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Project-Team MAGIQUE-3D

Advanced 3D Numerical Modeling in Geophysics

IN COLLABORATION WITH: Laboratoire de mathématiques et de leurs applications (LMAP)

RESEARCH CENTER
Bordeaux - Sud-Ouest

THEME
**Earth, Environmental and Energy
Sciences**

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- 3. - Environment and planet
 - 3.3. - Geosciences
 - 3.3.1. - Earth and subsoil
- 4. - Energy
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2. Overall Objectives

2.1. General setting

MAGIQUE-3D is a joint project-team between Inria and the Department of Applied Mathematics (LMA) of the University of Pau in partnership with CNRS. The mission of MAGIQUE-3D is to develop and validate efficient solution methodologies for solving complex three-dimensional geophysical problems, with a particular emphasis on problems arising in seismic imaging, in response to the local industrial and community needs. Indeed, as it is well known, the region of Pau has long-standing tradition in the Geosciences activities. However, in spite of the recent significant advances in algorithmic considerations as well as in computing platforms, the solution of most real-world problems in this field remains intractable. Hence, there is a scientific need of pressing importance to design new numerical methods for solving efficiently and accurately wave propagation problems defined in strongly heterogeneous domains.

MAGIQUE-3D program possesses an exceptional combination that is a prerequisite for accomplishing its mission: The investigator backgrounds, research interests, and technical skills complement to form a research team with a potential for significant impact on the computational infrastructure of geophysical sciences. The research record of MAGIQUE-3D group covers a large spectrum of accomplishments in the field of wave propagation including (a) the design, validation, and performance assessment of a class of DG-methods for solving efficiently high frequency wave problems, (b) the construction, convergence analysis, and performance assessment of various absorbing-type boundary conditions that are key ingredients for solving problems in infinite domains, and (c) the development of asymptotic models that are the primary candidate in the presence of heterogeneities that are small compared to the wave length. MAGIQUE-3D has built strong collaborations and partnerships with various institutions including (a) local industry (TOTAL), (b) national research centers (ONERA and CEA), and (c) international academic partnerships (e.g. Interdisciplinary Research Institute for the Sciences (IRIS) at California State University, Northridge, USA; University of Pays Basque at Bilbao, Spain; University of Novosibirsk, Russia).

3. Research Program

3.1. Introduction

Probing the invisible is a quest that is shared by a wide variety of scientists such as archaeologists, geologists, astrophysicists, physicists, etc... Magique-3D is involved in Geophysical imaging which aims at understanding the internal structure of the Earth from the propagation of waves. Both qualitative and quantitative information are required and two geophysical techniques can be used: **seismic reflection** and **seismic inversion**. Seismic reflection provides a qualitative description of the subsurface from reflected seismic waves by indicating the position of the reflectors while seismic inversion transforms seismic reflection data into a quantitative description of the subsurface. Both techniques are inverse problems based upon the numerical solution of

wave equations. Oil and Gas explorations have been pioneering application domains for seismic reflection and inversion and even if numerical seismic imaging is computationally intensive, oil companies promote the use of numerical simulations to provide synthetic maps of the subsurface. This is due to the tremendous progresses of scientific computing which have pushed the limits of existing numerical methods and it is now conceivable to tackle realistic 3D problems. However, mathematical wave modeling has to be well-adapted to the region of interest and the numerical schemes which are employed to solve wave equations have to be both accurate and scalable enough to take full advantage of parallel computing. Today, geophysical imaging tackles more and more realistic problems and we can contribute to this task by improving the modeling and by deriving advanced numerical methods for solving wave problems.

Magique-3D proposes to organize its research around three main axes:

1. Mathematical modeling of multi-physics involving wave equations;
2. Supercomputing for Helmholtz problems;
3. Construction of high-order hybrid schemes.

These three research fields will be developed with the main objective of solving inverse problems dedicated to geophysical imaging.

3.2. Mathematical modeling of multi-physics involving wave equations

Wave propagation modeling is of great interest for many applications like oil and gas exploration, non destructive testing, medical imaging, etc. It involves equations which can be solved in time or frequency domain and their numerical approximation is not easy to handle, in particular when dealing with real-world problems. In both cases, the propagation domain is either infinite or its dimensions are much greater than the characteristic wavelength of the phenomenon of interest. But since wave problems are hyperbolic, the physical phenomenon can be accurately described by computing solutions in a bounded domain including the sources which have generated the waves. Until now, we have mainly worked on imaging techniques based on acoustic or elastic waves and we have developed advanced finite element software packages which are used by Total for oil exploration. Nevertheless, research on modeling must go on because there are simulations which can still not be performed because their computational cost is much too high. This is particularly true for complex tectonics involving coupled wave equations. We then propose to address the issue of coupling wave equations problems by working on the mathematical construction of reduced systems. By this way, we hope to improve simulations of elastoacoustic and electroseismic phenomena and then, to perform numerical imaging of strongly heterogeneous media. Even in the simplest situation where the wavelengths are similar (elasto-acoustic coupling), the dimension of the discrete coupled problem is huge and it is a genuine issue in the prospect of solving 3D inverse problems.

The accurate numerical simulation of full wave problems in heterogeneous media is computationally intensive since it needs numerical schemes based on grids. The size of the cells depends on the propagation velocity of waves. When coupling wave problems, conversion phenomena may occur and waves with very different propagation velocity coexist. The size of the cells is then defined from the smallest velocity and in most of the real-world cases, the computational cost is crippling. Regarding existing computing capabilities, we propose to derive intermediate models which require less computational burden and provide accurate solutions for a wide-ranging class of problems including Elasto-acoustics and Electro-seismology.

When it comes to mathematical analysis, we have identified two tasks which could help us simulate realistic 3D multi-physics wave problems and which are in the scope of our *savoir-faire*. They are construction of approximate and multiscale models which are different tasks. The construction of approximate problems aims at deriving systems of equations which discrete formulation involves middle-sized matrices and in general, they are based on high frequency hypothesis. Multiscale models are based on a rigorous analysis involving a small parameter which does not depend on the propagation velocity necessarily.

Recently, we have conducted research on the construction of approximate models for offshore imaging. Elastic and acoustic wave equations are coupled and we investigate the idea of eliminating the computations inside water by introducing equivalent interface conditions on the sea bottom. We apply an On-Surface-Radiation-Condition (OSRC) which is obtained from the approximation of the acoustic Dirichlet-to-Neumann (DtN) operator [90], [68]. To the best of our knowledge, OSRC method has never been used for solving reduced coupling wave problems and preliminary promising results are available at [71]. We would like to investigate this technique further because we could form a battery of problems which can be solved quickly. This would provide a set of solutions which we could use as initial guess for solving inverse problems. But we are concerned with the performance of the OSRC method when wave conversions with different wavelengths occur. Anyway, the approximation of the DtN operator is not obvious when the medium is strongly heterogeneous and multiscale analysis might be more adapted. For instance, according to existing results in Acoustics and Electromagnetism for the modeling of wire antennas [80], multiscale analysis should turn out to be very efficient when the propagation medium includes well logs, fractures and faults which are very thin structures when compared to the wavelength of seismic waves. Moreover, multiscale analysis should perform well when the medium is strongly oscillating like porous media. It could thus provide an alternative to homogenization techniques which can be applied only when the medium is periodic. We thus propose to develop reduced multi-scale models by performing rigorous mathematical procedure based on regular and singular multiscale analysis. Our approach distinguishes itself from others because it focuses on the numerical representation of small structures by time-dependent problems. This could give rise to the development of new finite element methods which would combine DG approximations with XFEM (Extended Finite Element Method) which has been created for the finite element treatment of thin structures like cracks.

But Earth imaging must be more than using elasto-acoustic wave propagation. Electromagnetic waves can also be used and in collaboration with Prof. D. Pardo (Iker Basque Foundation and University of Bilbao), we conduct researches on passive imaging to probe boreholes. Passive imaging is a recent technique of imaging which uses natural electromagnetic fields as sources. These fields are generated by hydromagnetic waves propagating in the magnetosphere which transform into electromagnetic waves when they reach the ionosphere. This is a mid-frequency imaging technique which applies also to mineral and geothermal exploration, to predict seismic hazard or for groundwater monitoring. We aim at developing software package for resistivity inversion, knowing that current numerical methods are not able to manage 3D inversion. We have obtained results based on a Petrov-Galerkin approximation [65], but they are limited to 2D cases. We have thus proposed to reduce the 3D problem by using 1D semi-analytic approximation of Maxwell equations [95]. This work has just started in the framework of a PhD thesis and we hope that it will give us the possibility of imaging 3D problems.

Magique-3D would like to expand its know-how by considering electro-seismic problems which are in the scope of coupling electromagnetic waves with seismic waves. Electro-seismic waves are involved in porous media imaging which is a tricky task because it is based on the coupling of waves with very different wavelengths described by Biot equations and Maxwell equations. Biot equations govern waves in saturated porous media and they represent a complex physical phenomenon involving a slow wave which is very difficult to simulate numerically. In [88], interesting results have been obtained for the simulation of piezoelectric sensors. They are based on a quasi-static approximation of the Maxwell model coupled with Elastodynamics. Now, we are concerned with the capability of using this model for Geophysical Imaging and we believe that the derivation and/or the analysis of suitable modelings is necessary. Collaborations with Geophysicists are thus mandatory in the prospect of using both experimental and numerical approaches. We would like to collaborate with Prof. C. Bordes and Prof. D. Brito (Laboratory of Complex Fluids and their Reservoirs, CNRS and University of Pau) who have efficient experimental devices for the propagation of electromagnetic waves inside saturated porous media [70]. This collaboration should be easy to organize since Magique-3D has a long-term experience in collaborating with geophysicists. We then believe that we will not need a lot of time to get joint results since we can use our advanced software packages Hou10ni and Montjoie and our colleagues have already obtained data. Electro-seismology is a very challenging research domain for us and we would like to enforce our collaborations with IsTerre (Institute of Earth Science, University of Grenoble) and for that topic with Prof. S. Garambois who is an expert in Electro-seismology [97], [98], [85], [86]. A joint research program could gather Geophysicists from the University of Pau and from IsTerre and Magique-3D.

In particular, it would be interesting to compare simulations performed with Hou10ni, Montjoie, with the code developed by Prof. S. Garambois and to use experimental simulations for validation.

3.3. Supercomputing for Helmholtz problems

Probing invisible with harmonic equations is a need for many scientists and it is also a topic offering a wealth of interesting problems for mathematicians. It is well-known that Helmholtz equations discretization is very sensitive to the frequency scale which can be wide-ranging for some applications. For example, depth imaging is searching for deeper layers which may contain hydrocarbons and frequencies must be of a few tens of Hertz with a very low resolution. If it is to detect hidden objects, the depth of the explored region does not exceed a few tens of meters and frequencies close to the kiloHertz are used. High performing numerical methods should thus be stable for a widest as possible frequency range. In particular, these methods should minimize phenomena of numerical pollution that generate errors which increase faster with frequency than with the inverse of space discretization step. As a consequence, there is a need of mesh refinement, in particular at high frequency.

During the period 2010-2014, the team has worked extensively on high order discontinuous Galerkin (DG) methods. Like standard Finite Element Methods, they are elaborated with polynomial basis functions and they are very popular because they are defined locally for each element. It is thus easy to use basis polynomial functions with different degrees and this shows the perfect flexibility of the approximation in case of heterogeneous media including homogeneous parts. Indeed, low degree basis functions can be used in heterogeneous regions where a fine grid is necessary while high degree polynomials can be used for coarse elements covering homogeneous parts. In particular, Magique-3D has developed Hou10ni that solves harmonic wave equations with DG methods and curved elements. We found that both the effects of pollution and dispersion, which are very significant when a conventional finite element method is used, are limited [72]. However, bad conditioning is persisting and reliability of the method is not guaranteed when the coefficients vary considerably. In addition, the number of unknowns of the linear system is too big to hope to solve a realistic 3D problem. So it is important to develop approximation methods that require fewer degrees of freedom. Magique-3D wishes to invest heavily in the development of new approximation methods for harmonic wave equations. It is a difficult subject for which we want to develop different tasks, in collaboration with academic researchers with whom we are already working or have established contacts. Research directions that we would like to follow are the following.

First, we will continue our long-term collaboration with Prof. Rabia Djellouli. We want to continue to work on hybrid finite element methods that rely on basis functions composed of plane waves and polynomials. These methods have demonstrated good resistance to the phenomenon of numerical pollution [66], [67], but their capability of solving industrial problems has not been illustrated. This is certainly due to the absence of guideline for choosing the plane waves. We are thus currently working on the implementation of a methodology that makes the choice of plane waves automatic for a given simulation (fixed propagation domain, data source, etc.). This is up-front investigation and there is certainly a lot of remaining work before being applied to geophysical imaging. But it gives the team the opportunity to test new ideas while remaining in contact with potential users of the methods.

Then we want to work with Prof. A. Bendali on developing methods of local integral equations which allow calculation of numerical fluxes on the edges of elements. One could then use these fluxes in a DG method for reconstructing the solution throughout the volume of calculation. This research is motivated by recent results which illustrate the difficulties of the existing methods which are not always able to approximate the propagating modes (plane waves) and the evanescent modes (polynomials) that may coexist, especially when one considers realistic applications. Integral equations are direct tools for computing fluxes and they are known for providing very good accuracy. They thus should help to improve the quality of approximation of DG methods which are fully flux-dependent. In addition, local integral equations would limit calculations at the interfaces, which would have the effect of limiting the number of unknowns generally high, especially for DG methods. Again, it is a matter of long-term research which success requires a significant amount of mathematical analysis, and also the development of non-trivial code.

To limit the effects of pollution and dispersion is not the only challenge that the team wants to tackle. Our experience alongside Total has made us aware of the difficulties in constructing meshes that are essential to achieve our simulations. There are several teams at Inria working on mesh generation and we are in contact with them, especially with Gamma3 (Paris-Rocquencourt Research Center). These teams develop meshes increasingly sophisticated to take account of the constraints imposed by realistic industrial benchmarks. But in our opinion, issues which are caused by the construction of meshes are not the only downside. Indeed, we have in mind to solve inverse problems and in this case it is necessary to mesh the domain at each iteration of Newton-type solver. It is therefore interesting to work on methods that either do not use mesh or rely on meshes which are very easy to construct. Regarding meshless methods, we have begun a collaboration with Prof. Djellouli which allowed us to propose a new approach called Mesh-based Frontier Free Formulation (MF3). The principle of this method is the use of fundamental solutions of Helmholtz equations as basic functions. One can then reduce the volumic variational formulation to a surfacic variational formulation which is close to an integral equation, but which does not require the calculation of singularities. The results are very promising and we hope to continue our study in the context of the application to geophysical imaging. An important step to validate this method will be particularly its extension to 3D because the results we have achieved so far are for 2D problems.

Keeping in mind the idea of limiting the difficulties of mesh, we want to study the method of virtual elements. This method attracts us because it relies on meshes that can be made of arbitrarily-shaped polygon and meshes should thus be fairly straightforward. Existing works on the subject have been mainly developed by the University of Pavia, in collaboration with Los Alamos National Laboratory [69], [76], [75], [73], [77]. None of them mentions the feasibility of the method for industrial applications and to our knowledge, there are no results on the method of virtual elements applied to the wave equations. First, we aim at applying the method described in [74] to the scalar Helmholtz equation and explore opportunities to use discontinuous elements within this framework. Then hp-adaptivity could be kept, which is particularly interesting for wave propagation in heterogeneous media.

DG methods are known to require a lot of unknowns that can exceed the limits accepted by the most advanced computers. This is particularly true for harmonic wave equations that require a large number of discretization points, even in the case of a conventional finite element method. We therefore wish to pursue a research activity that we have just started in collaboration with the project-team Nachos (Sophia-Antipolis Méditerranée Research Center). In order to reduce the number of degrees of freedom, we are interested in "hybrid mixed" Discontinuous Galerkin methods that provides a two-step procedure for solving the Helmholtz equations [89], [94], [92]. First, Lagrange multipliers are introduced to represent the flux of the numerical solution through the interface (edge or face) between two elements. The Lagrange multipliers are solution to a linear system which is constructed locally element by element. The number of degrees of freedom is then strongly reduced since for a standard DG method, there is a need of considering unknowns including volumetric values inside the element. And obviously, the gain is even more important when the order of the element is high. Next, the solution is reconstructed from the values of the multipliers and the cost of this step is negligible since it only requires inverting small-sized matrices. We have obtained promising results in the framework of the PhD thesis of Marie Bonnasse Gahot and we want to apply it to the simulation of complex phenomena such as the 3D viscoelastic wave propagation.

Obviously, the success of all these works depends on our ability to consider realistic applications such as wave propagation in the Earth. And in these cases, it is quite possible that even if we manage to develop accurate less expensive numerical methods, the solution of inverse problems will still be computationally intensive. It is thus absolutely necessary that we conduct our research by taking advantage of the latest advances in high-performance computing. We have already initiated discussions with the project team HIEPACS (Bordeaux Sud-Ouest research Center) to test the performance of the latest features of Mumps <http://mumps.enseciht.fr/>, such as Low Rank Approximation or adaptation to hybrid CPU / GPU architectures and to Intel Xeon Phi, on realistic test cases. We are also in contact with the team Algorithm at Cerfacs (Toulouse) for the development of local integral equations solvers. These collaborations are essential for us and we believe that they will be decisive for the simulation of three-dimensional elastodynamic problems. However, our scientific contribution will be limited in this area because we are not experts in HPC.

3.4. Hybrid time discretizations of high-order

Most of the meshes we consider are composed of cells greatly varying in size. This can be due to the physical characteristics (propagation speed, topography, ...) which may require to refine the mesh locally, very unstructured meshes can also be the result of dysfunction of the mesher. For practical reasons which are essentially guided by the aim of reducing the number of matrix inversions, explicit schemes are generally privileged. However, they work under a stability condition, the so-called Courant Friedrichs Lewy (CFL) condition which forces the time step being proportional to the size of the smallest cell. Then, it is necessary to perform a huge number of iterations in time and in most of the cases because of a very few number of small cells. This implies to apply a very small time step on grids mainly composed of coarse cells and thus, there is a risk of creating numerical dispersion that should not exist. However, this drawback can be avoided by using low degree polynomial basis in space in the small meshes and high degree polynomials in the coarse meshes. By this way, it is possible to relax the CFL condition and in the same time, the dispersion effects are limited. Unfortunately, the cell-size variations are so important that this strategy is not sufficient. One solution could be to apply implicit and unconditionally stable schemes, which would obviously free us from the CFL constraint. Unfortunately, these schemes require inverting a linear system at each iteration and thus needs huge computational burden that can be prohibitive in 3D. Moreover, numerical dispersion may be increased. Then, as second solution is the use of local time stepping strategies for matching the time step to the different sizes of the mesh. There are several attempts [81], [78], [96], [91], [84] and Magique 3D has proposed a new time stepping method which allows us to adapt both the time step and the order of time approximation to the size of the cells. Nevertheless, despite a very good performance assessment in academic configurations, we have observed to our detriment that its implementation inside industrial codes is not obvious and in practice, improvements of the computational costs are disappointing, especially in a HPC framework. Indeed, the local time stepping algorithm may strongly affect the scalability of the code. Moreover, the complexity of the algorithm is increased when dealing with lossy media [87].

Recently, Dolean *et al* [83] have considered a novel approach consisting in applying hybrid schemes combining second order implicit schemes in the thin cells and second order explicit discretization in the coarse mesh. Their numerical results indicate that this method could be a good alternative but the numerical dispersion is still present. It would then be interesting to implement this idea with high-order time schemes to reduce the numerical dispersion. The recent arrival in the team of J. Chabassier should help us to address this problem since she has the expertise in constructing high-order implicit time scheme based on energy preserving Newmark schemes [79]. We propose that our work be organized around the two following tasks. The first one is the extension of these schemes to the case of lossy media because applying existing schemes when there is attenuation is not straightforward. This is a key issue because there is artificial attenuation when absorbing boundary conditions are introduced and if not, there are cases with natural attenuation like in visco-elastic media. The second one is the coupling of high-order implicit schemes with high-order explicit schemes. These two tasks can be first completed independently, but the ultimate goal is obviously to couple the schemes for lossy media. We will consider two strategies for the coupling. The first one will be based on the method proposed by Dolean *et al*, the second one will consist in using Lagrange multiplier on the interface between the coarse and fine grids and write a novel coupling condition that ensures the high order consistency of the global scheme. Besides these theoretical aspects, we will have to implement the method in industrial codes and our discretization methodology is very suitable for parallel computing since it involves Lagrange multipliers. We propose to organize this task as follows. There is first the crucial issue of a systematic distribution of the cells in the coarse/explicit and in the fine/implicit part. Based on our experience on local time stepping, we claim that it is necessary to define a criterion which discriminates thin cells from coarse ones. Indeed, we intend to develop codes which will be used by practitioners, in particular engineers working in the production department of Total. It implies that the code will be used by people who are not necessarily experts in scientific computing. Considering real-world problems means that the mesh will most probably be composed of a more or less high number of subsets arbitrarily distributed and containing thin or coarse cells. Moreover, in the prospect of solving inverse problems, it is difficult to assess which cells are thin or not in a mesh which varies at each iteration.

Another important issue is the load balancing that we can not avoid with parallel computing. In particular, we will have to choose one of these two alternatives: dedicate one part of processors to the implicit computations and the other one to explicit calculus or distribute the resolution with both schemes on all processors. A collaboration with experts in HPC is then mandatory since we are not expert in parallel computing. We will thus continue to collaborate with the team-projects Hiepac and Runtime with whom we have a long-term experience of collaborations. The load-balancing leads then to the issue of mesh partitioning. Main mesh partitioners are very efficient for the coupling of different discretizations in space but to the best of our knowledge, the case of non-uniform time discretization has never been addressed. The study of meshes being out of the scopes of Magique-3D, we will collaborate with experts on mesh partitioning. We get already on to François Pellegrini who is the principal investigator of Scotch (<http://www.labri.fr/perso/pelegrin/scotch>) and permanent member of the team project Bacchus (Inria Bordeaux Sud Ouest Research Center).

In the future, we aim at enlarging the application range of implicit schemes. The idea will be to use the degrees of freedom offered by the implicit discretization in order to tackle specific difficulties that may appear in some systems. For instance, in systems involving several waves (as P and S waves in porous elastic media, or coupled wave problems as previously mentioned) the implicit parameter could be adapted to each wave and optimized in order to reduce the computational cost. More generally, we aim at reducing numeric bottlenecks by adapting the implicit discretization to specific cases.

4. Application Domains

4.1. Seismic Imaging

The main objective of modern seismic processing is to find the best representation of the subsurface that can fit the data recorded during the seismic acquisition survey. In this context, the seismic wave equation is the most appropriate mathematical model. Numerous research programs and related publications have been devoted to this equation. An acoustic representation is suitable if the waves propagate in a fluid. But the subsurface does not contain fluids only and the acoustic representation is not sufficient in the general case. Indeed the acoustic wave equation does not take some waves into account, for instance shear waves, turning waves or the multiples that are generated after several reflections at the interfaces between the different layers of the geological model. It is then necessary to consider a mathematical model that is more complex and resolution techniques that can model such waves. The elastic or viscoelastic wave equations are then reference models, but they are much more difficult to solve, in particular in the 3D case. Hence, we need to develop new high-performance approximation methods.

Reflection seismics is an indirect measurement technique that consists in recording echoes produced by the propagation of a seismic wave in a geological model. This wave is created artificially during seismic acquisition surveys. These echoes (i.e., reflections) are generated by the heterogeneities of the model. For instance, if the seismic wave propagates from a clay layer to sand, one will observe a sharp reflected signal in the seismic data recorded in the field. One then talks about reflection seismics if the wave is reflected at the interface between the two media, or talks about seismic refraction if the wave is transmitted along the interface. The arrival time of the echo enables one to locate the position of this transition, and the amplitude of the echo gives information on some physical parameters of the two geological media that are in contact. The first petroleum exploration surveys were performed at the beginning of the 1920's and for instance, the Orchard Salt Dome in Texas (USA) was discovered in 1924 by the seismic-reflection method.

4.2. Modeling of Multiperforated plates in turboreactors

In the turbo-engine, the temperature can reach 2000 K inside the combustion chamber. To protect its boundary, "fresh" air at 800 K is injected through thousands of perforations. The geometry of the network of perforations is chosen in order to optimize the cooling and the mechanical properties of the chamber. It has been experimentally observed that these perforations have a negative impact on the stability of the combustion.

This is due to the interaction with an acoustic wave generated by the combustion. Due to the large number of holes (2000) and their small sizes (0.5 mm) with respect to the size of the combustion chamber (50 cm), it is not conceivable to rely on numerical computations (even with supercomputers) to predict the influence of these perforations.

In collaboration with ONERA, we develop new models which allow to take into account these multiperforated plates at the macroscopic scale.

4.3. Helioseismology

This collaboration with the Max Planck Institute for solar system, which started in 2014, aims at designing efficient numerical methods for the wave propagation problems that arise in helioseismology in the context of inverse problems. The final goal is to retrieve information about the structure of the sun i.e. inner properties such as density or pressure via the inversion of a wave propagation problem. Acoustic waves propagate inside the sun which, in a first approximation and regarding the time scales of physical phenomena, can be considered as a moving fluid medium with constant velocity of motion. Some other simplifications lead to computational saving, such as supposing a radial or axisymmetric geometry of the sun. Aeroacoustic equations must be adapted and efficiently solved in this context, this has been done in the finite elements code Montjoie 5.3. In other situations, a full 3D simulation is required and demands large computational resources. Ultimately, we aim at modeling the coupling with gravity potential and electromagnetic waves (MHD equations) in order to be able to better understand sun spots.

5. New Software and Platforms

5.1. Elasticus

SCIENTIFIC DESCRIPTION

Elasticus simulates acoustic and elastic wave propagation in 2D and in 3D, formulated as a first order system, using Discontinuous Galerkin Methods. The space discretization is based on two kind of basis functions, using Lagrange or Jacobi polynomials. Different kinds of fluxes (upwind and centered) are implemented, coupled with RK2 and RK4 time schemes.

FUNCTIONAL DESCRIPTION

Elasticus is a sequential library, independent of DIVA and developed in Fortran, to simulate wave propagation in geophysical environment, based on a DG method. It is meant to help PhD students and post-doctoral fellows to easily implement their algorithms in the library. Thus, readability of the code is privileged over optimization of its performances. Developed features should be easily transferred in the computing platform of Total. Contrary to DIVA which only computes approximate solutions with P1, P2 and P3 elements, Elasticus manages arbitrary orders for the spatial discretization with DG method.

IMPROVEMENT

The various kinds of fluxes and the RK4 time schemes were implemented by Simon Ettouati. The elasto-acoustic coupling was implemented by Elvira Shishenina in the framework of her Master internship, in collaboration with Simon Ettouati and Lionel Boillot. The TTI elastic kernel as well as the Absorbing Boundary Conditions were developed by Lionel Boillot.

- Participants: Simon Ettouati, Julien Diaz, Lionel Boillot and Elvira Shishenina.
- Partner: TOTAL
- Contact: Julien Diaz

5.2. Hou10ni

SCIENTIFIC DESCRIPTION

Hou10ni simulates acoustic and elastic wave propagation in time domain and in harmonic domain, in 2D and in 3D, formulated as a second order system. It is also able to model elasto acoustic coupling. It is based on the second order formulation of the wave equation and the space discretization is achieved using Interior Penalty Discontinuous Galerkin (IPDG) Method. Recently, the harmonic domain solver has been extended to handle Hybridizable Discontinuous Galerkin Methods.

FUNCTIONAL DESCRIPTION

This software simulates the propagation of waves in heterogeneous 2D and 3D media in time-domain and in frequency domain. It is based on an Interior Penalty Discontinuous Galerkin Method (IPDGM) and allows for the use of meshes composed of cells of various order (p-adaptivity in space).

IMPROVEMENT

The main improvements are related to the frequency domain part. The Hybridizable Discontinuous Galerkin was implemented in 2D and in 3D elastodynamics by Marie Bonnasse Gahot in the framework of her PhD thesis. The IPDG Method was implemented in 3D acoustics and for the 3D elasto-acoustic coupling by Conrad Hillairet in the framework of his Master thesis. The Perfectly Matched Layers were optimized (the length of the layers and the amplitude of the absorption parameters are now automatically computed) by Andrew Wang in the framework of his two months internship.

- Participants: Julien Diaz, Marie Bonnasse Gahot, Conrad Hillairet and Lionel Boillot
- Contact: Julien Diaz

5.3. Montjoie

SCIENTIFIC DESCRIPTION

Montjoie is designed for the efficient solution of time-domain and time-harmonic linear partial differential equations using high-order finite element methods. This code is mainly written for quadrilateral/hexahedral finite elements, partial implementations of triangular/tetrahedral elements are provided. The equations solved by this code, come from the "wave propagation" problems, particularly acoustic, electromagnetic, aeroacoustic, elastodynamic problems.

FUNCTIONAL DESCRIPTION

Montjoie is a code that provides a C++ framework for solving partial differential equations on unstructured meshes with finite element-like methods (continuous finite element, discontinuous Galerkin formulation, edge elements and facet elements). The handling of mixed elements (tetrahedra, prisms, pyramids and hexahedra) has been implemented for these different types of finite elements methods. Several applications are currently available : wave equation, elastodynamics, aeroacoustics, Maxwell's equations.

IMPROVEMENT

This year, new high-order schemes (internship of Guillaume Marty and thesis of Mamadou N'diaye) have been implemented. The code has been strongly modified in order to obtain a fast compilation. The software has been used by Chloe team (through the internship of Laurene Hume) and compared to COMSOL providing a similar efficiency.

- Participants: Marc Duruflé, Juliette Chabassier, Mamadou N'diaye, Guillaume Marty
- Contact: Marc Duruflé
- URL: <http://montjoie.gforge.inria.fr/>

5.4. TMBM-DG

SCIENTIFIC DESCRIPTION

TMBM-DG simulates acoustic and elastic wave propagation in 2D and in 3D, formulated as a first order system, using Discontinuous Galerkin Methods. The space discretization is based on two kinds of basis functions, using Lagrange or Jacobi polynomials coupled with RK2 and RK4 time schemes.

FUNCTIONAL DESCRIPTION

TMBM-DG is the follow up to DIVA-DG that we develop in collaboration with our partner Total. Its purpose is more general than DIVA-DG and should contains various DG schemes, basis functions and time schemes. It models wave propagation in acoustic media, elastic (isotropic and TTI) media and elasto-acoustic media, in two and three dimensions.

IMPROVEMENT

The first version of the code was recently developed jointly with our industrial partner Total. The main developer in *MAGIQUE-3D* is Lionel Boillot.

- Participants: Lionel Boillot, Julien Diaz and Simon Ettouati
- Partner: TOTAL
- Contact: Julien Diaz

6. New Results

6.1. Seismic Imaging and Inverse Problems

6.1.1. *hp-adaptive simulation and inversion of magnetotelluric measurements*

Participants: H el ene Barucq, Julien Alvarez Aramberri, David Pardo.

The magnetotelluric (MT) method is a passive exploration technique that aims at estimating the resistivity distribution of the Earth's subsurface, and therefore at providing an image of it. This process is divided into two different steps. The first one consists in recording the data. In a second step, recorded measurements are analyzed by employing numerical methods. In this work, we provide a rigorous mathematical setting in the context of the Finite Element Method (FEM) that helps to understand the MT problem and its inversion process. In order to recover a map of the subsurface based on 2D MT measurements, we employ for the first time in MTs a multigoal oriented self adaptive hp-Finite Element Method (FEM). We accurately solve both the full formulation as well as a secondary field formulation where the primary field is given by the solution of a 1D layered medium. To truncate the computational domain, we design a Perfectly Matched Layer (PML) that automatically adapts to high-contrast material properties that appear within the subsurface and on the air-ground interface. For the inversion process, we develop a first step of a Dimensionally Adaptive Method (DAM) by considering the dimension of the problem as a variable in the inversion. Additionally, this dissertation supplies a rigorous numerical analysis for the forward and inverse problems. Regarding the forward modelization, we perform a frequency sensitivity analysis, we study the effect of the source, the convergence of the hp-adaptivity, or the effect of the PML in the computation of the electromagnetic fields and impedance. As far as the inversion is concerned, we study the impact of the selected variable for the inversion process, the different information that each mode provides, and the gains of the DAM approach.

6.1.2. *Ultrasonic imaging of complex media*

Participants: H el ene Barucq, Juliette Chabassier, Marc Durufl e, Julien Diaz, S ebastien Tordeux, Ha Howard Faucher.

In 2015 we have begun a collaborating project with I2M (Physics Acoustics Department of Bordeaux 1 University). We aim at modeling and simulating efficiently the propagation of acoustic waves and later elastodynamic waves in highly heterogeneous media, the final goal is to use topological gradient imaging techniques. Classical techniques as finite elements can be too costly, we propose to design more efficient numerical techniques that exploit the fact that the wavelength is big with respect to the heterogeneities. For instance, we will use numerical upscaling, multiscale homogenization or asymptotic methods. A funding has been obtained for a PhD and a post doctoral position, that have both started in 2015. Our first step is to design a laboratory experiment and a simulation code in order to challenge the limits of the newly derived models and quantify their validity.

6.1.3. Impedance transmission conditions for the electric potential across a highly conductive casing

Participants: H el ene Barucq, Aralar Erdozain, David Pardo, Victor P eron.

In this study we present Impedance Transmission Conditions (ITCs) for the electric potential in the framework of borehole through-casing resistivity measurements. Such ITCs substitute the part of the domain corresponding to a highly conductive casing. The naturally small thickness of the casing makes it ideal for exhibiting ITCs. We numerically observe the delivered order of accuracy.

6.1.4. An efficient truncated SVD of large matrices based on the low-rank approximation for inverse geophysical problems

Participant: S ebastien Tordeux.

We have proposed a new algorithm to compute a truncated singular value decomposition of the Born matrix based on a low-rank arithmetic. Theoretical background to the low-rank SVD method has been investigated: the Born matrix of an acoustic problem can be approximated by a low-rank approximation derived thanks to a kernel independent multipole expansion. The new algorithm to compute T-SVD approximation consists of four steps, and they are described in detail. The largest singular values and their left and right singular vectors can be approximated numerically without performing any operation with the full matrix. The low-rank approximation is computed due to a dynamic panel strategy of cross approximation technique.

6.1.5. Handling clusters with a task-based runtime system: application to Geophysics

Participants: Emmanuel Agullo, H el ene Barucq, Lionel Boillot, George Bosilca, Julien Diaz.

The extreme complexity of hardware platforms makes them harder and harder to program. To fully exploit such machines, the High Performance Computing community often uses a MPI + X (X being pthreads, OpenMP, Cuda ...) programming models. We propose to use an alternative solution consisting of programming at a higher level of abstractions by describing a scientific, high performance computing application as a sequence of tasks whose execution is delegated to a runtime system. We compared MPI-based version and task-based version on Geophysics simulations, especially on the DIVA code of Total. Our previous studies demonstrated the task-based paradigm superiority on shared memory architectures (CPU or MIC), we are now working on distributed and heterogeneous architectures (CPUs+MICs) and, according to our preliminary results, the performances are still better than the MPI-version.

This work has been presented to the conferences PRACEdays [60], Rice Oil&Gas [43] and PASC [37].

6.2. Mathematical modeling of multi-physics involving wave equations

6.2.1. Elasto-acoustic coupling

Participants: H el ene Barucq, Lionel Boillot, Henri Calandra, Julien Diaz, Simon Ettouati, Conrad Hillairet, Elvira Shishenina.

In the framework of her Master thesis, Elvira Shishenina developed a Discontinuous Galerkin Method for the elastoacoustic coupling in time domain. The proposed solution methodology in general and can be applied to any kind of fluxes. We have implemented and validated in Elasticus a centered flux version and an upwind flux version in two dimensions. The time discretization is achieved thanks to Runge Kutta schemes of second and fourth orders.

In frequency domain, Conrad Hillairet developed a 3D elasto-coupling IPDG scheme, in the framework of his Master thesis. It has been implemented and validated in Hou10ni. Moreover, the code is able to handle p -adaptivity and we have proposed a strategy in order to determine the order of the cell as a function of the size of the cell and of the physical parameters. The results of this work have been presented to the Siam Conference on Geosciences in Stanford [39] and to the XXIV Congress on Differential Equations and Applications in Cadiz [32].

Finally, we have considered elastoacoustic coupling with curved interfaces and we have proposed a solution methodology based on Finite Element techniques, which allows for a flexible coupling between the fluid and the solid domain by using non-conforming meshes and curved elements. Since characteristic waves travel at different speeds through different media, specific levels of granularity for the mesh discretization are required on each domain, making impractical a possible conforming coupling in between. Advantageously, physical domains may be independently discretized in our framework due to the non-conforming feature. Consequently, an important increase in computational efficiency may be achieved compared to other implementations based on non-conforming techniques, namely by reducing the total number of degrees of freedom. Differently from other non-conforming approaches proposed so far, our technique is relatively simpler and requires only a geometrical adjustment at the coupling interface at a preprocessing stage, so that no extra computations are necessary during the time evolution of the simulation. On the other hand, as an advantage of using curvilinear elements, the geometry of the coupling interface between the two media of interest is faithfully represented up to the order of the scheme used. In other words, higher order schemes are in consonance with higher order approximations of the geometry. Concerning the time discretization, we analyzed both explicit and implicit schemes. These schemes are energy conserving and, for the explicit case, the stability is guaranteed by a CFL condition.

This work, which has been achieved in collaboration with Angel Rodriguez Rozas, former post-doc of the team, was published in *Journal of Computational Physics* [27].

6.2.2. *Atmospheric boundary conditions for helioseismology*

Participants: H el ene Barucq, Juliette Chabassier, Marc Durufl e, Victor P eron.

The sun does not have a clear boundary like a solid ball, but it has an atmosphere which can be modeled as an exponential decay of the density. We have studied the replacement of this atmosphere by an equivalent boundary condition in order to avoid meshing the atmosphere. When we assume that the exponential decay is large enough, asymptotic modeling can be performed with respect to this large parameter. Equivalent boundary conditions have been obtained for order 1, 2 and 3, and they substantially improve Dirichlet condition (order 0) for low frequencies. However for high frequencies, these conditions are no longer relevant. We have developed a first-order absorbing boundary condition adapted to an exponential decay of the density, this last condition provides good results for the tested range of frequency. These conditions have been used by the team of Laurent Gizon (Max Planck institute) to retrieve experimental dispersion curves, so called ‘‘power spectrum’’.

6.2.3. *Absorbing Boundary Conditions for 3D elastic TTI modeling*

Participants: H el ene Barucq, Lionel Boillot, Julien Diaz.

We propose stable low-order Absorbing Boundary Conditions (ABC) for elastic TTI modeling. Their derivation is justified in elliptic TTI media but it turns out that they are directly usable to non-elliptic TTI configurations. Numerical experiments are performed by using a new elastic tensor source formula which generates P-waves only in an elliptic TTI medium. Numerical results have been performed in 3D to illustrate the performance of the ABCs.

This work has been presented to the conferences PANACM [38] and SEG [33].

6.2.4. *The airfoil equation on near disjoint intervals : Approximate models and polynomial solutions*

Participants: Leandro Farina, Marcos Ferreira, Victor P eron.

In [26], the airfoil equation is considered over two disjoint intervals. Assuming the distance between the intervals is small an approximate solution is found and relationships between this approximation and the solution of the classical airfoil equation are obtained. Numerical results show the convergence of the approximation to the solution of the original problem. Polynomial solutions for an approximate model are obtained and a spectral method for the generalized airfoil equation on near disjoint intervals is proposed.

6.2.5. *Finite element subproblem method*

Participants: Patrick Dular, Christophe Geuzaine, Laurent Kr aehenb uhl, Victor P eron.

In [25], progressive refinements of inductors are done with a subproblem method, from their wire or filament representations with Biot-Savart models up to their volume finite-element models, from statics to dynamics. The reaction fields of additional magnetic and/or conducting regions are also considered. Accuracy improvements are efficiently obtained for local fields and global quantities, i.e., inductances, resistances, Joule losses, and forces.

6.2.6. *Asymptotic study for Stokes-Brinkman model with Jump embedded transmission conditions*

Participants: Philippe Angot, Gilles Carbou, Victor P eron.

In [18], one considers the coupling of a Brinkman model and Stokes equations with jump embedded transmission conditions. Assuming that the viscosity in the porous region is very small, we derive a Wentzel-Kramers-Brillouin (WKB) expansion in power series of the square root of this small parameter for the velocity and the pressure which are solution of the transmission problem. This WKB expansion is justified rigorously by proving uniform errors estimates.

6.2.7. *On the solution of the Laplace equation in 3-D domains with cracks and elliptical edges*

Participants: Victor P eron, Samuel Shannon, Zohar Yosibash.

An explicit asymptotic solution to the elasticity system in a three-dimensional domain in the vicinity of an elliptical crack front, or for an elliptical sharp V-notch is still unavailable. Towards its derivation we first consider in [30] the explicit asymptotic solutions of the Laplace equation in the vicinity of an elliptical singular edge in a three-dimensional domain. Both homogeneous Dirichlet and Neumann boundary conditions on the surfaces intersecting at the elliptical edge are considered. The dual singular solution is also provided to be used in a future study to extract the edges flux intensity functions by the quasi-dual function method. We show that just as for the circular edge case, the solution in the vicinity of an elliptical edge is composed of three series, with eigenfunctions being functions of two coordinates.

In [29] the singular solution of the Laplace equation with a straight-crack is represented by a series of eigenpairs, shadows and their associated edge flux intensity functions (EFIFs). We address the computation of the EFIFs associated with the integer eigenvalues by the quasi dual function method (QDFM). The QDFM is based on the dual eigenpairs and shadows, and we show that the dual shadows associated with the integer eigenvalues contain logarithmic terms. These are then used with the QDFM to extract EFIFs from p-version finite element solutions. Numerical examples are provided.

6.3. Supercomputing for Helmholtz problems

6.3.1. *High order methods for Helmholtz problems in highly heterogeneous media*

Participants: Th ophile Chaumont-Frelet, Henri Calandra, H el ne Barucq, Christian Gout.

The numerical solution of Helmholtz problems set in highly heterogeneous media is a tricky task. Classical high order discretizations fail to handle such propagation media, because they are not able to capture any of the scales of the velocity parameter. Indeed, they are build upon coarse meshes and therefore, if the velocity parameter is taken to be constant in each cell (through averaging, or local homogenization strategy), scale information is (at least partially) lost. We propose to overcome this difficulty by introducing a multiscale medium approximation strategy. The velocity parameter is not assumed to be constant on each cell, but on a submesh of each cell. If the submeshes are designed properly, the medium approximation method is equivalent to a quadrature formula, adapted to the medium. In particular, we show that this methodology has roughly the same computational cost as the classical finite element method. This new solution methodology has been presented in a paper under revision. We have performed a mathematical analysis of the multiscale medium approximation techniques to higher order discretization. First, we show that the heterogeneous Helmholtz problem is well-posed and derive stability estimates with respect to the right hand side, and with respect to variations of the velocity parameter, justifying the use of medium approximation. Those results are obtained assuming the velocity parameter is monotonous and that the propagation medium is closed by first order

absorbing boundary conditions. However, these hypothesis are not mandatory to discretize the problem. Second, we turn to the analysis of finite element schemes with subcell variations of the velocity. In particular, we show that even if the solution can be rough inside each cell because of velocity jumps, we are able to extend the asymptotic error estimates obtained in [93] to heterogeneous media with non-matching mesh in case of elements of order $1 \leq p \leq 3$. Third, we investigate numerically the stability of the scheme when the frequency is increasing to figure out optimal meshing conditions. We show that in simple media, the optimal homogeneous pre-asymptotic error estimates are still valid. However, in more complex cases, it looks like this condition is not sufficient anymore. Apart from showing that the homogeneous results are not always applicable to the heterogeneous Helmholtz equation, we are not able to give a clear answer to the question. Finally, we are able to conclude that high order methods are actually interesting: in our examples, $p = 4$ discretizations always yield a smaller linear system than lower order discretizations for the same precision.

6.3.2. Hybridizable Discontinuous Galerkin method for the elastic Helmholtz equations

Participants: Marie Bonnasse-Gahot, Henri Calandra, Julien Diaz, Stéphane Lanteri.

In the framework of the PhD thesis of Marie Bonnasse-Gahot, we have proposed an hybridizable discontinuous Galerkin method for solving the anisotropic elastodynamics wave equations in harmonic domain, in two and three dimensions. The method was implemented in Hou10ni and in the platform of Total. We have analyzed the performance of the proposed method in 2D on simple test case and compared it to classical DG methods. We have shown that the HDG method provides a more accurate solution for less computational cost provided that the order is high enough. We have illustrated the usefulness of the p -adaptivity in 2D, which allows to reach the accuracy of a global method of degree p for the costs of a global method of degree $p - 1$ or $p - 2$. This feature is already implemented in the 3D code. We now have to determine an accuracy criteria for assigning an order to a given cell, similar to the criteria we proposed in 2D.

For the numerical analysis of the scheme, we have shown that the HDG method could be rewritten as an upwind fluxes DG method and one of our perspectives is to use this equivalence in order to perform a dispersion analysis following the work of Ainsworth, Monk and Muniz [64].

We have shown that HDG could be used for 2D simulation on geophysical benchmark, and we will now implement the method in a Reverse Time Migration software, the ultimate goal being to couple HDG method with a full waveform inversion solver. In order to tackle more realistic test cases in 3D, it will be mandatory to improve the linear solver and we are now considering the use of an hybrid solver such as Maphys developed by the Inria team-project HIEPACS.

The results of this work have been presented at the ‘‘SIAM Conference on Geosciences’’ [48] and at the ‘‘Oil and Gas HPC Workshop’’ [49].

6.4. Hybrid time discretizations of high-order

6.4.1. High-order symmetric multistep schemes for wave equation

Participants: Juliette Chabassier, Marc Durufl e, Guillaume Marty.

We have studied high-order symmetric multistep schemes for the second-order formulation $y'' = f(t, y)$ during the internship of Guillaume Marty. The stability condition (CFL) can be optimized for explicit schemes since they have free parameters. However, this optimization procedure is not easy since the optimum is reached for forbidden values (values for which the high-order accuracy is no longer obtained). We have proposed acceptable values of free parameters for schemes of order 4, 6 and 8. These schemes have been tested for the wave equation, they suffer from a lack of robustness with respect to rounding numerical errors. The stability of implicit schemes has also been explored. For fourth-order schemes, a family of energy-conserving schemes has been obtained. However, we have not found unconditionally stable high-order schemes, which is well-known for the first-order formulation as Dahlquist’s barrier. It seems that for the second-order formulation, this barrier holds and only second-order accurate schemes are unconditionally stable. Implicit high-order schemes have a maximum CFL of $\sqrt{6}$, the same CFL as the standard θ -scheme with $\theta = \frac{1}{12}$. As a result, the implicit version of these schemes does not have a practical interest.

6.4.2. *High order conservative explicit and implicit schemes for wave equations.*

Participants: Juliette Chabassier, Sébastien Imperiale.

In 2015 we have studied the space/time convergence of a family of high order conservative explicit and implicit schemes for wave equations. An original proof of convergence has been proposed and provides an understanding of the lack of convergence of some schemes when the time step approaches its greatest admissible value for stability (CFL condition). An article is being written and will be submitted soon.

6.4.3. *Multi-level explicit local time-stepping methods for second-order wave equations*

Participants: Julien Diaz, Marcus Grote.

Local mesh refinement severely impedes the efficiency of explicit time-stepping methods for numerical wave propagation. Local time-stepping (LTS) methods overcome the bottleneck due to a few small elements by allowing smaller time-steps precisely where those elements are located. Yet when the region of local mesh refinement itself contains a sub-region of even smaller elements, any local time-step again will be overly restricted. To remedy the repeated bottleneck caused by hierarchical mesh refinement, multi-level local time-stepping methods are proposed, which permit the use of the appropriate time-step at every level of mesh refinement. Based on the LTS methods from Diaz and Grote [82], these multi-level LTS methods are explicit, yield arbitrarily high accuracy and conserve the energy.

The method was published in Computer Methods in Applied Mechanics and Engineering [24].

7. Bilateral Contracts and Grants with Industry

7.1. Contracts with TOTAL

- Depth Imaging Partnership (DIP)
Period: 2014 May - 2019 April , Management: Inria Bordeaux Sud-Ouest, Amount: 120000 euros/year.
- Construction de milieux équivalents en vue de la simulation d'ondes élastiques harmoniques en milieux fortement hétérogènes par des méthodes DG
Period: 2012 October - 2015 October, Management: Inria Bordeaux Sud-Ouest, Amount: 165000 euros
- Simulation de la propagation d'ondes élastiques et visco-élastiques en régime harmonique par des méthodes Galerkin discontinues d'ordre élevé en maillage non structuré adaptées au calcul haute-performance.
Period: 2012 October - 2015 October, Management: Inria Bordeaux Sud-Ouest, Amount: 165000 euros
- Méthodes d'inversion sismique dans le domaine fréquentiel
Period: 2014 October - 2017 December , Management: Inria Bordeaux Sud-Ouest, Amount: 180000 euros.

8. Partnerships and Cooperations

8.1. Regional Initiatives

8.1.1. *Partnership with I2M in Bordeaux supported by Conseil Régional d'Aquitaine*

title: Imaging complex materials.

Coordinator: H el ene Barucq

Other partners: I2M CNRS Université Bordeaux I

The detection, localization and monitoring of the defect evolution in composite materials, concrete and more generally heterogeneous materials is a challenging problem for Aeronautics and energy production. It is already possible to localize defects in homogeneous materials by using methods based on ultrasonic inspection and sometimes, they are usable in particular heterogeneous materials, most of the time in 2D. Classical methods rely on the correspondence between the distance and the propagation time of the wave traveling between the defect and the receivers. In complex media, such a correspondence may be lapsed, for instance when the velocity depends on the frequency (dispersion) or of the propagation direction (anisotropy). The defect signature can also be embedded in the acoustic field sent by the structure (multiple reflections). The complexity of the propagation in heterogeneous materials makes then difficult the accurate localization of the defect, in particular in 3D.

Topological imaging techniques can be applied to heterogeneous media. They can find the positions of defects from two simulations performed in a safe experimental medium. They have been developed at I2M laboratory to carry on 2D single/multi mode inspection in isotropic and anisotropic waveguides. They have also been applied to a highly reflecting medium observed with a single sensor. The objective of this work is to extend the technique to 3D problems. In particular, we are going to handle detection in composite plates and in highly heterogeneous media including a collection of small scatterers.

This project is supported by the Conseil Régional d'Aquitaine, for a duration of 2 years.

8.2. National Initiatives

8.2.1. *Depth Imaging Partnership*

Magique-3D maintains active collaborations with Total. In the context of Depth Imaging, Magique-3D coordinates research activities dealing with the development of high-performance numerical methods for solving wave equations in complex media. This project involves 2 other Inria Team-Projects (Hiepac and Nachos) which have complementary skills in mathematics, computing and in geophysics. DIP is fully funded by Total by the way of an outline agreement with Inria .

Since its beginning (2009), eight PhD students have been funded and Magique 3D has hired six of them, one being shared with the project team Nachos (<http://www-sop.inria.fr/nachos/>). Moreover, several internships have been realized. In 2014 the second phase of DIP has begun. Lionel Boillot has been hired as engineer to work on the DIP platform.

8.2.2. *Micro-local analysis of wave equations*

The numerical solution of wave equations most often requires to truncate the propagation domain to define a computational domain limited by an artificial boundary. Magique-3D is very involved in the construction and mathematical validation of boundary conditions which are set on the artificial boundary. Different techniques can be used for the design of such conditions and Magique-3D maintains a collaboration with Prof. Olivier Lafitte from the University of Paris 13 on the mathematical analysis of the Dirichlet-to-Neumann (DtN) operator for acoustic waves. This issue is addressed by applying micro-local analysis which enables us to consider the full DtN operator in the whole space of frequencies.

8.2.3. *Partnership with the department DMAE of ONERA*

title: Modeling of multiperforated plates

Coordinator: Sébastien Tordeux

Other partners: Department DMAE of ONERA

Abstract: In the aeronautic industry, there is a need of numerical models for the design of turboreactors of new generation. Magique-3D is cooperating with the department DMAE of ONERA to develop acoustic models of multiperforated plates which is an important component of the turboreactors.

This project is interdisciplinary, since it involves the experimental expertise of Estelle Piot (acoustician engineer of ONERA working on acoustic bench), the competences in mathematical modeling of Magique 3D. In parallel to the obtention of new theoretical results we are jointly developing a new numerical library based on the discontinuous Galerkin approximation which aims in interpreting experimental data.

This cooperation is formalized thanks to the common supervision of the PhD of Vincent Popie funded by ONERA and DGA and is a follow-up of the ANR APAM (2008-2011).

8.3. European Initiatives

8.3.1. FP7 & H2020 Projects

8.3.1.1. GEAGAM

Title: Geophysical Exploration using Advanced GALerkin Methods

Programm: H2020

Duration: January 2015 - January 2018

Coordinator: Universidad Del Pais Vasco (EHU UPV)

Partners:

Bcam - Basque Center for Applied Mathematics Asociacion (Spain)

Barcelona Supercomputing Center - Centro Nacional de Supercomputacion (Spain)

Total S.A. (France)

Universidad Del Pais Vasco/ Euskal Herriko Unibertsitatea (Spain)

Inria contact: Hélène BARUCQ

The main objective of this Marie Curie RISE action is to improve and exchange interdisciplinary knowledge on applied mathematics, high performance computing, and geophysics to be able to better simulate and understand the materials composing the Earth's subsurface. This is essential for a variety of applications such as CO₂ storage, hydrocarbon extraction, mining, and geothermal energy production, among others. All these problems have in common the need to obtain an accurate characterization of the Earth's subsurface, and to achieve this goal, several complementary areas will be studied, including the mathematical foundations of various high-order Galerkin multiphysics simulation methods, the efficient computer implementation of these methods in large parallel machines and GPUs, and some crucial geophysical aspects such as the design of measurement acquisition systems in different scenarios. Results will be widely disseminated through publications, workshops, post-graduate courses to train new researchers, a dedicated webpage, and visits to companies working in the area. In that way, we will perform an important role in technology transfer between the most advanced numerical methods and mathematics of the moment and the area of applied geophysics.

8.3.1.2. HPC4E

Title: HPC for Energy

Programm: H2020

Duration: December 2015 - December 2017

Coordinator: Barcelona Supercomputing Center

Inria contact: Stephane Lanteri

During the last years, High Performance Computing (HPC) resources have undergone a dramatic transformation, with an explosion on the available parallelism and the use of special purpose processors. There are international initiatives focusing on redesigning hardware and software in order to achieve the Exaflop (10^{18} flops) capability. This project aims at applying the new exascale HPC techniques to energy industry simulations, customizing them if necessary, and going beyond the state-of-the-art in the required HPC exascale simulations for different energy sources that are the present and the future of energy: wind energy production and design, efficient combustion systems for biomass-derived fuels (biogas), and exploration geophysics for hydrocarbon reservoirs.

8.4. International Initiatives

8.4.1. Inria International Partners

8.4.1.1. Declared Inria International Partners

8.4.1.1.1. MAGIC2

Title: Advance Modeling in Geophysics

International Partner (Institution - Laboratory - Researcher):

California State University at Northridge (United States) - Department of Mathematics -
Djellouli Rabia

The Associated Team MAGIC was created in January 2006 and renewed in January 2009. At the end of the program in December 2011, the two partners, *MAGIQUE-3D* and the California State University at Northridge (CSUN) decided to continue their collaboration and obtained the “Inria International Partner” label in 2013.

See also: <https://project.inria.fr/magic/>

The ultimate objective of this research collaboration is to develop efficient solution methodologies for solving inverse problems arising in various applications such as geophysical exploration, underwater acoustics, and electromagnetics. To this end, the research program will be based upon the following three pillars that are the key ingredients for successfully solving inverse obstacle problems. 1) The design of efficient methods for solving high-frequency wave problems. 2) The sensitivity analysis of the scattered field to the shape and parameters of heterogeneities/scatterers. 3) The construction of higher-order Absorbing Boundary Conditions.

8.5. International Research Visitors

8.5.1. Visits of International Scientists

8.5.1.1. Internships

Andrew Wang, graduate student from the Massachusetts Institute of Technology, visited *MAGIQUE-3D* for a two months internship in June and July 2015.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific events organisation

9.1.1.1. Member of the organizing committees

Hélène Barucq organized the Third Workshop of Strategic Action DIP in Pau, June 21-22, 2015, <http://dip.inria.fr/workshops/third-workshop-of-the-strategic-action-dip/>

Victor Péron and Hélène Barucq organized a series of courses at University of Pau in the framework of the GEAGAM project: Coding the FEM (A. Rodriguez), Seismic depth imaging (R. Baina), Discontinuous Galerkin methods for the simulation of wave propagation (J. Diaz), May 18-22, 2015

Victor Péron and Hélène Barucq organized the Workshop on Advanced Subsurface Visualization Methods: “Exploring the Earth”, Pau 26-27 May 2015, in the framework of the GEAGAM project <https://sites.google.com/site/geagamnetwork/workshop>.

Juliette Chabassier organized with Damien Fournier a Mini Symposium on Helioseismology at Waves 2015 conference (THE 12TH INTERNATIONAL CONFERENCE ON MATHEMATICAL AND NUMERICAL ASPECTS OF WAVE PROPAGATION), Karlsruhe, Germany, July 20-24, 2015. http://waves2015.math.kit.edu/conf_program.html#minisymp

9.1.2. Journal

9.1.2.1. Reviewer - Reviewing activities

In 2015, the members of the team have been reviewers for ESAIM : Mathematical Modelling and Numerical Analysis, New York Journal of Mathematics, SIAM Journal on Scientific Computing, SIAM Journal on Numerical Analysis, Acta Acustica united with Acustica, Journal of the Acoustical Society of America, Journal of Sound and Vibration, Journal of Computational Physics, Mathematics of Computation, Wave Motion, Geophysical Journal International, Journal of Computational Acoustics.

9.1.3. Research administration

Hélène Barucq was vice-chair of the Inria evaluation committee until June 2015. She participated to the national jury of Inria competitive selection for Senior Researchers (DR2) and to the local jury of Inria competitive selection for Young Graduate Scientists (CR2) in Bordeaux. She participated to the selection committee for Research Positions (Junior and Senior). She participated to the selection committee for an Assistant Professor position at the University of Pau. She is member of the board of the Laboratory of Mathematics of Pau and of the research federation IPRA which are both under the administrative supervision of CNRS. She is the scientific head of the project DIP.

Julien Diaz is elected member of the Inria Technical Committee and of the Inria Administrative Board. He is appointed member of the CDT (Commission de Développement Technologique) and of the Center Committee of Inria Bordeaux Sud-Ouest.

Sébastien Tordeux is elected member of the 26th section of the CNU (Conseil National des Universités).

Victor Péron is appointed member of the CJC (Commission Jeunes Chercheurs) of Inria Bordeaux Sud-Ouest.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Licence : Victor Péron, Mathématiques Appliquées, 15 Eq. TD, L1, UPPA, France

Licence : Victor Péron, Compléments d'analyse, 19,5 Eq. TD, L2, UPPA, France

Licence : Victor Péron, Calcul intégral, 19,5 Eq. TD, L3, UPPA, France

Master : Julien Diaz, Transformées, 24h Eq. TD, M1, EISTIA, France

Master : Marc Duruflé, Calcul scientifique en C++, 96h Eq. TD, M1, Bordeaux INP, France

Master : Marc Duruflé, Equations Différentielles, 20h Eq. TD, L3, Bordeaux INP, France

Master : Victor Péron et Sébastien Tordeux, Analyse numérique fondamentale, 87h Eq. TD, M1, UPPA, France

Master : Victor Péron, Analyse, 23h Eq. TD, M1, UPPA, France

Master : Sébastien Tordeux, Introduction aux phénomènes de propagation d'ondes, 38h Eq. TD, M2, UPPA, France

Master : Sébastien Tordeux, Introduction aux phénomènes de propagation d'ondes, 20h Eq. TD, M2, ENS Kouba, Algérie

9.2.2. Supervision

PhD : Jérôme Luquel, RTM en milieu hétérogène par équations d'ondes élastiques, UPPA, April 16th 2015, Hélène Barucq and Julien Diaz

PhD : Théophile Chaumont-Frelet, High Order Methods for Helmholtz Problems in Highly Heterogeneous Media, INSA Rouen, December 11th 2015, Hélène Barucq and Christian Gout (INSA Rouen).

PhD : Marie Bonnasse-Gahot, Simulation of elastic wave propagation in time harmonic domain using discontinuous Galerkin methods, Université de Nice Sophia Antipolis 15/12/2015, Julien Diaz and Stéphane Lantéri (EPI Nachos, Inria Sophia Antipolis-Méditerranée).

PhD : Julen Alvarez-Aramberri, *hp*-adaptive inversion of magnetotelluric measurements, University of Basque Country and UPPA, December 18th 2015, Hélène Barucq and David Pardo.

PhD in progress : Izar Azpiroz Irigorri, Approximation des problèmes d’Helmholtz couplés sur maillages virtuels , October 2014, Hélène Barucq, Julien Diaz and Rabia Djellouli.

PhD in progress : Vincent Darrigrand, Etude d’erreur pour des problèmes d’Helmholtz approchés par des techniques de Petrov-Galerkin , October 2013, Hélène Barucq and David Pardo.

PhD in progress : Aralar Erdozain, Fast inversion of 3D Borehole Resistivity Measurements using Model Reduction Techniques based on 1D Semi-Analytical Solutions, October 2013, Hélène Barucq, David Pardo and Victor Péron.

PhD in progress : Florian Faucher, Méthodes d’inversion sismique dans le domaine fréquentiel , October 2014, Hélène Barucq.

PhD in progress : Mamadou N’Diaye, “Analyse et développement de schémas temporels hybrides pour les équations hyperboliques du premier ordre”, January 2015, Hélène Barucq and Marc Duruflé.

PhD in progress : Elvira Shishenina, Approximations hybrides par éléments finis et éléments virtuels discontinus pour l’élasto-acoustique, October 2015, Hélène Barucq and Julien Diaz.

PhD in progress : Vincent Popie, Modélisation asymptotique de la réponse acoustique de plaques perforées dans un cadre linéaire avec étude des effets visqueux, 2012, Estelle Piot (ONERA) et Sébastien Tordeux.

PhD in progress : Hamza Alaoui Hafidi, Imagerie ultrasonore tridimensionnelle dans les milieux hétérogènes complexes, 2015, Encadrement : Marc Deschamps, Michel Castaings, Eric Ducasse, Samuel Rodriguez (I2M), Hélène Barucq, Marc Duruflé, Juliette Chabassier (Magique 3D).

9.2.3. *Juries*

Hélène Barucq : Antoine Rousseau (Université de Montpellier), “Modélisation mathématique et numérique de quelques problèmes issus des sciences de l’environnement”, HDR, December 3th 2015

Julien Diaz : Stojce Nakov (Université de Bordeaux) “Solveur hybrides très haute performance et multi seconds membres pour la simulation 3D en régime fréquentiel de propagation d’ondes dans des milieux avec hétérogénéité et topographie”, PhD thesis, December 14th 2015

Victor Péron : Julen Alvarez-Aramberri (UPV-EHU, Bilbao) “*hp*-Adaptative Simulation and Inversion of Magnetotelluric Measurements”, PhD thesis, December 18th 2015

10. Bibliography

Major publications by the team in recent years

- [1] C. AGUT, J. DIAZ. *Stability analysis of the Interior Penalty Discontinuous Galerkin method for the wave equation*, in "ESAIM: Mathematical Modelling and Numerical Analysis", 2013, vol. 47, n^o 3, pp. 903-932 [DOI : 10.1051/M2AN/2012061], <http://hal.inria.fr/hal-00759457>
- [2] M. AMARA, R. DJELLOULI, C. FARHAT. *Convergence analysis of a discontinuous Galerkin method with plane waves and Lagrange multipliers for the solution of Helmholtz problems*, in "SIAM Journal on Numerical Analysis", 2009, vol. 47, pp. 1038–1066

- [3] C. BALDASSARI, H. BARUCQ, H. CALANDRA, J. DIAZ. *Numerical performances of a hybrid local-time stepping strategy applied to the reverse time migration*, in "Geophysical Prospecting", September 2011, vol. 59, n^o 5, pp. 907-919 [DOI : 10.1111/j.1365-2478.2011.00975.x], <http://hal.inria.fr/hal-00627603/en>
- [4] H. BARUCQ, C. BEKKEY, R. DJELLOULI. *Full aperture reconstruction of the acoustic Far-Field Pattern from few measurements*, in "Communication in Computational Physics", 2012, vol. 11, pp. 647-659 [DOI : 10.4208/CICP.281209.150610s], <http://hal.inria.fr/inria-00527346>
- [5] H. BARUCQ, T. CHAUMONT FRELET, J. DIAZ, V. PÉRON. *Upscaling for the Laplace problem using a discontinuous Galerkin method*, in "Journal of Computational and Applied Mathematics", November 2012, <http://hal.inria.fr/hal-00757098>
- [6] H. BARUCQ, J. DIAZ, V. DUPRAT. *Long-Time Stability Analysis of Acoustic Absorbing Boundary Conditions for Regular-Shaped Surfaces*, in "Mathematical Models and Methods in Applied Sciences", 2013, vol. 23, n^o 11, pp. 2129-2154, <http://hal.inria.fr/hal-00759451>
- [7] H. BARUCQ, J. DIAZ, M. TLEMCANI. *New absorbing layers conditions for short water waves*, in "Journal of Computational Physics", 2010, vol. 229, pp. 58–72 [DOI : 10.1016/j.jcp.2009.08.033]
- [8] H. BARUCQ, R. DJELLOULI, C. BEKKEY. *A multi-step procedure for enriching limited two-dimensional acoustic far-field pattern measurements*, in "Journal of Inverse and Ill-Posed Problems", 2010, vol. 18, pp. 189-216
- [9] H. BARUCQ, R. DJELLOULI, E. ESTECAHANDY. *Characterization of the Fréchet derivative of the elasto-acoustic field with respect to Lipschitz domains*, in "Journal of Inverse and Ill-posed Problems", August 2013 [DOI : 10.1515/JIP-2012-0098], <http://hal.inria.fr/hal-00880508>
- [10] H. BARUCQ, A.-G. DUPOUY SAINT-GUIRONS, S. TORDEUX. *Non-reflecting boundary condition on ellipsoidal boundary*, in "Numerical Analysis and Applications", 2012, vol. 5, n^o 2, pp. 109-115, <http://hal.inria.fr/hal-00760458>
- [11] J. CHABASSIER, S. IMPERIALE. *Introduction and study of fourth order theta schemes for linear wave equations*, in "Journal of Computational and Applied Mathematics", January 2013, vol. 245, pp. 194-212 [DOI : 10.1016/j.cam.2012.12.023], <http://hal.inria.fr/hal-00873048>
- [12] P. DULAR, V. PÉRON, R. PERRUSSEL, L. KRÄHENBÜHL, C. GEUZAINÉ. *Perfect Conductor and Impedance Boundary Condition Corrections via a Finite Element Subproblem Method*, in "IEEE Transactions on Magnetism", February 2014, vol. 50, n^o 2, 7000504 [DOI : 10.1109/TMAG.2013.2284338], <https://hal.archives-ouvertes.fr/hal-00869987>
- [13] S. TORDEUX, A. BENDALI, P.-H. COCQUET. *Scattering of a scalar time-harmonic wave by N small spheres by the method of matched asymptotic expansions*, in "Numerical Analysis and Applications", 2012, vol. 5, n^o 2, pp. 116-123, <http://hal.inria.fr/hal-00760457>

Publications of the year

Doctoral Dissertations and Habilitation Theses

- [14] J. ALVAREZ ARAMBERRI. *hp-Adaptative Simulation and Inversion of Magnetotelluric Measurements*, University of Basque Country/University of Pau, December 2015
- [15] M. BONNASSE-GAHOT. *Simulation de la propagation d'ondes élastiques en domaine fréquentiel par des méthodes de Galerkin discontinues*, Université de Nice, December 2015
- [16] T. CHAUMONT FRELET. *Finite element approximation of Helmholtz problems with application to seismic wave propagation*, INSA de Rouen, December 2015, <https://tel.archives-ouvertes.fr/tel-01246244>
- [17] J. LUQUEL. *Imaging of complex media with elastic wave equations*, Université de Pau et des Pays de l'Adour, April 2015, <https://hal.inria.fr/tel-01217029>

Articles in International Peer-Reviewed Journals

- [18] P. ANGOT, G. CARBOU, V. PÉRON. *Asymptotic study for Stokes-Brinkman model with jump embedded transmission conditions*, in "Asymptotic Analysis", 2015, 25 p. , <https://hal.inria.fr/hal-01184429>
- [19] H. BARUCQ, R. DJELLOULI, E. ESTECAHANDY. *Fréchet differentiability of the elasto-acousticscattered field with respect to Lipschitz domains*, in "Mathematical Methods in the Applied Sciences", March 2015, 13 p. [DOI : 10.1002/MMA.3444], <https://hal.inria.fr/hal-01195633>
- [20] A. BENDALI, P.-H. COCQUET, S. TORDEUX. *Approximation by Multipoles of the Multiple Acoustic Scattering by Small Obstacles in Three Dimensions and Application to the Foldy Theory of Isotropic Scattering*, in "Archive for Rational Mechanics and Analysis", March 2016, vol. 219, n° 3 [DOI : 10.1007/s00205-015-0915-5], <https://hal-univ-tlse3.archives-ouvertes.fr/hal-01258966>
- [21] M. BONNET, A. BUREL, M. DURUFLÉ, P. JOLY. *Effective transmission conditions for thin-layer transmission problems in elastodynamics. The case of a planar layer model*, in "ESAIM: Mathematical Modelling and Numerical Analysis", 2016, vol. 50, pp. 43-75 [DOI : 10.1051/M2AN/2015030], <https://hal.archives-ouvertes.fr/hal-01144401>
- [22] J. CHABASSIER, S. IMPERIALE. *Fourth order energy-preserving locally implicit time discretization for linear wave equations*, in "International Journal for Numerical Methods in Engineering", 2015 [DOI : 10.1002/NME.5130], <https://hal.inria.fr/hal-01222072>
- [23] V. DARRIGRAND, D. PARDO, I. MUGA. *Goal-oriented adaptivity using unconventional error representations for the 1D Helmholtz equation*, in "Computers and Mathematics with Applications", May 2015, vol. 69, n° 9, pp. 964 - 979 [DOI : 10.1016/J.CAMWA.2015.03.006], <https://hal.archives-ouvertes.fr/hal-01140748>
- [24] J. DIAZ, M. J. GROTE. *Multi-level explicit local time-stepping methods for second-order wave equations*, in "Computer Methods in Applied Mechanics and Engineering", July 2015, vol. 291, pp. 240–265 [DOI : 10.1016/J.CMA.2015.03.027], <https://hal.inria.fr/hal-01184090>
- [25] P. DULAR, V. PERON, L. KRÄHENBÜHL, C. GEUZAINÉ. *Subproblem Finite-Element Refinement of Inductors From Wire to Static and Dynamic Volume Models*, in "IEEE Transactions on Magnetics", March 2015, vol. 51, n° 3, 7402704 p. [DOI : 10.1109/TMAG.2014.2360232], <https://hal.archives-ouvertes.fr/hal-01153089>

- [26] L. FARINA, M. FERREIRA, V. PÉRON. *The Airfoil equation on near disjoint intervals: Approximate models and polynomial solutions*, in "Journal of Computational and Applied Mathematics", 2016 [DOI : 10.1016/J.CAM.2015.11.024], <https://hal.inria.fr/hal-01253227>
- [27] A. RODRIGUEZ ROZAS, J. DIAZ. *Non-conforming curved finite element schemes for time-dependent elastic-acoustic coupled problems*, in "Journal of Computational Physics", January 2016, vol. 305, pp. 44–62 [DOI : 10.1016/J.JCP.2015.10.028], <https://hal.inria.fr/hal-01255188>
- [28] K. SCHMIDT, J. DIAZ, C. HEIER. *Non-conforming Galerkin finite element methods for local absorbing boundary conditions of higher order*, in "Computers and Mathematics with Applications", November 2015, vol. 70, n° 9, pp. 2252–2269 [DOI : 10.1016/J.CAMWA.2015.08.034], <https://hal.inria.fr/hal-01184251>
- [29] S. SHANNON, V. PERON, Z. YOSIBASH. *The Laplace equation in 3-D domains with cracks: Dual shadows with log terms and extraction of corresponding edge flux intensity functions*, in "Mathematical Methods in the Applied Sciences", 2015, 14 p. [DOI : 10.1002/SIM.0000], <https://hal.inria.fr/hal-01111593>
- [30] S. SHANNON, V. PÉRON, Z. YOSIBASH. *Singular asymptotic solution along an elliptical edge for the Laplace equation in 3-D*, in "Engineering Fracture Mechanics", 2015, 16 p. , <https://hal.inria.fr/hal-01097676>

Invited Conferences

- [31] H. BARUCQ, M. BERGOT, J. CHABASSIER, J. DIAZ. *Performance assessments of absorbing conditions for the reverse time-harmonic migration*, in "First Pan-American Congress on Computational Mechanic (PANACM 2015)", Buenos Aires, Argentina, April 2015, <https://hal.inria.fr/hal-01184102>
- [32] H. BARUCQ, J. DIAZ, R. DJELLOULI, E. ESTECAHANDY. *High-order Discontinuous Galerkin approximations for elasto-acoustic scattering problems*, in "XXIV Congress on Differential Equations and Applications / XIV Congress on Applied Mathematics (XXIV CEDYA / XIV CMA)", Cadiz, Spain, June 2015, <https://hal.inria.fr/hal-01184107>

International Conferences with Proceedings

- [33] L. BOILLOT, H. BARUCQ, J. DIAZ, H. CALANDRA. *Absorbing Boundary Conditions for 3D elastic TTI modeling*, in "SEG - Society of Exploration Geophysicists", Nouvelle-Orléans, United States, SEG Technical Program Expanded Abstracts 2015, October 2015, pp. 535-540 [DOI : 10.1190/SEGAM2015-5910360.1], <https://hal.inria.fr/hal-01223344>
- [34] V. POPIE, E. PIOT, S. TORDEUX, V. FRANÇOIS. *Theoretical and numerical investigation of acoustic response of a multiperforated plate for combustor liner*, in "ASME Turbo Expo 2015 : Turbine Technical Conference and Exposition", Montreal, Canada, June 2015, <https://hal.inria.fr/hal-01111467>

National Conferences with Proceedings

- [35] L. KRÄHENBÜHL, P. DULAR, V. PÉRON, R. PERRUSSEL, R. SABARIEGO, C. POIGNARD. *Impédances de surface en 2D : comparaison de méthodes de paramétrisation en δ* , in "Numélec", Nantes, France, M. FÉLACHI (editor), Actes de la 8ème Conférence Européenne sur les Méthodes Numériques en Electromagnétisme, Université de Nantes - IREENA, June 2015, <https://hal.archives-ouvertes.fr/hal-01199546>

Conferences without Proceedings

- [36] E. AGULLO, H. BARUCQ, L. BOILLOT, G. BOSILCA, H. CALANDRA, J. DIAZ. *Portable task-based programming for seismic wave propagation simulation in time domain*, in "HOSCAR – 5th Brazil-French workshop on High performance cOmputing and SCientific dAta management dRiven by highly demanding applications (Inria-CNPq)", Sophia Antipolis, France, September 2015, <https://hal.inria.fr/hal-01208461>
- [37] H. BARUCQ, L. BOILLOT, M. BONNASSE-GAHOT, H. CALANDRA, J. DIAZ, S. LANTERI. *Discontinuous Galerkin Approximations for Seismic Wave Propagation in a HPC Framework*, in "Platform for Advanced Scientific Computing Conference (PASC 15)", Zurich, Switzerland, June 2015, <https://hal.inria.fr/hal-01184106>
- [38] H. BARUCQ, L. BOILLOT, H. CALANDRA, J. DIAZ. *Absorbing Boundary Conditions for 3D Tilted Transverse Isotropic media*, in "First Pan-American Congress on Computational Mechanics (PANACM 2015)", Buenos Aires, Argentina, April 2015, <https://hal.inria.fr/hal-01184104>
- [39] H. BARUCQ, L. BOILLOT, H. CALANDRA, R. DJELLOULI, E. ESTECAHANDY. *High-Order IPDG Approximations for Elasto-Acoustic Problems*, in "SIAM Conference on Mathematical and Computational Issues in the Geosciences", Stanford, United States, June 2015, <https://hal.inria.fr/hal-01184110>
- [40] H. BARUCQ, H. CALANDRA, T. CHAUMONT FRELET, C. GOUT. *Helmholtz Equation in Highly Heterogeneous Media*, in "1st. Pan-American Congress on Computational Mechanics", Buenos Aires, Argentina, April 2015, <https://hal.inria.fr/hal-01217469>
- [41] H. BARUCQ, H. CALANDRA, T. CHAUMONT FRELET, C. GOUT. *Multiscale Medium Approximation for the Helmholtz equation. Application to geophysical benchmarks*, in "Third Workshop of the strategic action DIP", Pau, France, June 2015, <https://hal.inria.fr/hal-01217475>
- [42] H. BARUCQ, H. CALANDRA, T. CHAUMONT FRELET, C. GOUT. *Pollution analysis for high order discretizations of highly heterogeneous Helmholtz problems*, in "Exploring the earth", Pau, France, Team MAGIQUE-3D, May 2015, <https://hal.inria.fr/hal-01217484>
- [43] L. BOILLOT, E. AGULLO, G. BOSILCA, H. CALANDRA, H. BARUCQ, J. DIAZ. *Portable task-based programming for elastodynamics*, in "Rice - Oil & Gas HPC Workshop", Houston, United States, March 2015, <https://hal.inria.fr/hal-01128750>
- [44] L. BOILLOT, H. BARUCQ, J. DIAZ, H. CALANDRA. *Stable TTI Acousto-Elastic simulations*, in "Workshop DIP - Depth Imaging Partnership (Inria-TOTAL)", PAU, France, June 2015, <https://hal.inria.fr/hal-01188522>
- [45] L. BOILLOT, G. BOSILCA, E. AGULLO, H. CALANDRA. *Portable task-based programming for Seismic Imaging*, in "MATHIAS – TOTAL Symposium on Mathematics", Paris, France, October 2015, <https://hal.inria.fr/hal-01223339>
- [46] M. BONNASSE-GAHOT, H. CALANDRA, J. DIAZ, S. LANTERI. *Modeling of elastic Helmholtz equations by hybridizable discontinuous Galerkin method (HDG) for geophysical applications*, in "5th workshop France-Brazil HOSCAR", Nice, France, September 2015, <https://hal.inria.fr/hal-01207897>
- [47] M. BONNASSE-GAHOT, H. CALANDRA, J. DIAZ, S. LANTERI. *Modelling of seismic waves propagation in harmonic domain by hybridizable discontinuous Galerkin method (HDG)*, in "Workshop GEAGAMM", Pau, France, May 2015, <https://hal.inria.fr/hal-01207906>

- [48] M. BONNASSE-GAHOT, H. CALANDRA, J. DIAZ, S. LANTERI. *Performance Assessment on Hybridizable Dg Approximations for the Elastic Wave Equation in Frequency Domain*, in "SIAM Conference on Mathematical and Computational Issues in the Geosciences", Stanford, United States, June 2015, <https://hal.inria.fr/hal-01184111>
- [49] M. BONNASSE-GAHOT, H. CALANDRA, J. DIAZ, S. LANTERI. *Performance comparison between hybridizable DG and classical DG methods for elastic waves simulation in harmonic domain*, in "Workshop Oil & Gas Rice 2015", