedral Combinatorics and Graph Theory

Many fundamental combinatorial optimization problems can be modeled as the search for a specific structure in a graph. For example, ensuring connectivity in a network amounts to building a *tree* that spans all the nodes. Inquiring about its resistance to failure amounts to searching for a minimum

cardinality *cut* that partitions the graph. Selecting disjoint pairs of objects is represented by a so-called *matching*. Disjunctive choices can be modeled by edges in a so-called *conflict graph* where one searches for *stable sets* – a set of nodes that are not incident to one another. Polyhedral combinatorics is the study of combinatorial algorithms involving polyhedral considerations. Not only it leads to efficient algorithms, but also, conversely, efficient algorithms often imply polyhedral characterizations and related min-max relations. Developments of polyhedral properties of a fundamental problem will typically provide us with more interesting inequalities well suited for a branch-and-cut algorithm to more general problems. Furthermore, one can use the fundamental problems as new building bricks to decompose the more general problem at hand. For problem that let themselves easily be formulated in a graph setting, the graph theory and in particular graph decomposition theorem might help.

4 Application domains

4.1 Network Design and Routing Problems

We are actively working on problems arising in network topology design, implementing a survivability condition of the form “at least two paths link each pair of terminals”. We have extended polyhedral approaches to problem variants with bounded length requirements and re-routing restrictions [42]. Associated to network design is the question of traffic routing in the network: one needs to check that the network capacity suffices to carry the demand for traffic. The assignment of traffic also implies the installation of specific hardware at transient or terminal nodes.

To accommodate the increase of traffic in telecommunication networks, today’s optical networks use grooming and wavelength division multiplexing technologies. Packing multiple requests together in the same optical stream requires to convert the signal in the electrical domain at each aggregation of disaggregation of traffic at an origin, a destination or a bifurcation node. Traffic grooming and routing decisions along with wavelength assignments must be optimized to reduce opto-electronics system installation cost. We developed and compared several decomposition approaches [65, 68, 67] to deal with backbone optical network with relatively few nodes (around 20) but thousands of requests for which traditional multi-commodity network flow approaches are completely overwhelmed. We also studied the impact of imposing a restriction on the number of optical hops in any request route [66]. We also developed a branch-and-cut approach to a problem that consists in placing sensors on the links of a network for a minimum cost [51, 50].

The Dial-a-Ride Problem is a variant of the pickup and delivery problem with time windows, where the user inconvenience must be taken into account. In [58], ride time and customer waiting time are modeled through both constraints and an associated penalty in the objective function. We develop a column generation approach, dynamically generating feasible vehicle routes. Handling ride time constraints explicitly in the pricing problem solver requires specific developments. Our dynamic programming approach for pricing problem makes use of a heuristic dominance rule and a heuristic enumeration procedure, which in turns implies that our overall branch-and-price procedure is a heuristic. However, in practice our heuristic solutions are experimentally very close to exact solutions and our approach is numerically competitive in terms of computation times.

In [55, 56], we consider the problem of covering an urban area with sectors under additional constraints. We adapt the aggregation method to our column generation algorithm and focus on the problem of disaggregating the dual solution returned by the aggregated master problem.

We studied several time dependent formulations for the unit demand vehicle routing problem [34, 33]. We gave new bounding flow inequalities for a single commodity flow formulation of the problem. We described their impact by projecting them on some other sets of variables, such as variables issued of the Picard and Queyranne formulation or the natural set of design variables. Some inequalities obtained by projection are facet defining for the polytope associated with the problem. We are now running more numerical experiments in order to validate in practice the efficiency of our theoretical results.

We also worked on the p-median problem, applying the matching theory to develop an efficient algorithm in Y-free graphs and to provide a simple polyhedral characterization of the problem and therefore a simple linear formulation [64] simplifying results from Baiou and Barahona.

We considered the multi-commodity transportation problem. Applications of this problem arise in,

for example, rail freight service design, "less than truckload" trucking, where goods should be delivered between different locations in a transportation network using various kinds of vehicles of large capacity. A particularity here is that, to be profitable, transportation of goods should be consolidated. This means that goods are not delivered directly from the origin to the destination, but transferred from one vehicle to another in intermediate locations. We proposed an original Mixed Integer Programming formulation for this problem which is suitable for resolution by a Branch-and-Price algorithm and intelligent primal heuristics based on it.

For the problem of routing freight railcars, we proposed two algorithms based on the column generation approach. These algorithms have been tested on a set of real-life instances coming from a real Russian freight transportation company. Our algorithms have been faster on these instances than the current solution approach being used by the company.

4.2 Packing and Covering Problems

Realopt team has a strong experience on exact methods for cutting and packing problems. These problems occur in logistics (loading trucks), industry (wood or steel cutting), computer science (parallel processor scheduling).

We developed a branch-and-price algorithm for the Bin Packing Problem with Conflicts which improves on other approaches available in the literature [63]. The algorithm uses our methodological advances like the generic branching rule for the branch-and-price and the column based heuristic. One of the ingredients which contributes to the success of our method are fast algorithms we developed for solving the subproblem which is the Knapsack Problem with Conflicts. Two variants of the subproblem have been considered: with interval and arbitrary conflict graphs.

