IN PARTNERSHIP WITH:
Université des sciences et technologies de Lille (Lille 1)

Activity Report 2018

Project-Team RAPSODI

Reliable numerical approximations of dissipative systems.

IN COLLABORATION WITH: Laboratoire Paul Painlevé (LPP)

RESEARCH CENTER
Lille - Nord Europe

THEME
Numerical schemes and simulations

Table of contents

Project-Team RAPSODI

Creation of the Team: 2015 August 01, updated into Project-Team: 2017 November 01

Keywords:

Computer Science and Digital Science:

A6. - Modeling, simulation and control
A6.1. - Methods in mathematical modeling
A6.1.1. - Continuous Modeling (PDE, ODE)
A6.1.4. - Multiscale modeling
A6.1.5. - Multiphysics modeling
A6.2. - Scientific computing, Numerical Analysis & Optimization
A6.2.1. - Numerical analysis of PDE and ODE

Other Research Topics and Application Domains:

B3. - Environment and planet
B3.3. - Geosciences
B3.3.1. - Earth and subsoil
B3.4. - Risks
B3.4.2. - Industrial risks and waste
B4. - Energy
B4.2. - Nuclear Energy Production
B4.2.1. - Fission

1. Team, Visitors, External Collaborators

Research Scientists

Clément Cancès [Inria, Researcher, HDR]
Maxime Herda [Inria, Researcher, from Oct. 2018]
Simon Lemaire [Inria, Researcher]

Faculty Members

Caterina Calgaro Zotto [Université de Lille, Associate Professor]
Claire Chainais-Hillairet [Team leader, Université de Lille, Professor, HDR]
Emmanuel Creusé [Université de Lille, Professor, until Aug. 2018, HDR]
Ingrid Lacroix-Violet [Université de Lille, Associate Professor, HDR]
Benoît Merlet [Université de Lille, Professor, HDR]
Thomas Rey [Université de Lille, Associate Professor]

Post-Doctoral Fellows

Ahmed Ait Hammou Oulhaj [Université de Lille, from Mar. 2018 until Aug. 2018]
Florent Chave [Inria, from Dec. 2018]
David Maltese [Inria, until Sep. 2018]

PhD Students

Ahmed Ait Hammou Oulhaj [Inria, until Feb. 2018]
Benoît Gaudeul [École Normale Supérieure de Rennes, from Sep. 2018]
Claire Colin-Lecerf [Université de Lille]
Nicolas Peton [IFPEn, until Oct. 2018]

Antoine Zurek [Université de Lille]

Interns

Tarlan Ebrahimipourfaez [Inria, from Apr. 2018 until Jul. 2018]
Benoît Gaudeul [École Normale Supérieure de Rennes, from Apr. 2018 until Jul. 2018]
Cédric Marinel [Inria, from Jun. 2018 until Aug. 2018]

Administrative Assistant

Corinne Jamroz [Inria]

Visiting Scientists

Jürgen Fuhrmann [WIAS Berlin, from May 2018 until Jun. 2018]
Naoufel Staili [Faculté des Sciences de Meknès, until Apr. 2018]
Juliette Venel [Université Polytechnique Hauts-de-France, until Jul. 2018]

External Collaborator

Emmanuel Creusé [Université Polytechnique Hauts-de-France, Professor, from Sep. 2018, HDR]

2. Overall Objectives

2.1. Overall Objectives

Together with the diffusion of scientific computing, there has been a recent and impressive increase of the demand for numerical methods. The problems to be addressed are everyday more complex and require specific numerical algorithms. The quality of the results has to be accurately assessed, so that in-silico experiments results can be trusted. Nowadays, producing such reliable numerical results goes way beyond the abilities of isolated researchers, and must be carried out by structured teams.

The topics addressed by the RAPSODI project-team belong to the broad theme of numerical methods for the approximation of solutions of systems of partial differential equations (PDEs). Besides standard convergence properties, a good numerical method for approximating a physical problem has to satisfy at least the following three criteria:

1. preservation at the discrete level of some crucial features of the solution, such as positivity of solutions, conservation of prescribed quantities (e.g., mass, the decay of physically motivated entropies, etc.);
2. provide accurate numerical approximations at a reasonable computational cost (and ultimately maximize the accuracy at a fixed computational effort);
3. robustness with respect to physical conditions: the computational cost for a given accuracy should be essentially insensitive to change of physical parameters.

We aim to develop methods fulfilling the above quality criteria for physical models which all display a dissipative behavior, and that are motivated by industrial collaborations or multidisciplinary projects. In particular, we have identified a couple of specific situations we plan to investigate: models from corrosion science (in the framework of nuclear waste repository) [55], low-frequency electromagnetism [74], and mechanics of complex inhomogeneous fluids arising in avalanches [67] or in porous media [57].

Ideally, we should allow ourselves to design entirely new numerical methods. For some applications however (often in the context of industrial collaborations), the members of the team have to compose with existing codes. The numerical algorithms have thus to be optimized under this constraint.

2.2. Scientific Context

Some technological bottlenecks related to points (a)–(c) mentioned above are well identified. In particular, it appears that a good numerical method should handle general meshes, so that dynamic mesh adaptation strategies can be used in order to achieve (b). But it should also be of the highest possible order while remaining stable in the sense of (a), and robust in the sense of (c). There have been numerous research contributions on each point of (a)–(c) in the last decades, in particular for solving each difficulty apart, but combining them still leads to unsolved problems of crucial interest.

Let us mention for example the review paper by J. Droniou [83], where it is highlighted that all the linear methods for solving diffusion equations on general meshes suffer from the same lack of monotonicity and preserve neither the positivity of the solutions nor the decay of the entropy. Moreover, there is no complete convergence proof for the nonlinear methods exposed in [83]. The first convergence proof for a positivity preserving and entropy diminishing method designed to approximate transient dissipative equation on general meshes was proposed very recently in [69]. The idea and the techniques introduced in [69] should be extended to practical applications.

In systems of PDEs, the values of physical parameters often change the qualitative behavior of the solution. Then, one challenge in the numerical approximation of such systems is the design of methods which can be applied for a large range of parameters, as in particular in the regime of singular perturbations. Such schemes, called *asymptotic-preserving* (AP) schemes [96], are powerful tools as they permit the use of the same scheme for a given problem and for its limit with fixed discretization parameters. In many cases, the AP property of numerical schemes is just empirically established, without any rigorous proof. We aim to extend the techniques recently introduced in [64] for the drift-diffusion system, and based on the control of the numerical dissipation of entropy, to other dissipative systems in order prove the AP property of numerical schemes.

The question of the robustness of the numerical methods with respect to the physical parameters is also fundamental for fluid mixtures models. The team already developed such schemes for the variable density Navier–Stokes system [66] or [67]. We aim to propose new ones for more complex models with the same philosophy in mind. On the one hand, we will be interested in high-order schemes, which must be as simple as possible in view of 3D practical implementations. Let us stress that combining high order accuracy and stability is very challenging. On the other hand, the optimization of the computation will have to be considered, in particular with the development of some *a posteriori* error estimators. Impressive progresses have been achieved in this field [79], allowing important computational savings without compromising the accuracy of the results. Recently, we successfully applied this strategy to the Reissner-Mindlin model arising in solid mechanics [76], the dead-oil model for porous media flows [70] or the Maxwell equations in their quasi-static approximation for some eddy current problems [74] and [75]. We aim to develop new *a posteriori* estimators for other dissipative systems, like fluid mixtures models.

In a nutshell, our goal is to take advantage of and extend the most recent breakthroughs of the mathematical community to tackle in an efficient way some application-guided problems coming either from academics or from industrial partners. To this end, we shall focus on the following objectives, which are necessary for the applications we have in mind:

1. *Design and numerical analysis of structure-preserving numerical methods.*
2. *Computational optimization.*

3. Research Program

3.1. Design and analysis of structure-preserving schemes

3.1.1. Numerical analysis of nonlinear numerical methods

Up to now, the numerical methods dedicated to degenerate parabolic problems that the mathematicians are able to analyze almost all rely on the use of mathematical transformations (like e.g. the Kirchhoff's transform). It forbids the extension of the analysis to complex realistic models. The methods used in the industrial codes for solving such complex problems rely on the use of what we call NNM, i.e., on methods that preserve all the nonlinearities of the problem without reducing them thanks to artificial mathematical transforms. Our aim is to take advantage of the recent breakthrough proposed by C. Cancès & C. Guichard [69], [4] to develop efficient new numerical methods with a full numerical analysis (stability, convergence, error estimates, robustness w.r.t. physical parameters, ...).

3.1.2. Design and analysis of asymptotic-preserving schemes

There has been an extensive effort in the recent years to develop numerical methods for diffusion equations that are robust with respect to heterogeneities, anisotropy, and the mesh (see for instance [83] for an extensive discussion on such methods). On the other hand, the understanding of the role of nonlinear stability properties in the asymptotic behaviors of dissipative systems increased significantly in the last decades (see for instance [71], [101]).

Recently, C. Chainais-Hillairet and co-authors [64], [72] and [73] developed a strategy based on the control of the numerical counterpart of the physical entropy to develop and analyze AP numerical methods. In particular, these methods show great promises for capturing accurately the behavior of the solutions to dissipative problems when some physical parameter is small with respect to the discretization characteristic parameters, or in the long-time asymptotic. Since it requires the use of nonlinear test functions in the analysis, strong restrictions on the physics (isotropic problems) and on the mesh (Cartesian grids, Voronoï boxes...) are required in [64], [72] and [73]. The schemes proposed in [69] and [4] allow to handle nonlinear test functions in the analysis without restrictions on the mesh and on the anisotropy of the problem. Combining the nonlinear schemes *à la* [69] with the methodology of [64], [72], [73] would provide schemes that are robust both with respect to the meshes and to the parameters. Therefore, they would be also robust under adaptive mesh refinement.

3.1.3. Design and stability analysis of numerical methods for low-Mach models

We aim at extending the range of the `NS2DDV-M` software by introducing new physical models, like for instance the low-Mach model, which gives intermediate solutions between the compressible Navier–Stokes model and the incompressible Navier–Stokes one. This model was introduced in [99] as a limiting system which describes combustion processes at low Mach number in a confined region. Within this scope, we will propose a theoretical study for proving the existence of weak solutions for a particular class of models for which the dynamic viscosity of the fluid is a specific function of the density. We will propose also the extension of a combined Finite Volume-Finite Element method, initially developed for the simulation of incompressible and variable density flows, to this class of models.

3.2. Optimizing the computational efficiency

3.2.1. High-order nonlinear numerical methods

The numerical experiments carried out in [69] show that in case of very strong anisotropy, the convergence of the proposed NNM becomes too slow (less than first order). Indeed, the method appears to strongly overestimate the dissipation. In order to make the method more competitive, it is necessary to estimate the dissipation in a more accurate way. Preliminary numerical results show that second order accuracy in space can be achieved in this way. One also aims to obtain (at least) second order accuracy in time without jeopardizing the stability. For many problems, this can be done by using so-called two-step backward differentiation formulas (BDF2) [87].

Concerning the inhomogeneous fluid models, we aim to investigate new methods for the mass equation resolution. Indeed, we aim at increasing the accuracy while maintaining some positivity-like properties and the efficiency for a wide range of physical parameters. To this end, we will consider Residual Distribution schemes, that appear as an alternative to Finite Volume methods. Residual Distribution schemes enjoy very compact stencils. Therefore, their extension from 2D to 3D yield reasonable difficulties. These methods appeared twenty years ago, but recent extensions to unsteady problems [102], [95], with high-order accuracy [49], [48], or for parabolic problems [46], [47] make them very competitive. Relying on these breakthroughs, we aim at designing new Residual Distribution schemes for fluid mixture models with high-order accuracy while preserving the positivity of the solutions.

3.2.2. A posteriori error control

The question of the *a posteriori* error estimators will also have to be addressed in this optimization context. Since the pioneering papers of Babuska and Rheinboldt more than thirty years ago [54], *a posteriori* error estimators have been widely studied. We will take advantage of the huge corresponding bibliography database in order to optimize our numerical results.

For example, we would like to generalize the results we derived for the harmonic magnetodynamic case (e.g. [74] and [75]) to the temporal magnetodynamic one, for which space/time *a posteriori* error estimators have to be developed. A space/time refinement algorithm should consequently be proposed and tested on academic as well as industrial benchmarks.

We also want to develop *a posteriori* estimators for the variable density Navier–Stokes model or some of its variants. To do so, several difficulties have to be tackled: the problem is nonlinear, unsteady, and the numerical method [66], [67] we developed combines features from Finite Elements and Finite Volumes. Fortunately, we do not start from scratch. Some recent references are devoted to the unsteady Navier–Stokes model in the Finite Element context [61], [105]. In the Finite Volume context, recent references deal with unsteady convection-diffusion equations [104], [52], [81] and [70]. We want to adapt some of these results to the variable density Navier–Stokes system, and to be able to design an efficient space-time remeshing algorithm.

3.2.3. Efficient computation of pairwise interactions in large systems of particles

Many systems are modeled as a large number of punctual individuals (N) which interact pairwise which means $N(N-1)/2$ interactions. Such systems are ubiquitous, they are found in chemistry (Van der Waals interaction between atoms), in astrophysics (gravitational interactions between stars, galaxies or galaxy clusters), in biology (flocking behavior of birds, swarming of fishes) or in the description of crowd motions. Building on the special structure of convolution-type of the interactions, the team develops computation methods based on the Non Uniform Fast Fourier Transform [90]. This reduces the $O(N^2)$ naive computational cost of the interactions to $O(N \log N)$, allowing numerical simulations involving millions of individuals.

4. Application Domains

4.1. Porous media flows

Porous media flows are of great interest in many contexts, like, e.g., oil engineering, water resource management, nuclear waste repository management, or carbon dioxyde sequestration. We refer to [57], [56] for an extensive discussion on porous media flow models.

From a mathematical point of view, the transport of complex fluids in porous media often leads to possibly degenerate parabolic conservation laws. The porous rocks can be highly heterogeneous and anisotropic. Moreover, the grids on which one intends to solve numerically the problems are prescribed by the geological data, and might be non-conformal with cells of various shapes. Therefore, the schemes used for simulating such complex flows must be particularly robust.

4.2. Corrosion and concrete carbonation

The team is interested in the theoretical and numerical analysis of mathematical models describing degradation of materials as concrete carbonation and corrosion. The study of such models is an important environmental and industrial issue. Atmospheric carbonation degrades reinforced concretes and limits the lifetime of civil engineering structures. Corrosion phenomena issues occur for instance in the reliability of nuclear power plants and the nuclear waste repository. The study of the long time evolution of these phenomena is of course fundamental in order to predict the lifetime of the structures.

From a mathematical point of view, the modeling of concrete carbonation (see [51]) as the modeling of corrosion in an underground repository (DPCM model developed by Bataillon *et al.* [55]) lead to systems of PDEs posed on moving domains. The coupling between convection-diffusion-reaction equations and moving boundary equations leads to challenging mathematical questions.

4.3. Complex fluid flows

The team is interested in some numerical methods for the simulation of systems of PDEs describing complex flows, like for instance, mixture flows, granular gases, rarefied gases, or quantum fluids.

Variable-density, low-Mach flows have been widely studied in the recent literature because of their applicability in various phenomena such as flows in high-temperature gas reactors, meteorological flows, flows with convective and/or conductive heat transfer or combustion processes. In such cases, the resolution of the full compressible Navier–Stokes system is not adapted, because of the sound waves speed. The Boussinesq incompressible model is not a better alternative for such low-speed phenomena, because the compressibility effects can not be totally cancelled due to large variations of temperature and density. Consequently, some models have been formally derived, leading to the filtering of the acoustic waves by the use of some formal asymptotic expansions and two families of methods have been developed in the literature in order to compute these flows. We are interested in particular in the so-called pressure-based methods which are more robust than density-based solvers, although their range of validity is in general more limited.

Kinetic theory of molecular gases models a gas as a system of elastically colliding spheres, conserving mechanical energy during impact. Once initialized, it takes a molecular gas not more than few collisions per particle to relax to its equilibrium state, characterized by a Maxwellian velocity distribution and a certain homogeneous density (in the absence of external forces). A granular gas is a system of dissipatively colliding, macroscopic particles (grains). This slight change in the microscopic dynamics (converting energy into heat) cause drastic changes in the behavior of the gas: granular gases are open systems, which exhibits self-organized spatio-temporal cluster formations, and has no equilibrium distribution. They can be used to model silos, avalanches, pollen or planetary rings.

The quantum models can be used to describe superfluids, quantum semiconductors, weakly interacting Bose gases or quantum trajectories of Bohmian mechanics. They have attracted considerable attention in the last decades, due in particular to the development of the nanotechnology applications. To describe quantum phenomena, there exists a large variety of models. In particular there exist three different levels of description: microscopic, mesoscopic and macroscopic. The quantum Navier–Stokes equations deal with a macroscopic description in which the quantum effects are taken into account through a third order term called the quantum Bohm potential. This Bohm potential arises from the fluid dynamical formulation of the single-state Schrödinger equation. The non-locality of quantum mechanics is approximated by the fact that the equations of state do not only depend on the particle density but also on its gradient. These equations were employed to model field emissions from metals and steady-state tunneling in metal- insulator- metal structures and to simulate ultra-small semiconductor devices.

4.4. Stratigraphy

The knowledge of the geology is a prerequisite before simulating flows within the subsoil. Numerical simulations of the geological history thanks to stratigraphy numerical codes allow to complete the knowledge of the geology where experimental data are lacking. Stratigraphic models consist in a description of the erosion and sedimentation phenomena at geological scales.

The characteristic time scales for the sediments are much larger than the characteristic time scales for the water in the river. However, the (time-averaged) water flux plays a crucial role in the evolution of the stratigraphy. Therefore, defining appropriate models that take the coupling between the rivers and the sediments into account is fundamental and challenging. Once the models are at hand, efficient numerical methods must be developed.

4.5. Low-frequency electromagnetism

Numerical simulation is nowadays an essential tool in order to design electromagnetic systems, by estimating the electromagnetic fields generated in a wide variety of devices. An important challenge for many applications is to quantify the intensity of the electric field induced in a conductor by a current generated in its neighborhood. In the low-frequency regime, we can for example quote the study of the impact on the human body of a high-tension line or, for higher frequencies, the one of a smartphone. But the ability to simulate accurately some electromagnetic fields is also very useful for non-destructive control, in the context of the maintenance of nuclear power stations for example. The development of efficient numerical tools, among which *a posteriori* error estimators, is consequently necessary to reach a high precision of calculation in order to provide estimations as reliable as possible.

5. Highlights of the Year

5.1. Highlights of the Year

In 2018, the RAPSODI project-team was strongly involved in the organization of scientific events. In particular, in the framework of the LabEx CEMPI thematic semester on Numerical Analysis and PDEs, the following events were organized by RAPSODI members:

- the Mathematics-Enterprises Study Week, co-organized at LILLIAD Learning Center by E. Creusé from January 29 to February 2;
- the third edition of the ABPDE conference (on Asymptotic Behavior of systems of PDEs arising in physics and biology), co-organized at LILLIAD Learning Center by C. Cancès, C. Chainais-Hillairet, I. Lacroix-Violet, and T. Rey on August 28-31;
- the second edition of the One-day conference on Calculus of Variations, co-organized at Laboratoire Paul Painlevé by I. Lacroix-Violet and B. Merlet on October 12;
- the fifth edition of the Lille days on Numerical Analysis (dedicated to domain decomposition and its applications to PDEs), co-organized at Laboratoire Paul Painlevé by C. Calgaro Zotto and E. Creusé on November 13-14.

A research school on Mathematics for Nuclear Energy was also co-organized at the Roscoff Marine Station by C. Cancès on July 2-6, in partnership with the GdR MaNu. Let us as well mention the organization in the CANUM (national NUMerical Analysis Congress) at Cap d'Agde from May 28 to June 1 of three mini-symposia by members of the team: one by C. Cancès on cross-diffusion systems, one by S. Lemaire on polytopal discretization methods, and one co-organized by T. Rey on kinetic models. Team contributions finally include the co-organization by E. Creusé of the Maths Jobs Forum that was held in Paris on December 13, and the co-organization by A. Zurek of the Young Mathematicians Regional Tournament that was held in Laboratoire Paul Painlevé on April 14-15.

6. New Software and Platforms

6.1. Platform NS2DDV-M

`NS2DDV-M` is a Matlab code, developed by C. Calgaro Zotto, E. Creusé, and A. Mouton (CNRS research engineer at Université de Lille), for the simulation of homogeneous and inhomogeneous fluid flows by a combined Finite Volume-Finite Element method. The code is freely distributed, to allow for easy comparisons with concurrent codes on benchmark test-cases, and to promote new collaborations in the domain.
In 2018, a new version (v. 2.0) has been released, which contains a detailed documentation as well as some new functionalities, such as some post-processing tools and parallel computation capabilities.

7. New Results

7.1. Numerical simulation of concrete carbonation

In [20], C. Chainais-Hillairet, B. Merlet, and A. Zurek introduce and study a Finite Volume scheme for a concrete carbonation model proposed by Aiki and Muntean in [50]. This model consists in a system of two weakly coupled parabolic equations in a varying domain whose length is governed by an ordinary differential equation. The numerical scheme is obtained by a Euler discretization in time and a Scharfetter–Gummel discretization in space. The convergence of the scheme is established and the existence of a solution to the model is obtained as a by-product. Finally, some numerical experiments are performed to show the efficiency of the scheme.

In [45], A. Zurek studies the long-time regime of the moving interface appearing in the concrete carbonation model. He proves that the approximate free boundary, given by an implicit-in-time Finite Volume scheme, increases in time following a $\sqrt{t}$-law. This result is illustrated by numerical experiments.

7.2. Modeling and numerical simulation of complex fluids

In the context of C. Colin-Lecerf's PhD, C. Calgaro Zotto, C. Colin-Lecerf, and E. Creusé derive in [35] a combined Finite Volume-Finite Element scheme for a low-Mach model, in which a temperature field obeying an energy law is taken into account. The continuity equation is solved, whereas the state equation linking temperature, density, and thermodynamic pressure is imposed implicitly. Since the velocity field is not divergence-free, the projection method solving the momentum equation has to be adapted. This combined scheme preserves some steady states, and ensures a discrete maximum principle on the density. Numerical results are provided and compared to other approaches using purely Finite Element schemes, on a benchmark consisting in particular in a transient injection flow [58], [89], [53], as well as in the natural convection of a flow in a cavity [97], [93], [89], [53].

The theoretical study of the low-Mach limit system is a vast subject that has been considered by many authors. In particular, in [86], Embid establishes the local-in-time existence of classical solutions in Sobolev spaces. In [77], Danchin and Liao study the well-posedness issue in the critical Besov spaces, locally and globally, assuming that the initial density is close to a constant and that the initial velocity is small enough. Levermore *et al.* [98] consider the so-called ghost effect system, which is quite similar to the low-Mach system with thermal stress term added to the right-hand-side of the momentum equation, and they prove the local well-posedness of classical solutions for the Cauchy problem. In [94], Huang and Tan prove a local well-posedness result for strong solutions and also the existence and uniqueness of a global strong solution for the two-dimensional case. In [14], C. Calgaro Zotto, C. Colin-Lecerf, E. Creusé *et al.* investigate a specific low-Mach model for which the dynamic viscosity of the fluid is a specific function of the density. The model is reformulated in terms of the temperature and velocity, with nonlinear temperature equation, and strong solutions are considered. In addition to a local-in-time existence result for strong solutions, some convergence rates of the error between the approximation and the exact solution are obtained, following the same approach as Guillén-González *et al.* [91], [92].

Diffuse interface models, such as the Kazhikhov–Smagulov model, allow to describe some phase transition phenomena. In [15], C. Calgaro Zotto and co-workers investigate theoretically the combined Finite Volume-Finite Element scheme. They construct a fully discrete numerical scheme for approximating the two-dimensional Kazhikhov–Smagulov model, using a first-order time discretization and a splitting in time to allow the construction of the combined scheme. Consequently, at each time step, one only needs to solve two decoupled problems, the first one for the density (using the Finite Volume method) and the second one for the velocity and pressure (using the Finite Element method). The authors prove the stability of the combined scheme and the convergence towards the global-in-time weak solution of the model.

In [27], I. Lacroix-Violet *et al.* present the construction of global weak solutions to the quantum Navier–Stokes equation, for any initial value with bounded energy and entropy. The construction is uniform with respect to the Planck constant. This allows to perform the semi-classical limit to the associated compressible Navier–Stokes equation. One of the difficulties of the problem is to deal with the degenerate viscosity, together with the lack of integrability on the velocity. The method is based on the construction of weak solutions that are renormalized in the velocity variable. The existence and stability of these solutions do not need the Mellet–Vasseur inequality.

In [34], I. Lacroix-Violet *et al.* generalize to the Navier–Stokes–Korteweg (with density-dependent viscosities satisfying the BD relation) and Euler–Korteweg systems a recent relative entropy proposed in [65]. As a concrete application, this helps justifying mathematically the convergence between global weak solutions of the quantum Navier–Stokes system and dissipative solutions of the quantum Euler system when the viscosity coefficient tends to zero. The results are based on the fact that Euler–Korteweg systems and corresponding Navier–Stokes–Korteweg systems can be reformulated through an augmented system. As a by-product of the analysis, Lacroix-Violet *et al.* show that this augmented formulation helps to define relative entropy estimates for the Euler–Korteweg systems in a simpler way and with less hypotheses compared to recent works [82], [88].

7.3. Stratigraphic modeling and simulation

Stratigraphy is a discipline of physics that aims at predicting the geological composition of the subsoil. In [44], N. Peton, C. Cancès *et al.* propose a new water flow driven forward stratigraphic model with the following particularities. First, the water surface flow is modelled at the continuous level, in opposition to what is currently done in this community. Second, the model incorporates a constraint on the erosion rate. A stable numerical scheme is proposed to simulate the model.

7.4. Numerical simulation in low-frequency electromagnetism

In [24], [28], E. Creusé and co-workers investigate the behavior of some Finite Element error estimators in the context of low-frequency elecromagnetism simulations, to underline the main differences in some practical situations. In addition, a more theoretical contribution is developed in [23], to prove the equivalence of some usual discrete gauge conditions. Once again, their numerical behaviors are compared on some characteristic benchmarks.

7.5. Asymptotic analysis

In [33], C. Cancès and co-workers derive the porous medium equation as the hydrodynamic limit of an interacting particle system which belongs to the family of exclusion processes with nearest neighbor exchanges. The main outcome of this work is to allow regions with vanishing density, where the dynamics turns out to degenerate. The convergence builds on a generalization of the entropy method and on suitable regularization of the dynamics.

In [29], A. Ait Hammou Oulhaj, C. Cancès, C. Chainais-Hillairet *et al.* study analytically and numerically the large time behavior of the solutions to a two-phase extension of the porous medium equation, which models the so-called seawater intrusion problem. They identify the self-similar solutions that correspond to steady states of a rescaled version of the problem. They finally provide numerical illustrations of the stationary states and exhibit numerical convergence rates.

In [13], C. Chainais-Hillairet *et al.* propose a new proof of existence of a solution to the scheme introduced in [63] which does not require any assumption on the time step. The result relies on the application of a topological degree argument which is based on the positivity and on uniform-in-time upper bounds of the approximate densities. They also establish uniform-in-time lower bounds satisfied by the approximate densities. These uniform-in-time upper and lower bounds ensure the exponential decay of the scheme towards the thermal equilibrium as shown in [63].

In [38], C. Chainais-Hillairet and M. Herda study the large-time behavior of solutions to Finite Volume discretizations of convection-diffusion equations or systems endowed with non-homogeneous Dirichlet and Neumann type boundary conditions. Their results concern various linear and nonlinear models such as Fokker–Planck equations, porous media equations, or drift-diffusion systems for semiconductors. For all of these models, some relative entropy principle is satisfied and implies exponential decay to the stationary state. They show that in the framework of Finite Volume schemes on orthogonal meshes, a large class of two-point monotone fluxes preserve this exponential decay of the discrete solution to the discrete steady state of the scheme.

In [32], M. Herda, T. Rey *et al.* are interested in the asymptotic analysis of a Finite Volume scheme for one-dimensional linear kinetic equations, with either Fokker–Planck or linearized BGK collision operator. Thanks to appropriate uniform estimates, they establish that the proposed scheme is asymptotic-preserving in the diffusive limit. Moreover, they adapt to the discrete framework the hypocoercivity method proposed by [80] to prove the exponential return to equilibrium of the approximate solution. They obtain decay estimates that are uniform in the diffusive limit. Finally, they present an efficient implementation of the proposed numerical schemes, and perform numerous numerical simulations assessing their accuracy and efficiency in capturing the correct asymptotic behaviors of the models.

In [26], M. Herda *et al.* consider various sets of Vlasov–Fokker–Planck equations modeling the dynamics of charged particles in a plasma under the effect of a strong magnetic field. For each of them, in a regime where the strength of the magnetic field is effectively stronger than that of collisions, they first formally derive asymptotically reduced models. In this regime, strong anisotropic phenomena occur; while equilibrium along magnetic field lines is asymptotically reached the asymptotic models capture a nontrivial dynamics in the perpendicular directions. They do check that in any case the obtained asymptotic model defines a well-posed dynamical system and when self-consistent electric fields are neglected they provide a rigorous mathematical justification of the formally derived systems. In this last step they provide a complete control on solutions by developing anisotropic hypocoercive estimates.

7.6. Structure-preserving numerical methods

The design and the analysis of numerical methods preserving at the discrete level the key features of the continuous models is one of the core tasks of the RAPSODI project-team. C. Cancès was invited to write a review paper [16] on energy stable numerical methods for complex porous media flows. The paper addresses three different approaches: monotonicity-based numerical methods like two-point flux approximation Finite Volumes, as well as two methods based on multi-point flow approximation that are either based on upwinding or on positive local dissipation tensors.

Concerning methods based on upwinding, A. Ait Hammou Oulhaj, C. Cancès, and C. Chainais-Hillairet extend in [12] the nonlinear Control Volume Finite Element scheme of [69] to the discretization of Richards equation modeling unsaturated flows in porous media. This strategy is also applied in [30] by A. Ait Hammou Oulhaj and D. Maltese for the simulation of seawater intrusion in the subsoil nearby coastal regions. The scheme proposed in [30] is still convergent if the porous medium is anisotropic, in opposition to the energy-diminishing scheme analyzed in [11] by A. Ait Hammou Oulhaj, which is designed to be accurate in the long-time regime studied in [29]. Besides, an implicit Euler-Finite Volume scheme for a degenerate cross-diffusion system describing the ion transport through biological membranes is analyzed in [17] by C. Cancès, C. Chainais-Hillairet *et al.* The strongly coupled equations for the ion concentrations include drift terms involving the electric potential, which is coupled to the concentrations through the Poisson equation. The cross-diffusion system possesses a formal gradient flow structure revealing nonstandard degeneracies, which lead to considerable mathematical difficulties. The Finite Volume scheme is based on two-point flux approximations with “double” upwind mobilities. It preserves the structure of the continuous model like non-negativity, upper bounds, and entropy dissipation.

Concerning methods based on positive local dissipation tensors, C. Cancès, C. Chainais-Hillairet *et al.* propose in [18] a nonlinear Discrete Duality Finite Volume scheme to approximate the solutions of drift diffusion equations. The scheme is built to preserve at the discrete level even on severely distorted meshes the energy/energy dissipation relation. In [37], C. Cancès and co-workers propose a Finite Element scheme for the numerical approximation of degenerate parabolic problems in the form of a nonlinear anisotropic Fokker–Planck equation. The scheme is energy-stable, only involves physically motivated quantities in its definition, and is able to handle general unstructured grids. Its convergence is rigorously proven thanks to compactness arguments, under very general assumptions. Although the scheme is based on Lagrange Finite Elements of degree 1, it is locally conservative after a local post-processing giving rise to an equilibrated flux. This also allows to derive a guaranteed *a posteriori* error estimate for the approximate solution. Numerical experiments are presented in order to give evidence of a very good behavior of the proposed scheme in various situations involving strong anisotropy and drift terms.

C. Cancès *et al.* derive in [36] a model of degenerate Cahn–Hilliard type for the phase segregation in incompressible multiphase flows. The model is obtained as the Wasserstein gradient flow of a Ginzburg–Landau energy with the constraint that the sum of the volume fractions must stay equal to 1. The resulting model differs from the classical degenerate Cahn–Hilliard model (see [106], [85]) and is closely related to a model proposed by E and collaborators [84], [100]. Besides the derivation of the model, the convergence of a minimizing movement scheme is proven in [36]. The Wasserstein gradient flow structure of the PDE system governing multiphase flows in porous media has recently been highlighted in [68]. The model can thus be approximated by means of the minimizing movement (or JKO) scheme, that C. Cancès *et al.* solve in [19] thanks to the ALG2-JKO scheme proposed in [60]. The numerical results are compared to a classical upstream mobility Finite Volume scheme, for which strong stability properties can be established.

In [42], S. Lemaire builds a bridge between the Hybrid High-Order [78] and Virtual Element [59] methods, which are the two main new-generation approaches to the arbitrary-order approximation of PDEs on meshes with general, polytopal cells. The Virtual Element method writes in functional terms and is naturally conforming; at the opposite, the Hybrid High-Order method writes in algebraic terms and is naturally nonconforming. It has been remarked a few years ago that the Hybrid High-Order method can be viewed as a nonconforming version of the Virtual Element method. In [42], S. Lemaire ends up unifying the Hybrid High-Order and Virtual Element approaches by showing that the Virtual Element method can be reformulated as a (newborn) conforming Hybrid High-Order method. This parallel has interesting consequences: it allows important simplifications in the *a priori* analysis of Virtual Element methods, and sheds new light on the differences between conforming and nonconforming Virtual Element methods, in particular in terms of mesh assumptions.

In [31], I. Lacroix-Violet *et al.* are interested in the numerical integration in time of nonlinear Schrödinger equations using different methods preserving the energy or a discrete analog of it. In particular, they give a rigorous proof of the order of the relaxation method (presented in [62] for cubic nonlinearities) and they propose a generalized version that allows to deal with general power law nonlinearities. Numerical simulations for different physical models show the efficiency of these methods.

7.7. Cost reduction for numerical methods

In [22], S. Lemaire *et al.* design and analyze (in the periodic setting) nonconforming multiscale methods for highly oscillatory elliptic problems, which are applicable on coarse grids that may feature general polytopal cells. Two types of methods are introduced: a Finite Element-type method, that generalizes classical nonconforming multiscale Finite Element methods to general meshes and to arbitrary-order polynomial cell boundary conditions, and a Virtual Element-type method, that allows, up to the computation of an adequate projection, to compute less oscillatory basis functions for equivalent precision. The Virtual Element-type method is based on the Hybrid High-Order framework [78]. As standard with such multiscale approaches, the general workflow of the method splits into an offline, massively parallelizable stage, where all fine-scale computations are performed, and the online, fully-coarse-scale stage.

In [25], T. Rey *et al.* extend the Fast Kinetic Scheme (FKS) originally constructed for solving the BGK equation, to the more challenging case of the Boltzmann equation. The scheme combines a robust and fast method for treating the transport part based on an innovative Lagrangian technique, supplemented with conservative fast spectral schemes to treat the collisional operator by means of an operator splitting approach.

This approach along with several implementation features related to the parallelization of the algorithm permits to construct an efficient simulation tool which is numerically tested against exact and reference solutions on classical problems arising in rarefied gas dynamics.

In [43], T. Rey *et al.* present high-order, fully explicit time integrators for nonlinear collisional kinetic equations, including the full Boltzmann equation. The methods, called projective integration, first take a few small steps with a simple, explicit method (forward Euler) to damp out the stiff components of the solution. Then, the time derivative is estimated and used in a Runge–Kutta method of arbitrary order. The procedure can be recursively repeated on a hierarchy of projective levels to construct telescopic projective integration methods. We illustrate the method with numerical results in one and two spatial dimensions.

7.8. Applied calculus of variations

In [41], B. Merlet *et al.* study a variational problem which models the behavior of topological singularities on the surface of a biological membrane in P_β-phase (see [103]). The problem combines features of the Ginzburg–Landau model in 2D and of the Mumford–Shah functional. As in the classical Ginzburg–Landau theory, a prescribed number of point vortices appear in the moderate energy regime; the model allows for discontinuities, and the energy penalizes their length. The novel phenomenon here is that the vortices have a fractional degree $1/m$ with m prescribed. Those vortices must be connected by line discontinuities to form clusters of total integer degrees. The vortices and line discontinuities are therefore coupled through a topological constraint. As in the Ginzburg–Landau model, the energy is parameterized by a small length scale $\varepsilon > 0$. B. Merlet *et al.* perform a complete Γ-convergence analysis of the model as $\varepsilon \downarrow 0$ in the moderate energy regime. Then, they study the structure of minimizers of the limit problem. In particular, the line discontinuities of a minimizer solve a variant of the Steiner problem.

In [21], B. Merlet *et al.* consider a generalization of branched transport in arbitrary dimension and codimension: minimize the h-mass of some oriented k-dimensional branched surface in $\mathbf{R}^n$ with some prescribed boundary. Attached to the surface is a multiplicity $m(x)$ which is not necessarily an integer and is a conserved quantity (Kirchhoff current law is satisfied at branched points). The h-mass is defined as the integral of a cost $h(|m(x)|)$ over the branched surface. As usual in branched transportation, the cost function is a lower-semicontious, sublinear increasing function with $h(0) = 0$ (for instance $h(m) = \sqrt{1 + am^2}$ if $m \neq 0$ and $h(0) = 0$). For numerical purpose, it is convenient to approximate the measure defined by the k-dimensional surfaces by smooth functions in $\mathbf{R}^n$. In this spirit, B. Merlet *et al.* propose phase field approximations of the branched surfaces and of their energy in the spirit of the Ambrosio–Tortorelli functional. The convergence of these approximations towards the original k-dimensional branched transport problem is established in [21] in the sense of Γ-convergence. Next, considering the cost $h(m) = \sqrt{1 + am^2}$ and sending a to 0, a phase field approximation of the Plateau problem is obtained. Numerical experiments show the efficiency of the method. These numerical results are exceptional as they are obtained without any guess on the topology of the minimizing k-surface (as opposed to methods based on parameterizations of the k-surface). In [39], B. Merlet *et al.* establish new results on the approximation of k-dimensional surfaces (k-rectifiable currents) by polyhedral surfaces with convergence in h-mass and with preservation of the boundary (the approximating polyhedral surface has the same boundary as the limit). This approximation result is required in the convergence study of [21].

7.9. Approximation theory

In [40], M. Herda *et al.* propose a new iterative algorithm for the calculation of sum of squares decompositions of polynomials, reformulated as positive interpolation. The method is based on the definition of a dual functional G from values at interpolation points. The domain of G, the boundary of the domain and the behavior of G at infinity are analyzed in details. In the general case, G is closed convex. For univariate polynomials in the context of the Lukacs representation, G is coercive and strictly convex which yields a unique critical point, corresponding to a sum of squares decomposition of G. Various descent algorithms are evoked. Numerical examples are provided, for univariate and bivariate polynomials.

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

C. Cancès supervised the PhD thesis of N. Peton at IFPEn from October 15, 2015 to October 12, 2018. The bilateral contract enters the framework-agreement between Inria and IFPEn.

9. Partnerships and Cooperations

9.1. National Initiatives

9.1.1. ANR

C. Chainais-Hillairet is a member of the ANR MOONRISE project. The MOONRISE project aims at exploring modeling, mathematical, and numerical issues originating from the presence of high oscillations in nonlinear PDEs mainly from the physics of nanotechnologies and from the physics of plasmas.

Title: MOdels, Oscillations, and NumeRIcal SchEmes

Type: Fondements du numérique (DS0705) - 2014

ANR reference: ANR-14-CE23-0007

Coordinator: F. Méhats (Université de Rennes 1)

Duration: October 2014 - June 2019

C. Chainais-Hillairet and T. Rey are members of the ANR MOHYCON project. The MOHYCON project is related to the analysis and simulation of multiscale models of semiconductors. As almost all current electronic technology involves the use of semiconductors, there is a strong interest for modeling and simulating the behavior of such devices, which was recently reinforced by the development of organic semiconductors used for example in solar panels or in mobile phones and television screens (among others).

Title: multiscale MOdels and HYbrid numerical methods for semiCONductors

Type: Société de l'information et de la communication (DS07) - 2017

ANR reference: ANR-17-CE40-0027

Coordinator: M. Bessemoulin-Chatard (CNRS and Université de Nantes)

Duration: January 2018 - December 2020

9.1.2. LabEx CEMPI

Title: Centre Européen pour les Mathématiques, la Physique et leurs Interactions

Coordinator: S. De Bièvre (Université de Lille)

Duration: January 2012 - December 2019

Partners: Laboratoire Paul Painlevé and Laser Physics department (PhLAM), Université de Lille

The "Laboratoire d'Excellence" Centre Européen pour les Mathématiques, la Physique et leurs Interactions (CEMPI), a project of the Laboratoire de Mathématiques Paul Painlevé and the Laboratoire de Physique des Lasers, Atomes et Molécules (PhLAM), was created in the context of the "Programme d'Investissements d'Avenir" in February 2012.

The association Painlevé-PhLAM creates in Lille a research unit for fundamental and applied research and for training and technological development that covers a wide spectrum of knowledge stretching from pure and applied mathematics to experimental and applied physics.

One of the three focus areas of CEMPI research is the interface between mathematics and physics. This focus area encompasses three themes. The first is concerned with key problems of a mathematical, physical and technological nature coming from the study of complex behavior in cold atoms physics and non-linear optics, in particular fibre optics. The two other themes deal with fields of mathematics such as algebraic geometry, modular forms, operator algebras, harmonic analysis and quantum groups that have promising interactions with several branches of theoretical physics.

9.2. International Research Visitors

9.2.1. Visits of International Scientists

The RAPSODI project-team invited several scientists in 2018. The following people came for long visits:

- J. Venel (Université Polytechnique Hauts-de-France) visited Inria Lille until July;
- J. Fuhrmann (WIAS Berlin) was invited for 1 month between May and June, thanks to a support of the LabEx CEMPI;
- A. Vasseur (UT Austin) was invited for 1 month in June, also thanks to a support of the LabEx CEMPI.

The following people came for shorter visits:

- S. Krell (Université de Nice) came in Lille on February 12-15;
- M. Rodrigues (Université de Rennes 1) came in Lille on December 3-7;
- M. Breden (TU Munich) came on December 17-21.

9.2.2. Internships

N. Staili, PhD student in the Faculté des Sciences de Meknès, came in Lille for a three-month visit between January and April.

9.2.3. Visits to International Teams

C. Cancès visited D. Matthes (TU Munich) during 1 week on December 3-7 to collaborate on the variational derivation of multiphase flow models.

9.2.4. Research Stays Abroad

A. Zurek spent 2 months (October-November) in the research team of A. Jüngel at TU Vienna to collaborate on the numerical simulation of a biofilm model. He was supported by the Institut Français d'Autriche and by EKINOX CNRS grant (Laboratoire Paul Painlevé).

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific Events Organization

10.1.1.1. General Chair, Scientific Chair

Four scientific events were organized by RAPSODI members in the framework of the LabEx CEMPI thematic semester on Numerical Analysis and PDEs:

- the Mathematics-Enterprises Study Week, co-organized at LILLIAD Learning Center by E. Creusé from January 29 to February 2;
- the third edition of the ABPDE conference (on Asymptotic Behavior of systems of PDEs arising in physics and biology), co-organized at LILLIAD Learning Center by C. Cancès, C. Chainais-Hillairet, I. Lacroix-Violet, and T. Rey on August 28-31;
- the second edition of the One-day conference on Calculus of Variations, co-organized at Laboratoire Paul Painlevé by I. Lacroix-Violet and B. Merlet on October 12;
- the fifth edition of the Lille days on Numerical Analysis (dedicated to domain decomposition and its applications to PDEs), co-organized at Laboratoire Paul Painlevé by C. Calgaro Zotto and E. Creusé on November 13-14.

C. Cancès co-organized a research school on Mathematics for Nuclear Energy at the Roscoff Marine Station on July 2-6, in partnership with the GdR MaNu. C. Chainais-Hillairet was part of the scientific board for this event.

In the CANUM (national NUMerical Analysis Congress) at Cap d'Agde from May 28 to June 1, three mini-symposia were organized by members of the team: one by C. Cancès on cross-diffusion systems, one by S. Lemaire on polytopal discretization methods, and one co-organized by T. Rey on kinetic models.

E. Creusé co-organized the Maths Jobs Forum that was held in Paris on December 13.

A. Zurek co-organized the Young Mathematicians Regional Tournament that was held in Laboratoire Paul Painlevé on April 14-15.

10.1.1.2. Member of the Organizing Committees

The whole team RAPSODI was involved in the organization of the ABPDE III conference.

10.1.2. Journal

10.1.2.1. Member of the Editorial Boards

C. Chainais-Hillairet is a member of the editorial board of the North-Western European Journal of Mathematics and of the International Journal on Finite Volumes.

10.1.2.2. Reviewer - Reviewing Activities

RAPSODI team members are regular reviewers for all the main international journals in numerical analysis and PDEs.

10.1.3. Invited Talks

C. Cancès was an invited speaker in the International workshop on PDEs, optimal transport, and applications, held on October 17-20 in Essaouira. He was also one of the speakers in the mini-symposium on polytopal discretization methods organized by S. Lemaire at CANUM (Cap d'Agde, May 28-June 1). He finally gave several seminars in Amiens, Orsay, Strasbourg, and Munich.

C. Chainais-Hillairet was invited to give a talk in a mini-symposium on finite volume methods at the fifteenth edition of the International Conference Zaragoza-Pau on Mathematics and its Applications held in Jaca on September 10-12. She also gave a seminar in Lyon (ICJ).

B. Gaudeul presented a poster at the AMaSiS (Applied Mathematics and Simulation for Semiconductors) conference held in Berlin on October 8-10.

M. Herda gave a talk in the same conference in Berlin, and another one for the Day of the Nord-Pas-de-Calais Mathematics Research Federation on October 3 in Lille. He was also one of the speakers in the mini-symposium on kinetic models co-organized by T. Rey at CANUM (Cap d'Agde, May 28-June 1).

I. Lacroix-Violet gave several seminars in Marseille (I2M), Dijon (IMB), Montpellier (IMAG), and Rennes.

S. Lemaire was invited to give a talk in a mini-symposium on polytopal discretization methods in the biennial Congress of the Italian Society of Applied and Industrial Mathematics (SIMAI) held in Rome on July 2-6. He also gave two seminars in Montpellier (IMAG), and Inria Paris, and gave a talk for the ANEDP team day at the Laboratoire Paul Painlevé.

D. Maltese gave a talk at CANUM (Cap d'Agde, May 28-June 1).

B. Merlet was an invited speaker in the congress Geometric Measure Theory in Verona held on June 11-15, and in the Workshop in Calculus of Variations organized at Paris-Diderot on June 25-27.

T. Rey was an invited speaker for the MAFRAN Days (Mathematical Frontiers in the Analysis of Many-particle Systems) held in Cambridge on September 24-26, and in the mini-workshop Innovative Trends in the Numerical Analysis and Simulation of Kinetic Equations organized in Oberwolfach on December 16-22. He also gave several seminars in Montpellier (IMAG), Paris-Dauphine (CEREMADE), Imperial College London, and Paris-Descartes.

A. Zurek was invited to give a talk in a special session on the mathematical problems arising from materials and biological science in the twelfth edition of the AIMS Conference on Dynamical Systems, Differential Equations, and Applications held in Taipei on July 5-9. He also presented a poster at CANUM (Cap d'Agde, May 28-June 1).

10.1.4. Research Administration

C. Cancès is the head of the MaNu research group (GdR MaNu) funded by the Institute for Mathematical Sciences and its Interactions (INSMI) of the French National Center for Research (CNRS).

E. Creusé has in charge to develop some actions promoted by AMIES (Agency for the Mathematics in Interaction with the Enterprise and the Society). More particularly, his action in 2018 was devoted to several characteristic points: management of some PEPS (First Support for Exploratory Projects), discussions to initiate collaborations between academic researchers in mathematics and industrial partners, participation to the monthly AMIES meeting, organization of the Mathematics-Enterprises Study Week in Lille in January as well as of the Maths Jobs Forum in Paris in December.

I. Lacroix-Violet, B. Merlet, and T. Rey are elected members of the Conseil du Laboratoire Paul Painlevé. I. Lacroix-Violet is also a member of the Jury de domaine.

T. Rey is in charge of the organization of the weekly seminar of the ANEDP team of the Laboratoire Paul Painlevé. He is also a member of the team of the Opération Postes.

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

RAPSODI team members are strongly involved in teaching at the Université de Lille. C. Calgaro Zotto is in charge of the Master of Mathematical Engineering. B. Merlet is in charge of the Master 2 of Scientific Computing. E. Creusé was responsible of the “Cursus Master en Ingénierie Mathématiques” until August. He is since September director of the Mathematics Department of the Université Polytechnique Hauts-de-France. C. Cancès gave lectures at Centrale Lille. S. Lemaire gave lectures in the Master 2 of Scientific Computing.

C. Chainais-Hillairet gave 16h of courses in the international Summer School “The way to become a mathematician” in the Harbin Institute of Technology in July.

10.2.2. Supervision

Post-doc: F. Chave arrived in December to work on high-order polytopal discretization methods for electromagnetism; supervisors: E. Creusé and S. Lemaire.

PhD: L. Ferrari defended his PhD thesis on “Phase field approximations for branched transport problems” on October 5; advisors: A. Chambolle (CNRS & CMAP, École Polytechnique) and B. Merlet.

PhD: N. Peton defended his PhD thesis on “Numerical methods for a stratigraphic model with nonlinear diffusion and moving frontier areas” on October 12; advisors: C. Cancès, Q.-H. Tran (IFPEn), and S. Wolf (IFPEn).

PhD in progress: C. Colin-Lecerf, on the “Analyse numérique et simulations de modèles multi-fluides”, since 10/01/2015; advisors: C. Calgaro Zotto and E. Creusé.

PhD in progress: A. Zurek, on the “Numerical and theoretical analysis of models describing the corrosion of materials”, since 10/01/2016; advisors: C. Chainais-Hillairet and B. Merlet.

PhD in progress: B. Gaudeul, on the “Numerical approximation of cross-diffusion systems arising in physics and biology”, since 09/01/2018; advisors: C. Cancès and C. Chainais-Hillairet.

Master internship: A. El Keurti, on the “Study of upwind finite volume schemes for nonlocal transport”, from November 2017 to July 2018; advisor: T. Rey.

Master internship: A. Latrech, on the “Numerical study of local minimizers of a Bose–Einstein energy functional in 2D”, from January to July; advisors: G. Dujardin (Inria Lille) and I. Lacroix-Violet.

Master internship: T. Ebrahimipourfaez, on “Entropy-diminishing finite volume schemes for cross-diffusion systems”, from April to July; advisor: C. Chainais-Hillairet.

Master internship: B. Gaudeul, on “Numerical schemes for a Nernst–Planck–Poisson model”, from April to July; advisor: C. Chainais-Hillairet.

Master internship: C. Marinel, on the “Implementation and parallelization of a finite element code with *a posteriori* error estimators”, from June to August; advisor: E. Creusé.

Master internship in progress: N. Aghouzzaf, on the “Design of high-order numerical time integrators for kinetic equations”, since December; advisor: T. Rey.

10.2.3. Juries

E. Creusé reported on O. Gorynina’s PhD thesis, defended on February 22, 2018 at Université Bourgogne-Franche Comté. Title: Éléments finis adaptatifs pour l’équation des ondes instationnaire.

B. Merlet was a member of the jury of the PhD thesis of A. Julia, defended on October 9, 2018 at Sorbonne Paris Cité. Title: Functions with bounded variations on a current.

C. Calgaro Zotto and C. Cancès are members of the Jury de l’Agrégation de Mathématiques, which is a national hiring committee for the highest level of high-school teachers.

C. Cancès was part of the selection committee for an associate professor (MCF) position at the Laboratoire de Mathématiques d’Orsay.

C. Chainais-Hillairet was part of the selection committee for a full professor (PR) position at Aix-Marseille Université.

10.3. Popularization

C. Calgaro Zotto is in charge of the communication of the Laboratoire Paul Painlevé. She organizes various events which promote mathematics among young people:

- les “Mathématiques itinérantes”;
- la “Semaine des mathématiques”;
- la collection “Stages scientifiques en Seconde”.

Members of the team participate regularly to these actions.

S. Lemaire gave a talk at Inria Lille for the internal scientific popularization event “30 minutes of Science” in June.

11. Bibliography

Major publications by the team in recent years

[1] M. BESSEMOULIN-CHATARD, C. CHAINAIS-HILLAIRET. *Exponential decay of a finite volume scheme to the thermal equilibrium for drift–diffusion systems*, in "Journal of Numerical Mathematics", 2017, vol. 25, n^{o} 3, pp. 147-168 [*DOI :* 10.1515/JNMA-2016-0007], https://hal.archives-ouvertes.fr/hal-01250709

[2] C. CALGARO, E. CREUSÉ, T. GOUDON, S. KRELL. *Simulations of non homogeneous viscous flows with incompressibility constraints*, in "Mathematics and Computers in Simulation", 2017, vol. 137, pp. 201-225, https://hal.archives-ouvertes.fr/hal-01246070

[3] C. CANCÈS, T. GALLOUËT, L. MONSAINGEON. *Incompressible immiscible multiphase flows in porous media: a variational approach*, in "Analysis & PDE", 2017, vol. 10, n^{o} 8, pp. 1845–1876 [*DOI :* 10.2140/APDE.2017.10.1845], https://hal.archives-ouvertes.fr/hal-01345438

[4] C. Cancès, C. Guichard. *Numerical analysis of a robust free energy diminishing Finite Volume scheme for parabolic equations with gradient structure*, in "Foundations of Computational Mathematics", 2017, vol. 17, n^{o} 6, pp. 1525-1584, https://hal.archives-ouvertes.fr/hal-01119735

[5] C. Chainais-Hillairet, B. Merlet, A. Vasseur. *Positive Lower Bound for the Numerical Solution of a Convection-Diffusion Equation*, in "FVCA8 2017 - International Conference on Finite Volumes for Complex Applications VIII", Lille, France, Springer, June 2017, pp. 331-339 [*DOI :* 10.1007/978-3-319-57397-7_26], https://hal.archives-ouvertes.fr/hal-01596076

[6] D. A. Di Pietro, A. Ern, S. Lemaire. *An arbitrary-order and compact-stencil discretization of diffusion on general meshes based on local reconstruction operators*, in "Computational Methods in Applied Mathematics", June 2014, vol. 14, n^{o} 4, pp. 461-472 [*DOI :* 10.1515/CMAM-2014-0018], https://hal.archives-ouvertes.fr/hal-00978198

[7] G. Dimarco, R. Loubère, J. Narski, T. Rey. *An efficient numerical method for solving the Boltzmann equation in multidimensions*, in "Journal of Computational Physics", 2018, vol. 353, pp. 46-81 [*DOI :* 10.1016/J.JCP.2017.10.010], https://hal.archives-ouvertes.fr/hal-01357112

[8] F. Filbet, M. Herda. *A finite volume scheme for boundary-driven convection-diffusion equations with relative entropy structure*, in "Numerische Mathematik", 2017, vol. 137, n^{o} 3, pp. 535-577, https://hal.archives-ouvertes.fr/hal-01326029

[9] I. Lacroix-Violet, A. Vasseur. *Global weak solutions to the compressible quantum Navier–Stokes equation and its semi-classical limit*, in "Journal de Mathématiques Pures et Appliquées", 2018, vol. 114, pp. 191-210, https://hal.archives-ouvertes.fr/hal-01347943

[10] B. Merlet. *A highly anisotropic nonlinear elasticity model for vesicles I. Eulerian formulation, rigidity estimates and vanishing energy limit*, in "Arch. Ration. Mech. Anal.", 2015, vol. 217, n^{o} 2, pp. 651–680 [*DOI :* 10.1007/S00205-014-0839-5], https://hal.archives-ouvertes.fr/hal-00848547

Publications of the year

Articles in International Peer-Reviewed Journals

[11] A. Ait Hammou Oulhaj. *Numerical analysis of a finite volume scheme for a seawater intrusion model with cross-diffusion in an unconfined aquifer*, in "Numerical Methods for Partial Differential Equations", May 2018 [*DOI :* 10.1002/NUM.22234], https://hal.archives-ouvertes.fr/hal-01432197

[12] A. Ait Hammou Oulhaj, C. Cancès, C. Chainais-Hillairet. *Numerical analysis of a nonlinearly stable and positive Control Volume Finite Element scheme for Richards equation with anisotropy*, in "ESAIM: Mathematical Modelling and Numerical Analysis", 2018, vol. 52, n^{o} 4, pp. 1532-1567 [*DOI :* 10.1051/M2AN/2017012], https://hal.archives-ouvertes.fr/hal-01372954

[13] M. Bessemoulin-Chatard, C. Chainais-Hillairet. *Uniform-in-time Bounds for approximate Solutions of the drift-diffusion System*, in "Numerische Mathematik", 2018, https://hal.archives-ouvertes.fr/hal-01659418

[14] C. Calgaro, c. colin, E. Creusé, E. Zahrouni. *Approximation by an iterative method of a low Mach model with temperature dependent viscosity*, in "Mathematical Methods in the Applied Sciences", 2018 [*DOI :* 10.1002/MMA.5342], https://hal.archives-ouvertes.fr/hal-01801242

[15] C. CALGARO, M. EZZOUG, E. ZAHROUNI. *Stability and convergence of an hybrid finite volume-finite element method for a multiphasic incompressible fluid model*, in "Communications on Pure and Applied Analysis", March 2018, vol. 17, n^{o} 2, pp. 429-448, https://hal.archives-ouvertes.fr/hal-01586201

[16] C. CANCÈS. *Energy stable numerical methods for porous media flow type problems*, in "Oil & Gas Science and Technology - Revue d'IFP Energies nouvelles", 2018, vol. 73, 78 p. [*DOI :* 10.2516/OGST/2018067], https://hal.archives-ouvertes.fr/hal-01953395

[17] C. CANCÈS, C. CHAINAIS-HILLAIRET, A. GERSTENMAYER, A. JÜNGEL. *Convergence of a Finite-Volume Scheme for a Degenerate Cross-Diffusion Model for Ion Transport*, in "Numerical Methods for Partial Differential Equations", 2018, https://arxiv.org/abs/1801.09408 , https://hal.archives-ouvertes.fr/hal-01695129

[18] C. CANCÈS, C. CHAINAIS-HILLAIRET, S. KRELL. *Numerical analysis of a nonlinear free-energy diminishing Discrete Duality Finite Volume scheme for convection diffusion equations*, in "Computational Methods in Applied Mathematics", 2018, vol. 18, n^{o} 3, pp. 407-432, https://arxiv.org/abs/1705.10558 - Special issue on "Advanced numerical methods: recent developments, analysis and application" [*DOI :* 10.1515/CMAM-2017-0043], https://hal.archives-ouvertes.fr/hal-01529143

[19] C. CANCÈS, T. GALLOUËT, M. LABORDE, L. MONSAINGEON. *Simulation of multiphase porous media flows with minimizing movement and finite volume schemes*, in "European Journal of Applied Mathematics", 2018 [*DOI :* 10.1017/S0956792518000633], https://hal.archives-ouvertes.fr/hal-01700952

[20] C. CHAINAIS-HILLAIRET, B. MERLET, A. ZUREK. *Convergence of a finite volume scheme for a parabolic system with a free boundary modeling concrete carbonation*, in "ESAIM: Mathematical Modelling and Numerical Analysis", June 2018, vol. 52, n^{o} 2, pp. 457-480, https://hal.archives-ouvertes.fr/hal-01477543

[21] A. CHAMBOLLE, L. A. D. FERRARI, B. MERLET. *Variational approximation of size-mass energies for k-dimensional currents*, in "ESAIM: Control, Optimisation and Calculus of Variations", April 2018, https://arxiv.org/abs/1710.08808 , https://hal.archives-ouvertes.fr/hal-01622540

[22] M. CICUTTIN, A. ERN, S. LEMAIRE. *A Hybrid High-Order method for highly oscillatory elliptic problems*, in "Computational Methods in Applied Mathematics", 2018 [*DOI :* 10.1515/CMAM-2018-0013], https://hal.archives-ouvertes.fr/hal-01467434

[23] E. CREUSÉ, P. DULAR, S. NICAISE. *About the gauge conditions arising in Finite Element magnetostatic problems*, in "Computers and Mathematics with Applications", 2018, https://hal.archives-ouvertes.fr/hal-01955649

[24] E. CREUSÉ, Y. LE MENACH, S. NICAISE, F. PIRIOU, R. TITTARELLI. *Two Guaranteed Equilibrated Error Estimators for Harmonic Formulations in Eddy Current Problems*, in "Computers and Mathematics with Applications", 2018, https://hal.archives-ouvertes.fr/hal-01955692

[25] G. DIMARCO, R. LOUBÈRE, J. NARSKI, T. REY. *An efficient numerical method for solving the Boltzmann equation in multidimensions*, in "Journal of Computational Physics", 2018, vol. 353, pp. 46-81 [*DOI :* 10.1016/J.JCP.2017.10.010], https://hal.archives-ouvertes.fr/hal-01357112

[26] M. HERDA, L. M. M. RODRIGUES. *Anisotropic Boltzmann-Gibbs dynamics of strongly magnetized Vlasov-Fokker-Planck equations*, in "Kinetic and Related Models ", 2018, https://arxiv.org/abs/1610.05138 , https://hal.archives-ouvertes.fr/hal-01382854

[27] I. LACROIX-VIOLET, A. VASSEUR. *Global weak solutions to the compressible quantum navier-stokes equation and its semi-classical limit*, in "Journal de Mathématiques Pures et Appliquées", 2018, vol. 114, pp. 191-210, https://arxiv.org/abs/1607.06646 , https://hal.archives-ouvertes.fr/hal-01347943

[28] R. TITTARELLI, Y. LE MENACH, F. PIRIOU, E. CREUSÉ, S. NICAISE, J.-P. DUCREUX. *Comparison of Numerical Error Estimators for Eddy Current Problems solved by FEM*, in "IEEE Transactions on Magnetics", 2018, vol. 54, n^o 3, https://hal.archives-ouvertes.fr/hal-01645591

Other Publications

[29] A. AIT HAMMOU OULHAJ, C. CANCÈS, C. CHAINAIS-HILLAIRET, P. LAURENÇOT. *Large time behavior of a two phase extension of the porous medium equation*, March 2018, https://arxiv.org/abs/1803.10476 - 28 pages, 14 figures, https://hal.archives-ouvertes.fr/hal-01752759

[30] A. AIT HAMMOU OULHAJ, D. MALTESE. *Positive nonlinear Control Volume Finite Element scheme for an anisotropic seawater intrusion model with cross-diffusion in an unconfined aquifer*, November 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01906872

[31] C. BESSE, S. DESCOMBES, G. DUJARDIN, I. LACROIX-VIOLET. *Energy preserving methods for nonlinear schrodinger equations*, December 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01951527

[32] M. C. BESSEMOULIN-CHATARD, M. HERDA, T. REY. *Hypocoercivity and diffusion limit of a finite volume scheme for linear kinetic equations*, December 2018, 36 pages, 9 figures, 2 tables, https://hal.archives-ouvertes.fr/hal-01957832

[33] O. BLONDEL, C. CANCÈS, M. SASADA, M. SIMON. *Convergence of a Degenerate Microscopic Dynamics to the Porous Medium Equation*, April 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01710628

[34] D. BRESCH, M. GISCLON, I. LACROIX-VIOLET. *On Navier-Stokes-Korteweg and Euler-Korteweg Systems: Application to Quantum Fluids Models*, March 2018, https://arxiv.org/abs/1703.09460 - working paper or preprint, https://hal.archives-ouvertes.fr/hal-01496960

[35] C. CALGARO, C. COLIN, E. CREUSÉ. *A combined Finite Volumes -Finite Elements method for a low-Mach model*, December 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01574894

[36] C. CANCÈS, D. MATTHES, F. NABET. *A two-phase two-fluxes degenerate Cahn-Hilliard model as constrained Wasserstein gradient flow*, 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01665338

[37] C. CANCÈS, F. NABET, M. VOHRALÍK. *Convergence and a posteriori error analysis for energy-stable finite element approximations of degenerate parabolic equations*, October 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01894884

[38] C. CHAINAIS-HILLAIRET, M. HERDA. *Large-time behavior of a family of finite volume schemes for boundary-driven convection-diffusion equations*, October 2018, https://arxiv.org/abs/1810.01087 - working paper or preprint, https://hal.archives-ouvertes.fr/hal-01885015

[39] A. CHAMBOLLE, L. A. D. FERRARI, B. MERLET. *Strong approximation in h-mass of rectifiable currents under homological constraint*, June 2018, https://arxiv.org/abs/1806.05046 - working paper or preprint, https://hal.archives-ouvertes.fr/hal-01813234

[40] B. DESPRÉS, M. HERDA. *Iterative Calculation of Sum Of Squares*, December 2018, https://arxiv.org/abs/1812.02444 - working paper or preprint, https://hal.archives-ouvertes.fr/hal-01946539

[41] M. GOLDMAN, B. MERLET, V. MILLOT. *A Ginzburg-Landau model with topologically induced free discontinuities*, December 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01643795

[42] S. LEMAIRE. *Bridging the Hybrid High-Order and Virtual Element methods*, November 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01902962

[43] W. MELIS, T. REY, G. SAMAEY. *Projective and telescopic projective integration for the nonlinear BGK and Boltzmann equations*, December 2018, https://arxiv.org/abs/1712.06362 - 35 pages, 2 annexes, 12 figures, https://hal.archives-ouvertes.fr/hal-01666346

[44] N. PETON, C. CANCÈS, D. GRANJEON, Q.-H. TRAN, S. WOLF. *Numerical scheme for a water flow-driven forward stratigraphic model*, September 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01870347

[45] A. S. F. ZUREK. *Numerical approximation of a concrete carbonation model: study of the* $\sqrt{t}$*-law of propagation*, July 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01839277

References in notes

[46] R. ABGRALL. *A review of residual distribution schemes for hyperbolic and parabolic problems: the July 2010 state of the art*, in "Commun. Comput. Phys.", 2012, vol. 11, n^o 4, pp. 1043–1080, http://dx.doi.org/10.4208/cicp.270710.130711s

[47] R. ABGRALL, G. BAURIN, A. KRUST, D. DE SANTIS, M. RICCHIUTO. *Numerical approximation of parabolic problems by residual distribution schemes*, in "Internat. J. Numer. Methods Fluids", 2013, vol. 71, n^o 9, pp. 1191–1206, http://dx.doi.org/10.1002/fld.3710

[48] R. ABGRALL, A. LARAT, M. RICCHIUTO. *Construction of very high order residual distribution schemes for steady inviscid flow problems on hybrid unstructured meshes*, in "J. Comput. Phys.", 2011, vol. 230, n^o 11, pp. 4103–4136, http://dx.doi.org/10.1016/j.jcp.2010.07.035

[49] R. ABGRALL, A. LARAT, M. RICCHIUTO, C. TAVÉ. *A simple construction of very high order non-oscillatory compact schemes on unstructured meshes*, in "Comput. & Fluids", 2009, vol. 38, n^o 7, pp. 1314–1323, http://dx.doi.org/10.1016/j.compfluid.2008.01.031

[50] T. AIKI, A. MUNTEAN. *Existence and uniqueness of solutions to a mathematical model predicting service life of concrete structure*, in "Adv. Math. Sci. Appl.", 2009, vol. 19, pp. 109-129

[51] T. AIKI, A. MUNTEAN. *A free-boundary problem for concrete carbonation: front nucleation and rigorous justification of the* $\sqrt{t}$*-law of propagation*, in "Interfaces Free Bound.", 2013, vol. 15, n^o 2, pp. 167–180, http://dx.doi.org/10.4171/IFB/299

[52] B. AMAZIANE, A. BERGAM, M. EL OSSMANI, Z. MGHAZLI. *A posteriori estimators for vertex centred finite volume discretization of a convection-diffusion-reaction equation arising in flow in porous media*, in "Internat. J. Numer. Methods Fluids", 2009, vol. 59, n^o 3, pp. 259–284, http://dx.doi.org/10.1002/fld.1456

[53] M. AVILA, J. PRINCIPE, R. CODINA. *A finite element dynamical nonlinear subscale approximation for the low Mach number flow equations*, in "J. Comput. Phys.", 2011, vol. 230, n^o 22, pp. 7988–8009, http://dx.doi.org/10.1016/j.jcp.2011.06.032

[54] I. BABUŠKA, W. C. RHEINBOLDT. *Error estimates for adaptive finite element computations*, in "SIAM J. Numer. Anal.", 1978, vol. 15, n^o 4, pp. 736–754

[55] C. BATAILLON, F. BOUCHON, C. CHAINAIS-HILLAIRET, C. DESGRANGES, E. HOARAU, F. MARTIN, S. PERRIN, M. TUPIN, J. TALANDIER. *Corrosion modelling of iron based alloy in nuclear waste repository*, in "Electrochim. Acta", 2010, vol. 55, n^o 15, pp. 4451–4467

[56] J. BEAR, Y. BACHMAT. *Introduction to modeling of transport phenomena in porous media*, Springer, 1990, vol. 4

[57] J. BEAR. *Dynamic of Fluids in Porous Media*, American Elsevier, New York, 1972

[58] A. BECCANTINI, E. STUDER, S. GOUNAND, J.-P. MAGNAUD, T. KLOCZKO, C. CORRE, S. KUDRIAKOV. *Numerical simulations of a transient injection flow at low Mach number regime*, in "Internat. J. Numer. Methods Engrg.", 2008, vol. 76, n^o 5, pp. 662–696, http://dx.doi.org/10.1002/nme.2331

[59] L. BEIRÃO DA VEIGA, F. BREZZI, A. CANGIANI, G. MANZINI, L. D. MARINI, A. RUSSO. *Basic principles of virtual element methods*, in "Math. Models Methods Appl. Sci. (M3AS)", 2013, vol. 23, n^o 1, pp. 199–214

[60] J.-D. BENAMOU, G. CARLIER, M. LABORDE. *An augmented Lagrangian approach to Wasserstein gradient flows and applications*, in "Gradient flows: from theory to application", ESAIM Proc. Surveys, EDP Sci., Les Ulis, 2016, vol. 54, pp. 1–17, https://doi.org/10.1051/proc/201654001

[61] S. BERRONE, V. GARBERO, M. MARRO. *Numerical simulation of low-Reynolds number flows past rectangular cylinders based on adaptive finite element and finite volume methods*, in "Comput. & Fluids", 2011, vol. 40, pp. 92–112, http://dx.doi.org/10.1016/j.compfluid.2010.08.014

[62] C. BESSE. *Analyse numérique des systèmes de Davey–Stewartson*, Université Bordeaux 1, 1998

[63] M. BESSEMOULIN-CHATARD, C. CHAINAIS-HILLAIRET. *Exponential decay of a finite volume scheme to the thermal equilibrium for drift-diffusion systems*, in "J. Numer. Math.", 2017, vol. 25, n^o 3, pp. 147–168, https://doi.org/10.1515/jnma-2016-0007

[64] M. BESSEMOULIN-CHATARD, C. CHAINAIS-HILLAIRET, M.-H. VIGNAL. *Study of a fully implicit scheme for the drift-diffusion system. Asymptotic behavior in the quasi-neutral limit*, in "SIAM, J. Numer. Anal.", 2014, vol. 52, n^o 4, http://epubs.siam.org/toc/sjnaam/52/4

[65] D. BRESCH, P. NOBLE, J.-P. VILA. *Relative entropy for compressible Navier-Stokes equations with density dependent viscosities and various applications*, in "LMLFN 2015—low velocity flows—application to low Mach and low Froude regimes", ESAIM Proc. Surveys, EDP Sci., Les Ulis, 2017, vol. 58, pp. 40–57

[66] C. CALGARO, E. CREUSÉ, T. GOUDON. *An hybrid finite volume-finite element method for variable density incompressible flows*, in "J. Comput. Phys.", 2008, vol. 227, n^o 9, pp. 4671–4696

[67] C. CALGARO, E. CREUSÉ, T. GOUDON. *Modeling and simulation of mixture flows: application to powder-snow avalanches*, in "Comput. & Fluids", 2015, vol. 107, pp. 100–122, http://dx.doi.org/10.1016/j.compfluid.2014.10.008

[68] C. CANCÈS, T. O. GALLOUËT, L. MONSAINGEON. *Incompressible immiscible multiphase flows in porous media: a variational approach*, in "Anal. PDE", 2017, vol. 10, n^o 8, pp. 1845–1876, https://doi.org/10.2140/apde.2017.10.1845

[69] C. CANCÈS, C. GUICHARD. *Convergence of a nonlinear entropy diminishing Control Volume Finite Element scheme for solving anisotropic degenerate parabolic equations*, in "Mathematics of Computation", 2016, vol. 85, n^o 298, pp. 549-580, https://hal.archives-ouvertes.fr/hal-00955091

[70] C. CANCÈS, I. S. POP, M. VOHRALÍK. *An a posteriori error estimate for vertex-centered finite volume discretizations of immiscible incompressible two-phase flow*, in "Math. Comp.", 2014, vol. 83, n^o 285, pp. 153–188, http://dx.doi.org/10.1090/S0025-5718-2013-02723-8

[71] J. A. CARRILLO, A. JÜNGEL, P. A. MARKOWICH, G. TOSCANI, A. UNTERREITER. *Entropy dissipation methods for degenerate parabolic problems and generalized Sobolev inequalities*, in "Monatsh. Math.", 2001, vol. 133, n^o 1, pp. 1–82, http://dx.doi.org/10.1007/s006050170032

[72] C. CHAINAIS-HILLAIRET. *Entropy method and asymptotic behaviours of finite volume schemes*, in "Finite volumes for complex applications. VII. Methods and theoretical aspects", Springer Proc. Math. Stat., Springer, Cham, 2014, vol. 77, pp. 17–35

[73] C. CHAINAIS-HILLAIRET, A. JÜNGEL, S. SCHUCHNIGG. *Entropy-dissipative discretization of nonlinear diffusion equations and discrete Beckner inequalities*, in "Modelisation Mathématique et Analyse Numérique", 2016, vol. 50, n^o 1, pp. 135-162, https://hal.archives-ouvertes.fr/hal-00924282

[74] E. CREUSÉ, S. NICAISE, Z. TANG, Y. LE MENACH, N. NEMITZ, F. PIRIOU. *Residual-based a posteriori estimators for the* $\mathbf{A}-\phi$ *magnetodynamic harmonic formulation of the Maxwell system*, in "Math. Models Methods Appl. Sci.", 2012, vol. 22, n^o 5, 1150028, 30 p. , http://dx.doi.org/10.1142/S021820251150028X

[75] E. CREUSÉ, S. NICAISE, Z. TANG, Y. LE MENACH, N. NEMITZ, F. PIRIOU. *Residual-based a posteriori estimators for the* $\mathbf{T}/\Omega$ *magnetodynamic harmonic formulation of the Maxwell system*, in "Int. J. Numer. Anal. Model.", 2013, vol. 10, n^o 2, pp. 411–429

[76] E. CREUSÉ, S. NICAISE, E. VERHILLE. *Robust equilibrated a posteriori error estimators for the Reissner-Mindlin system*, in "Calcolo", 2011, vol. 48, n^o 4, pp. 307–335, http://dx.doi.org/10.1007/s10092-011-0042-0

[77] R. DANCHIN, X. LIAO. *On the well-posedness of the full low Mach number limit system in general critical Besov spaces*, in "Commun. Contemp. Math.", 2012, vol. 14, n^o 3, 1250022, 47 p. , https://doi.org/10.1142/S0219199712500228

[78] D. A. Di Pietro, A. Ern, S. Lemaire. *An arbitrary-order and compact-stencil discretization of diffusion on general meshes based on local reconstruction operators*, in "Comput. Methods Appl. Math.", 2014, vol. 14, n$^{\text{O}}$ 4, pp. 461–472, https://doi.org/10.1515/cmam-2014-0018

[79] D. A. Di Pietro, M. Vohralík. *A Review of Recent Advances in Discretization Methods, a Posteriori Error Analysis, and Adaptive Algorithms for Numerical Modeling in Geosciences*, in "Oil & Gas Science and Technology-Rev. IFP", June 2014, pp. 1-29, (online first)

[80] J. Dolbeault, C. Mouhot, C. Schmeiser. *Hypocoercivity for linear kinetic equations conserving mass*, in "Trans. Amer. Math. Soc.", 2015, vol. 367, n$^{\text{O}}$ 6, pp. 3807–3828, https://doi.org/10.1090/S0002-9947-2015-06012-7

[81] V. Dolejší, A. Ern, M. Vohralík. *A framework for robust a posteriori error control in unsteady nonlinear advection-diffusion problems*, in "SIAM J. Numer. Anal.", 2013, vol. 51, n$^{\text{O}}$ 2, pp. 773–793, http://dx.doi.org/10.1137/110859282

[82] D. Donatelli, E. Feireisl, P. Marcati. *Well/ill posedness for the Euler–Korteweg–Poisson system and related problems*, in "Comm. Partial Differential Equations", 2015, vol. 40, pp. 1314-1335

[83] J. Droniou. *Finite volume schemes for diffusion equations: introduction to and review of modern methods*, in "Math. Models Methods Appl. Sci.", 2014, vol. 24, n$^{\text{O}}$ 8, pp. 1575-1620

[84] W. E, P. Palffy-Muhoray. *Phase separation in incompressible systems*, in "Phys. Rev. E", Apr 1997, vol. 55, pp. R3844–R3846, https://link.aps.org/doi/10.1103/PhysRevE.55.R3844

[85] C. M. Elliott, H. Garcke. *On the Cahn-Hilliard equation with degenerate mobility*, in "SIAM J. Math. Anal.", 1996, vol. 27, n$^{\text{O}}$ 2, pp. 404–423, http://dx.doi.org/10.1137/S0036141094267662

[86] P. Embid. *Well-posedness of the nonlinear equations for zero Mach number combustion*, in "Comm. Partial Differential Equations", 1987, vol. 12, n$^{\text{O}}$ 11, pp. 1227–1283, https://doi.org/10.1080/03605308708820526

[87] E. Emmrich. *Two-step BDF time discretisation of nonlinear evolution problems governed by monotone operators with strongly continuous perturbations*, in "Comput. Methods Appl. Math.", 2009, vol. 9, n$^{\text{O}}$ 1, pp. 37–62

[88] J. Giesselmann, C. Lattanzio, A.-E. Tzavaras. *Relative energy for the Korteweg theory and related Hamiltonian flows in gas dynamics*, in "Arch. Rational Mech. Analysis", 2017, vol. 223, pp. 1427-1484

[89] V. Gravemeier, W. A. Wall. *Residual-based variational multiscale methods for laminar, transitional and turbulent variable-density flow at low Mach number*, in "Internat. J. Numer. Methods Fluids", 2011, vol. 65, n$^{\text{O}}$ 10, pp. 1260–1278, http://dx.doi.org/10.1002/fld.2242

[90] L. Greengard, J.-Y. Lee. *Accelerating the nonuniform fast Fourier transform*, in "SIAM Rev.", 2004, vol. 46, n$^{\text{O}}$ 3, pp. 443–454, http://dx.doi.org/10.1137/S003614450343200X

[91] F. Guillén-González, P. Damázio, M. A. Rojas-Medar. *Approximation by an iterative method for regular solutions for incompressible fluids with mass diffusion*, in "J. Math. Anal. Appl.", 2007, vol. 326, n$^{\text{O}}$ 1, pp. 468–487, http://dx.doi.org/10.1016/j.jmaa.2006.03.009

[92] F. GUILLÉN-GONZÁLEZ, M. SY. *Iterative method for mass diffusion model with density dependent viscosity*, in "Discrete Contin. Dyn. Syst. Ser. B", 2008, vol. 10, n$^{\text{o}}$ 4, pp. 823–841, http://dx.doi.org/10.3934/dcdsb.2008.10.823

[93] V. HEUVELINE. *On higher-order mixed FEM for low Mach number flows: application to a natural convection benchmark problem*, in "Internat. J. Numer. Methods Fluids", 2003, vol. 41, n$^{\text{o}}$ 12, pp. 1339–1356, http://dx.doi.org/10.1002/fld.454

[94] F. HUANG, W. TAN. *On the strong solution of the ghost effect system*, in "SIAM J. Math. Anal.", 2017, vol. 49, n$^{\text{o}}$ 5, pp. 3496–3526, https://doi.org/10.1137/16M106964X

[95] M. E. HUBBARD, M. RICCHIUTO. *Discontinuous upwind residual distribution: a route to unconditional positivity and high order accuracy*, in "Comput. & Fluids", 2011, vol. 46, pp. 263–269, http://dx.doi.org/10.1016/j.compfluid.2010.12.023

[96] S. JIN. *Efficient asymptotic-preserving (AP) schemes for some multiscale kinetic equations*, in "SIAM, J. Sci. Comput.", 1999, vol. 21, pp. 441-454

[97] P. LE QUÉRÉ, R. MASSON, P. PERROT. *A Chebyshev collocation algorithm for 2D non-Boussinesq convection*, in "Journal of Computational Physics", 1992, vol. 103, pp. 320-335

[98] S. LEVERMORE, W. SUN, K. TRIVISA. *Local well-posedness of a ghost effect system*, in "Indiana Univ. J.", 2011

[99] A. MAJDA, J. SETHIAN. *The derivation and numerical solution of the equations for zero Mach number combustion*, in "Combustion Science and Technology", 1985, vol. 42, pp. 185–205

[100] F. OTTO, W. E. *Thermodynamically driven incompressible fluid mixtures*, in "J. Chem. Phys.", 1997, vol. 107, n$^{\text{o}}$ 23, pp. 10177-10184, https://doi.org/10.1063/1.474153

[101] F. OTTO. *The geometry of dissipative evolution equations: the porous medium equation*, in "Comm. Partial Differential Equations", 2001, vol. 26, n$^{\text{o}}$ 1-2, pp. 101–174

[102] M. RICCHIUTO, R. ABGRALL. *Explicit Runge-Kutta residual distribution schemes for time dependent problems: second order case*, in "J. Comput. Phys.", 2010, vol. 229, n$^{\text{o}}$ 16, pp. 5653–5691, http://dx.doi.org/10.1016/j.jcp.2010.04.002

[103] D. RUPPEL, E. SACKMANN. *On defects in different phases of two-dimensional lipid bilayers*, in "J. Phys. France", 1983, vol. 44, n$^{\text{o}}$ 9, pp. 1025-1034 [*DOI :* 10.1051/JPHYS:019830044090102500], https://jphys.journaldephysique.org/articles/jphys/abs/1983/09/jphys_1983__44_9_1025_0/jphys_1983__44_9_1025_0.html

[104] M. VOHRALÍK. *Residual flux-based a posteriori error estimates for finite volume and related locally conservative methods*, in "Numer. Math.", 2008, vol. 111, n$^{\text{o}}$ 1, pp. 121–158, http://dx.doi.org/10.1007/s00211-008-0168-4

[105] J. DE FRUTOS, B. GARCÍA-ARCHILLA, J. NOVO. *A posteriori error estimations for mixed finite-element approximations to the Navier-Stokes equations*, in "J. Comput. Appl. Math.", 2011, vol. 236, n^{o} 6, pp. 1103–1122, http://dx.doi.org/10.1016/j.cam.2011.07.033

[106] P. G. DE GENNES. *Dynamics of fluctuations and spinodal decomposition in polymer blends*, in "J. Chem. Phys.", 1980, vol. 72, pp. 4756-4763