We also developed a branch-and-price algorithm for a variant of the bin-packing problem where the items are fragile. In [24] we studied empirically different branching schemes and different algorithms for solving the subproblems.

We studied a variant of the knapsack problem encountered in inventory routing problem [53]: we faced a multiple-class integer knapsack problem with setups [52] (items are partitioned into classes whose use implies a setup cost and associated capacity consumption). We showed the extent to which classical results for the knapsack problem can be generalized to this variant with setups and we developed a specialized branch-and-bound algorithm.

We studied the orthogonal knapsack problem, with the help of graph theory [45, 43], [46, 44]. Fekete and Schepers proposed to model multi-dimensional orthogonal placement problems by using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to represent interval graphs. We propose to represent these graphs by matrices with consecutive ones on each row. We proposed a branch-and-bound algorithm for the 2D knapsack problem that uses our 2D packing feasibility check. We are currently developing exact optimization tools for glass-cutting problems in a collaboration with Saint-Gobain [27]. This 2D-3stage-Guillotine cut problems are very hard to solve given the scale of the instance we have to deal with. Moreover one has to issue cutting patterns that avoid the defaults that are present in the glass sheet that are used as raw material. There are extra sequencing constraints regarding the production that make the problem even more complex.

We have also organized a European challenge on packing with society Renault. This challenge was about loading trucks under practical constraints.

4.3 Planning, Scheduling, and Logistic Problems

Inventory routing problems combine the optimization of product deliveries (or pickups) with inventory control at customer sites. We considered an industrial application where one must construct the planning of single product pickups over time; each site accumulates stock at a deterministic rate; the stock is emptied on each visit. We have developed a branch-and-price algorithm where periodic plans are generated for vehicles by solving a multiple choice knapsack subproblem, and the global planning

of customer visits is coordinated by the master program [54]. We previously developed approximate solutions to a related problem combining vehicle routing and planning over a fixed time horizon (solving instances involving up to 6000 pick-ups and deliveries to plan over a twenty day time horizon with specific requirements on the frequency of visits to customers [48]).

Together with our partner company GAPSO from the associate team SAMBA, we worked on the equipment routing task scheduling problem [57] arising during port operations. In this problem, a set of tasks needs to be performed using equipments of different types with the objective to maximize the weighted sum of performed tasks.

We participated to the project on an airborne radar scheduling. For this problem, we developed fast heuristics [41] and exact algorithms [26]. A substantial research has been done on machine scheduling problems. A new compact MIP formulation was proposed for a large class of these problems [25]. An exact decomposition algorithm was developed for the NP-hard maximizing the weighted number of late jobs problem on a single machine [59]. A dominant class of schedules for malleable parallel jobs was discovered in the NP-hard problem to minimize the total weighted completion time [61]. We proved that a special case of the scheduling problem at cross docking terminals to minimize the storage cost is polynomially solvable [62], [60].

Another application area in which we have successfully developed MIP approaches is in the area of tactical production and supply chain planning. In [23], we proposed a simple heuristic for challenging multi-echelon problems that makes effective use of a standard MIP solver. [22] contains a detailed investigation of what makes solving the MIP formulations of such problems challenging; it provides a survey of the known methods for strengthening formulations for these applications, and it also pinpoints the specific substructure that seems to cause the bottleneck in solving these models. Finally, the results of [28] provide demonstrably stronger formulations for some problem classes than any previously proposed. We are now working on planning phytosanitary treatments in vineries.

We have been developing robust optimization models and methods to deal with a number of applications like the above in which uncertainty is involved. In [39, 38], we analyzed fundamental MIP models that incorporate uncertainty and we have exploited the structure of the stochastic formulation of the problems in order to derive algorithms and strong formulations for these and related problems. These results appear to be the first of their kind for structured stochastic MIP models. In addition, we have engaged in successful research to apply concepts such as these to health care logistics [29]. We considered train timetabling problems and their re-optimization after a perturbation in the network [21, 32]. The question of formulation is central. Models of the literature are not satisfactory: continuous time formulations have poor quality due to the presence of discrete decision (re-sequencing or re-routing); arc flow in time-space graph blow-up in size (they can only handle a single line timetabling problem). We have developed a discrete time formulation that strikes a compromise between these two previous models. Based on various time and network aggregation strategies, we develop a 2-stage approach, solving the contiguous time model having fixed the precedence based on a solution to the discrete time model.

Currently, we are conducting investigations on a real-world planning problem in the domain of energy production, in the context of a collaboration with EDF [37, 36, 35]. The problem consists in scheduling maintenance periods of nuclear power plants as well as production levels of both nuclear and conventional power plants in order to meet a power demand, so as to minimize the total production cost. For this application, we used a Dantzig-Wolfe reformulation which allows us to solve realistic instances of the deterministic version of the problem [40]. In practice, the input data comprises a number of uncertain parameters. We deal with a scenario-based stochastic demand with help of a Benders decomposition method. We are working on Multistage Robust Optimization approaches to take into account other uncertain parameters like the duration of each maintenance period, in a dynamic optimization framework. The main challenge addressed in this work is the joint management of different reformulations and solving techniques coming from the deterministic (Dantzig-Wolfe decomposition, due to the large scale nature of the problem), stochastic (Benders decomposition, due to the number of demand scenarios) and robust (reformulations based on duality and/or column and/or row generation due to maintenance extension scenarios) components of the problem [30].

5 Social and environmental responsibility

5.1 Footprint of research activities

Our research involves a large amount of computational experiments.

5.2 Impact of research results

The objective of our research is to reduce the quantity of energy/material used to realize some large projects, including energy production and distribution, chemical treatments, and distribution of goods.

6 Highlights of the year

2020 was marked by the covid crisis and its impact on the overall society and its activity. The world of research has also been greatly affected:

- faculty members have seen their teaching load increase significantly;
- PhD students and post-docs have often had to deal with a worsening of their working conditions;
- most scientific collaborations have been greatly affected, with several of international activities cancelled or postponed to dates still to be defined.

On the bright side, a major publication proposing the first generic exact solver for vehicle routing and related problems has been published [9] in *Mathematical Programming*, one of the top journals in the area.

7 New software and platforms

7.1 New software

7.1.1 BaPCod

Name: A generic Branch-And-Price-And-Cut Code

Keywords: Column Generation, Branch-and-Price, Branch-and-Cut, Mixed Integer Programming, Mathematical Optimization, Benders Decomposition, Dantzig-Wolfe Decomposition, Extended Formulation

Functional Description: BaPCod is a prototype code that solves Mixed Integer Programs (MIP) by application of reformulation and decomposition techniques. The reformulated problem is solved using a branch-and-price-and-cut (column generation) algorithms, Benders approaches, network flow and dynamic programming algorithms. These methods can be combined in several hybrid algorithms to produce exact or approximate solutions (primal solutions with a bound on the deviation to the optimum).

Release Contributions: Correction of numerous bugs.

URL: https://realopt.bordeaux.inria.fr/?page_id=2

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Partners: Université de Bordeaux, CNRS, IPB, Universidade Federal Fluminense

7.1.2 VRPSolver

Name: VRPSolver

Keywords: Column Generation, Vehicle routing, Numerical solver

Scientific Description: Major advances were recently obtained in the exact solution of Vehicle Routing Problems (VRPs). Sophisticated Branch-Cut-and-Price (BCP) algorithms for some of the most classical VRP variants now solve many instances with up to a few hundreds of customers. However, adapting and reimplementing those successful algorithms for other variants can be a very demanding task. This work proposes a BCP solver for a generic model that encompasses a wide class of VRPs. It incorporates the key elements found in the best recent VRP algorithms: ng-path relaxation, rank-1 cuts with limited memory, and route enumeration, all generalized through the new concept of "packing set". This concept is also used to derive a new branch rule based on accumulated resource consumption and to generalize the Ryan and Foster branch rule. Extensive experiments on several variants show that the generic solver has an excellent overall performance, in many problems being better than the best existing specific algorithms. Even some non-VRPs, like bin packing, vector packing and generalized assignment, can be modeled and effectively solved.

Functional Description: This solver allows one to model and solve to optimality many combinatorial optimization problems, belonging to the class of vehicle routing, scheduling, packing and network design problems. The problem is formulated using variables, linear objective function, linear and integrality constraints, definition of graphs, resources, and mapping between graph arcs and variables. A complex Branch-Cut-and-Price algorithm is used to solve the model. A new concept of elementarity and packing sets is used to pass an additional information to the solver, so that several state-of-the-art Branch-Cut-and-Price components can be used to improve radically the efficiency of the solver. The interface of the solver is implemented in Julia using JuMP package. To simplify the installation and usage, the solver is distributed as a docker image. The solver can be used only for academic purposes.

Release Contributions: Version 0.4 brings introduction of elementarity sets, bug corrections, as well as update of dependencies.

News of the Year: 2020 - version 0.4 2019 - solver release, versions 0.1, 0.2, 0.3

URL: <https://vrpsolver.math.u-bordeaux.fr/>

Publication: [hal-02178171v2](https://arxiv.org/abs/1908.02171v2)

Contact: Ruslan Sadykov

Participants: Ruslan Sadykov, Eduardo Uchoa Barboza, Artur Alves Pessoa, Eduardo Queiroga, Teobaldo Bulhões, Laurent Facq

Partners: Universidade Federal Fluminense, Universidade Federal da Paraíba

8 New results

8.1 Algorithms for optimization under uncertainty

We introduce a new exact algorithm, called Benders by batch algorithm, based on Benders decomposition to solve two-stage stochastic linear programs. This algorithm is based on based on the multicut formulation of Benders decomposition and solves only a few number of subproblems at each iteration. We propose two primal stabilization methods for the algorithm and perform an extensive computational study on six large-scale benchmarks of the stochastic optimization literature. Results show the efficiency of the method compared to five classical alternative algorithms and significant time saving provided by its primal stabilization. We show acceleration factors up to 10 times faster than the best method from the literature we compare to, and up to 800 times faster than IBM ILOG CPLEX 12.10 built-in Benders decomposition.

We have studied a class of two-stage robust binary optimization problems with objective uncertainty where recourse decisions are restricted to be mixed-binary [3]. For these problems, we present a deterministic equivalent formulation through the convexification of the recourse feasible region. We then explore this formulation under the lens of a relaxation, showing that the specific relaxation we propose can be solved using the branch-and-price algorithm. We present conditions under which this relaxation is exact, and describe alternative exact solution methods when this is not the case. Despite the two-stage nature of the problem, we provide NP-completeness results based on our reformulations. Finally, we present various applications in which the methodology we propose can be applied. We compare our exact methodology to those approximate methods recently proposed in the literature under the name K-adaptability. Our computational results show that our methodology is able to produce better solutions in less computational time compared to the K-adaptability approach, as well as to solve bigger instances than those previously managed in the literature.

8.2 Machine scheduling problems

Minimizing the weighted number of tardy jobs is a classical and intensively studied scheduling problem. In [15], we develop a two-stage robust approach, where exact weights are known after accepting to perform the jobs, and before sequencing them on the machine. This assumption allows diverse recourse decisions to be taken in order to better adapt one's tactical plan. The contribution of this paper is twofold: first, we introduce a new scheduling problem and model it as a min-max-min optimization problem with mixed-integer recourse by extending existing models proposed for the classical problem where all the costs are assumed to be known. Second, we take advantage of the special structure of the problem to propose two solution approaches based on results from the recent robust optimization literature, namely finite adaptability (Bertsimas and Caramanis, 2010) and a convexification-based approach (Arslan and Detienne, 2020). We also study the cost of finding anchored solutions, where the sequence of jobs has to be decided before the uncertainty is revealed. Computational experiments to analyze the effectiveness of our approaches are reported.

Work [4] deals with a very generic class of scheduling problems with identical/uniform/unrelated parallel machine environment. It considers well-known attributes such as release dates or sequence-dependent setup times and accepts any objective function defined over job completion times. Non-regular objectives are also supported. We introduce a branch-cut-and-price algorithm for such problems that makes use of non-robust cuts, i.e., cuts which change the structure of the pricing problem. This is the first time that such cuts are employed for machine scheduling problems. The algorithm also embeds other important techniques such as strong branching, reduced cost fixing and dual stabilization. Computational experiments over literature benchmarks showed that the proposed algorithm is indeed effective and could solve many instances to optimality for the first time.

8.3 Generic solver for vehicle routing and similar problems

Major advances were recently obtained in the exact solution of Vehicle Routing Problems (VRPs). Sophisticated Branch-Cut-and-Price (BCP) algorithms for some of the most classical VRP variants now solve many instances with up to a few hundreds of customers. However, adapting and reimplementing those successful algorithms for other variants can be a very demanding task. Work [9] proposes a BCP solver for a generic model that encompasses a wide class of VRPs. It incorporates the key elements found in the best recent VRP algorithms: ng-path relaxation, rank-1 cuts with limited memory, and route enumeration; all generalized through the new concept of "packing set". This concept is also used to derive a new branch rule based on accumulated resource consumption and to generalize the Ryan and Foster branch rule. Extensive experiments on several variants show that the generic solver has an excellent overall performance, in many problems being better than the best existing specific algorithms. Even some non-VRPs, like bin packing, vector packing and generalized assignment, can be modeled and effectively solved.

The Shortest Path Problem with Resource Constraints (SPPRC) arises as a subproblem in state-of-the-art Branch-Cut-and-Price algorithms for vehicle routing problems, including the BCP solver described just above. In [11], we propose a variant of the bi-directional label correcting algorithm in which the labels are stored and extended according to the so-called bucket graph. Such organization of labels helps

to decrease significantly the number of dominance checks and the running time of the algorithm. We also show how the forward/backward route symmetry can be exploited and how to eliminate arcs from the bucket graph using reduced costs. The proposed algorithm can be especially beneficial for vehicle routing instances with large vehicle capacity and/or with time window constraints. Computational experiments were performed on instances from the distance constrained vehicle routing problem, including multi-depot and site-dependent variants, on the vehicle routing problem with time windows, and on the "nightmare" instances of the heterogeneous fleet vehicle routing problem. Significant improvements over the best algorithms in the literature were achieved and many instances could be solved for the first time.

8.4 Vehicle routing applications

8.4.1 Classic vehicle routing problems

In [7], we examine the robust counterpart of the classical Capacitated Vehicle Routing Problem (CVRP). We consider two types of uncertainty sets for the customer demands: the classical budget polytope introduced by Bertsimas and Sim (2003), and a partitioned budget polytope proposed by Gounaris et al. (2013). We show that using the set-partitioning formulation it is possible to reformulate our problem as a deterministic heterogeneous vehicle routing problem. Thus, many state-of-the-art techniques for exactly solving deterministic VRPs can be applied for the robust counterpart, and a modern branch-and-cut-and-price algorithm can be adapted to our setting by keeping the number of pricing subproblems strictly polynomial. More importantly, we introduce new techniques to significantly improve the efficiency of the algorithm. We present analytical conditions under which a pricing subproblem is infeasible. This result is general and can be applied to other combinatorial optimization problems with knapsack uncertainty. We also introduce robust capacity cuts which are provably stronger than the ones known in the literature. Finally, a fast iterated local search algorithm is proposed to obtain heuristic solutions for the problem. Using our branch-and-cut-and-price algorithm incorporating existing and new techniques, we are able to solve to optimality all but one open instances from the literature.

In [10], we are interested in the exact solution of the vehicle routing problem with back-hauls (VRPB), a classical vehicle routing variant with two types of customers: linehaul (delivery) and backhaul (pickup) ones. We propose two branch-cut-and-price (BCP) algorithms for the VRPB. The first of them follows the traditional approach with one pricing subproblem, whereas the second one exploits the linehaul/backhaul customer partitioning and defines two pricing sub-problems. The methods incorporate elements of state-of-the-art BCP algorithms, such as rounded capacity cuts, limited-memory rank-1 cuts, strong branching, route enumeration, arc elimination using reduced costs and dual stabilization. Computational experiments show that the proposed algorithms are capable of obtaining optimal solutions for all existing benchmark instances with up to 200 customers, many of them for the first time. It is observed that the approach involving two pricing subproblems is more efficient computationally than the traditional one. Moreover, new instances are also proposed for which we provide tight bounds. Also, we provide results for benchmark instances of the heterogeneous fixed fleet VRPB and the VRPB with time windows.

In [17], we propose a partial optimization metaheuristic under special intensification conditions (POPMUSIC) for the classical capacitated vehicle routing problem (CVRP). The proposed approach uses a branch-cut-and-price algorithm as a powerful heuristic to solve subproblems whose dimensions are typically between 25 and 200 customers. The whole algorithm can be seen as the application of local search over very large neighborhoods, starting from a single initial solution. The main computational experiments were carried out on instances having between 302 and 1000 customers. Using initial solutions generated by some of the best available metaheuristics for the problem, POPMUSIC was able to obtain consistently better solutions for long runs of up to 32 hours. In a final experiment, starting from the best known solutions available in CVRP library (CVRPLIB), POPMUSIC was able to find new best solutions for several instances, including some very large ones.

8.4.2 Fixed route vehicle charging problem

Electric vehicles offer a pathway to more sustainable transportation, but their adoption entails new challenges not faced by their petroleum-based counterparts. One of the most challenging tasks in vehicle routing problems addressing these challenges is determining how to make good charging decisions for

an electric vehicle traveling a given route. This is known as the fixed route vehicle charging problem. An exact and efficient algorithm for this task was introduced in a recent work [31]. The algorithm has been used and extended by [47] to account for specific features (time windows, deterministic waiting times). Its implementation is sufficiently complex to deter researchers from adopting it. In [14], we introduce `frvcpy`, an open-source Python package implementing this algorithm. Our aim with the package is to make it easier for researchers to solve electric vehicle routing problems, facilitating the development of optimization tools that may ultimately enable the mass adoption of electric vehicles.

8.4.3 Two-echelon vehicle routing problems

Guillaume Marques successfully defended his thesis [12] on solution approaches for two-echelon vehicle routing problems. This thesis includes the following two works.

In [6], we propose a branch-cut-and-price algorithm for the two-echelon capacitated vehicle routing problem in which delivery of products from a depot to customers is performed using intermediate depots called satellites. Our algorithm incorporates significant improvements recently proposed in the literature for the standard capacitated vehicle routing problem such as bucket graph based labeling algorithm for the pricing problem, automatic stabilization, limited memory rank-1 cuts, and strong branching. In addition, we make some specific problem contributions. First, we introduce a new route based formulation for the problem which does not use variables to determine product flows in satellites. Second, we introduce a new branching strategy which significantly decreases the size of the branch-and-bound tree. Third, we introduce a new family of satellite supply inequalities, and we empirically show that it improves the quality of the dual bound at the root node of the branch-and-bound tree. Finally, extensive numerical experiments reveal that our algorithm can solve to optimality all literature instances with up to 200 customers and 10 satellites for the first time and thus double the size of instances which could be solved to optimality.

The previous work has been to the case when delivery to each client should be performed within a specific time window. In [16], we consider the variant of the problem with precedence constraints for unloading and loading freight at satellites. This variant allows for storage and consolidation of freight at satellites. Thus, the total transportation cost may decrease in comparison with the alternative variant with exact freight synchronization at satellites. We suggest a mixed integer programming formulation for the problem with an exponential number of route variables and an exponential number of precedence constraints which link first-echelon and second-echelon routes. Routes at the second echelon connecting satellites and clients may consist of multiple trips and visit several satellites. A branch-cut-and-price algorithm is proposed to solve efficiently the problem. This is the first exact algorithm in the literature for the multi-trip variant of the problem. We also present a post-processing procedure to check whether the solution can be transformed to avoid freight consolidation and storage without increasing its transportation cost. Our algorithm significantly outperforms another recent one for the single-trip variant of the problem. We also show that all single-trip literature instances solved to optimality admit optimal solutions of the same cost for both variants of the problem either with precedence constraints or with exact synchronization constraints.

Given the emergence of two-echelon distribution systems in several practical contexts, this paper tackles, at the strategic level, a distribution network design problem under uncertainty. This problem is characterized by the two-echelon stochastic multi-period capacitated location-routing problem (2E-SM-CLRP). In the first echelon, one has to decide the number and location of warehouse platforms as well as the intermediate distribution platforms for each period; while fixing the capacity of the links between them. In the second echelon, the goal is to construct vehicle routes that visit ship-to locations (SLs) from operating distribution platforms under a stochastic and time-varying demand and varying costs. This problem is modeled as a two-stage stochastic program with integer recourse, where the first-stage includes location and capacity decisions to be fixed at each period over the planning horizon, while routing decisions of the second echelon are determined in the recourse problem. In [13], we propose a logic-based Benders decomposition approach to solve this model. In the proposed approach, the location and capacity decisions are taken by solving the Benders master problem. After these first-stage decisions are fixed, the resulting sub-problem is a capacitated vehicle-routing problem with capacitated multiple depots (CVRP-CMD) that is solved by a branch-cut-and-price algorithm. Computational experiments show that instances of realistic size can be solved optimally within a reasonable time and provide relevant

managerial insights on the design problem.

8.5 Cutting and packing problems

In [18], we introduce and motivate a variant of the bin packing problem where bins are assigned to time slots, and minimum and maximum lags are required between some pairs of items. We suggest two integer programming formulations for the problem: a compact one, and a stronger formulation with an exponential number of variables and constraints. We propose a branch-cut-and-price approach which exploits the latter formulation. For this purpose, we devise separation algorithms based on a mathematical characterization of feasible assignments for two important special cases of the problem. Computational experiments are reported for instances inspired from a real-case application of chemical treatment planning in vineyards, as well as for literature instances for special cases of the problem. The experimental results show the efficiency of our branch-cut-and-price approach, as it outperforms the compact formulation of newly proposed instances, and is able to obtain improved lower and upper bounds for literature instances.

In [8], we propose branch-cut-and-price algorithms for the classic bin packing problem and also for the following related problems: vector packing, variable sized bin packing and variable sized bin packing with optional items. The algorithms are defined as models for VRPSolver, a generic solver for vehicle routing problems. In that way, a simple parameterization enables the use of several branch-cut-and-price advanced elements: automatic stabilization by smoothing, limited-memory rank-1 cuts, enumeration, hierarchical strong branching and limited discrepancy search diving heuristics. As an original theoretical contribution, we prove that the branching over accumulated resource consumption, that does not increase the difficulty of the pricing subproblem, is sufficient for those bin packing models. Extensive computational results on instances from the literature show that the VRPSolver models have a performance that is very robust over all those problems, being often superior to the existing exact algorithms on the hardest instances. Several instances could be solved to optimality for the first time.

We have developed an approach to solve the temporal knapsack problem (TKP) based on a very large size dynamic programming formulation [5]. In this generalization of the classical knapsack problem, selected items enter and leave the knapsack at fixed dates. We solve the TKP with a dynamic program of exponential size, which is solved using a method called Successive Sublimation Dynamic Programming (SSDP). This method starts by relaxing a set of constraints from the initial problem, and iteratively reintroduces them when needed. We show that a direct application of SSDP to the temporal knapsack problem does not lead to an effective method, and that several improvements are needed to compete with the best results from the literature.

9 Bilateral contracts and grants with industry

9.1 Bilateral contracts with industry

We have a contract with RTE to develop strategies inspired from stochastic gradient methods to speed-up Benders' decomposition. The PhD thesis of Xavier Blanchot is part of this contract.

We had a contract with Thales Avionique to study a robust scheduling problem.

9.2 Bilateral grants with industry

Our joint project with Atoptima start-up "Solution methods for the inventory routing problem: application to waste collection in the urban environment" has been supported in 2020 by Nouvelle Aquitaine region (appel à projet "Recherche et Enseignement Supérieur"). The project is financing one half of a PhD thesis.

10 Partnerships and cooperations

10.1 International initiatives

10.1.1 Participation in other international programs

We have obtained an ANR PRCI grant in relation with Sobolev Institute in Novosibirsk (Russia).

10.1.2 Visits of international scientists

Two visits have been cancelled due to the pandemic (a six months visit by a doctoral student from Brazil and a seven months sabbatical visit by a professor from Canada).

10.2 Regional initiatives

We have obtained a grant from Région Nouvelle Aquitaine to work on inventory-routing problems.

11 Dissemination

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

We were part of the organization team for Dataquitaine 2020, which gathered 500 participants from Nouvelle Aquitaine.

Member of the conference program committees

- Pierre Pesneau : member of the program committee (and reviewer) of ISCO 2020 (International Symposium on Combinatorial Optimization), Monreal, Canada (held Online).
- François Clautiaux is member of the program committee of ROADEF, the French OR conference.

11.1.2 Journal

Member of the editorial boards François Clautiaux is a member of the editorial board of OJMO (Open Journal on Mathematical Optimization).

Ruslan Sadykov is an associate editor of EJCO (EURO Journal of Computational Optimization).

Reviewer - reviewing activities

- Aurélien Froger: European Journal of Operational Research, INFORMS Journal on Computing, Transportation Research Part B: Methodological, Transportation Science
- Pierre Pesneau : European Journal of Operational Research, European Journal on Computational Optimization, Discrete Optimization
- Ruslan Sadykov : SN Operations Research Forum, INFORMS Journal on Optimization, Transportation Science, Open Journal of Mathematical Optimization, INFORMS Journal on Computing, Omega, European Journal on Operations Research, Networks, RAIRO - Operations Research
- François Clautiaux : Computers and Operations Research, European Journal of Operational Research, INFORMS Journal on Computing, Mathematical Programming C

11.1.3 Invited talks

Boris Detienne: Invited talk at the 21st ROADEF conference, in Montpellier (19-21/02/2020)

11.1.4 Leadership within the scientific community

François Clautiaux is president of the French Operations Research Society ROADEF (more than 500 members).

11.1.5 Scientific expertise

Boris Detienne has been expert for the European Science Foundation.

11.2 Teaching - Supervision - Juries

11.2.1 Teaching

Boris Detienne is **head of the Master Program in Operations Research** of the University of Bordeaux.

Pierre Pesneau is **head of the Master of Engineering in Mathematical Optimization** (CMI OPTIM) of the University of Bordeaux.

François Clautiaux is **head of the Master in Applied Mathematics** (180 students) of the University of Bordeaux.

- Licence : François Clautiaux, Projet d'optimisation, L3, Université de Bordeaux, France
- Licence : François Clautiaux, Grands domaines de l'optimisation, L1, Université de Bordeaux, France
- Master : François Clautiaux, Introduction à la programmation en variables entières, M1, Université de Bordeaux, France
- Master : François Clautiaux, Integer Programming, M2, Université de Bordeaux, France
- Master : François Clautiaux, Algorithmes pour l'optimisation en nombres entiers, M1, Université de Bordeaux, France
- Master : François Clautiaux, Programmation linéaire, M1, Université de Bordeaux, France
- Master: Boris Detienne, Combinatoire et routage, ENSEIRB INPB
- Licence : Boris Detienne, Optimisation, L2, Université de Bordeaux
- Licence : Boris Detienne, Groupe de travail applicatif, L3, Université de Bordeaux
- Master : Boris Detienne, Optimisation continue, M1, Université de Bordeaux
- Master : Boris Detienne, Integer Programming, M2, Université de Bordeaux
- Master : Boris Detienne, Optimisation dans l'incertain, M2, Université de Bordeaux
- Licence : Aurélien Froger, Groupe de travail applicatif, L3, Université de Bordeaux, France
- Master : Aurélien Froger, Optimisation dans les graphes, M1, Université de Bordeaux, France
- Master : Aurélien Froger, Gestion des opérations et planification de la production, M2, Université de Bordeaux, France
- Master : Ruslan Sadykov, Introduction to Constraint Programming, M2, Université de Bordeaux, France
- Licence : Pierre Pesneau, Grands domaines de l'optimisation, L1, Université de Bordeaux, France
- Licence : Pierre Pesneau, Programmation pour le calcul scientifique, L2, Université de Bordeaux, France
- Licence : Pierre Pesneau, Optimisation, L2, Université de Bordeaux, France

- Master : Pierre Pesneau, Introduction à la programmation en variables entières, M1, Université de Bordeaux, France
- Master : Pierre Pesneau, Programmation linéaire, M1, Université de Bordeaux, France
- Master : Pierre Pesneau, Projet Algorithmes de flot, M1, Université de Bordeaux, France
- Master : Pierre Pesneau, Integer Programming, M2, Université de Bordeaux, France

11.2.2 Supervision

- PhD: Guillaume Marques, Planification de tournées de véhicules avec transbordement en logistique urbaine : approches basées sur les méthodes exactes de l'optimisation mathématique, 2017-2020 Ruslan Sadykov (dir).
- PhD: Mohamed Benkirane, "Optimisation des moyens dans la recombinaison commerciale de dessertes TER" 2016-2020, François Clautiaux (dir), Boris Detienne (dir).
- PhD in progress : Gaël Guillot, Aggregation and disaggregation methods for hard combinatorial problems, from November 2017, François Clautiaux (dir) and Boris Detienne (dir).
- PhD in progress : Orlando Rivera Letelier, Bin Packing Problem with Generalized Time Lags, from May 2018, François Clautiaux (dir) and Ruslan Sadykov (co-dir), a co-tutelle with Universidad Adolfo Ibáñez, Peñalolén, Santiago, Chile.
- PhD in progress: Xavier Blanchot, "Accélération de la Décomposition de Benders à l'aide du Machine Learning : Application à de grands problèmes d'optimisation stochastique two-stage pour les réseaux d'électricité" from September 2019, François Clautiaux (dir), Aurélien Froger (co-dir).
- PhD in progress: Johan Levêque, "Conception de réseaux de distributions urbains mutualisées en mode doux", from September 2018, François Clautiaux (dir), Gautier Stauffer (co-dir).
- PhD in progress: Mellila Kechir, "Optimization of supply-chain optimization using IoT concepts", from september 2020, François Clautiaux (dir), Walid Klibli (co-dir).
- PhD in progress: Isaac Balster, "Solution methods for the inventory routing problem: application to waste collection in the urban environment", from November 2020, Ruslan Sadykov (dir).
- PhD in progress Daniel Khachay, "Exact algorithms for vehicle routing problems", from September 2020, Ruslan Sadykov (dir).

11.2.3 Juries

- François Clautiaux: Walid Klibli (Bordeaux, HDR, jury member), Marko Mladenovic (Valenciennes, PhD, reviewer), Matthieu Guillot (Grenoble, PhD, reviewer), Lucie Pansart (Grenoble, PhD, reviewer), Guillaume Marques (Bordeaux, PhD, jury member).
- Aurélien Froger: Laura Catalina Echeverri Guzman (Tours, PhD, jury member).
- Ruslan Sadykov: jury member for Young Researcher (CRN and ISFP) positions at Inria Bordeaux Sud-Ouest, Guillaume Marques (Bordeaux, jury member).

11.3 Popularization

11.3.1 Articles and contents

François Clautiaux was part of the content management team for the special issue "Operations Research" of *Tangente* (popularization of mathematics).

François Clautiaux and Pierre Pesneau : popularization paper in *Tangente* (topic: Integer Linear Programming).

12 Scientific production

12.1 Major publications

- [1] A. A. Pessoa, R. Sadykov, E. Uchoa and F. Vanderbeck. 'A Generic Exact Solver for Vehicle Routing and Related Problems'. In: *Mathematical Programming* 183 (2020), pp. 483–523. DOI: [10.1007/s10107-020-01523-z](https://doi.org/10.1007/s10107-020-01523-z). URL: <https://hal.inria.fr/hal-02178171>.
- [2] R. Sadykov, A. A. Pessoa and E. Uchoa. 'A Bucket Graph Based Labelling Algorithm for Vehicle Routing'. In: *Transportation Science*. Ahead of Print (Oct. 2020). DOI: [10.1287/trsc.2020.0985](https://doi.org/10.1287/trsc.2020.0985). URL: <https://hal.inria.fr/hal-02378624>.

12.2 Publications of the year

International journals

- [3] A. N. Arslan and B. Detienne. 'Decomposition-based approaches for a class of two-stage robust binary optimization problems'. In: *INFORMS Journal on Computing* (2021). URL: <https://hal.inria.fr/hal-02190059>.
- [4] T. Bulhoes, R. Sadykov, A. Subramanian and E. Uchoa. 'On the exact solution of a large class of parallel machine scheduling problems'. In: *Journal of Scheduling* 23 (2020), pp. 411–429. DOI: [10.1007/s10951-020-00640-z](https://doi.org/10.1007/s10951-020-00640-z). URL: <https://hal.inria.fr/hal-01958180>.
- [5] F. Clautiaux, B. Detienne and G. Guillot. 'An iterative dynamic programming approach for the temporal knapsack problem'. In: *European Journal of Operational Research* (2021). URL: <https://hal.archives-ouvertes.fr/hal-02044832>.
- [6] G. Marques, R. Sadykov, J.-C. Deschamps and R. Dupas. 'An improved branch-cut-and-price algorithm for the two-echelon capacitated vehicle routing problem'. In: *Computers and Operations Research* 114 (2020), p. 104833. DOI: [10.1016/j.cor.2019.104833](https://doi.org/10.1016/j.cor.2019.104833). URL: <https://hal.archives-ouvertes.fr/hal-02112287>.
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- [9] A. A. Pessoa, R. Sadykov, E. Uchoa and F. Vanderbeck. 'A Generic Exact Solver for Vehicle Routing and Related Problems'. In: *Mathematical Programming* 183 (2020), pp. 483–523. DOI: [10.1007/s10107-020-01523-z](https://doi.org/10.1007/s10107-020-01523-z). URL: <https://hal.inria.fr/hal-02178171>.
- [10] E. Queiroga, Y. Frota, R. Sadykov, A. Subramanian, E. Uchoa and T. Vidal. 'On the exact solution of vehicle routing problems with backhauls'. In: *European Journal of Operational Research* 287.1 (2020), pp. 76–89. DOI: [10.1016/j.ejor.2020.04.047](https://doi.org/10.1016/j.ejor.2020.04.047). URL: <https://hal.inria.fr/hal-02379008>.
- [11] R. Sadykov, A. A. Pessoa and E. Uchoa. 'A Bucket Graph Based Labelling Algorithm for Vehicle Routing'. In: *Transportation Science*. Ahead of Print (5th Oct. 2020). DOI: [10.1287/trsc.2020.0985](https://doi.org/10.1287/trsc.2020.0985). URL: <https://hal.inria.fr/hal-02378624>.

Doctoral dissertations and habilitation theses

- [12] G. Marques. 'Two-echelon vehicle routing problems in city logistics : approaches based on exact methods of mathematical optimization'. Université de Bordeaux, 26th Nov. 2020. URL: <https://tel.archives-ouvertes.fr/tel-03052822>.

Reports & preprints

- [13] I. Ben Mohamed, W. Klibi, R. Sadykov, H. Şen and F. Vanderbeck. *The Two-Echelon Stochastic Multi-period Capacitated Location-Routing Problem*. 3rd Nov. 2020. URL: <https://hal.inria.fr/hal-02987266>.
- [14] N. D. Kullman, A. Froger, J. E. Mendoza and J. C. Goodson. *frucpy: An Open-Source Solver for the Fixed Route Vehicle Charging Problem*. 18th Oct. 2020. URL: <https://hal.archives-ouvertes.fr/hal-02496381>.
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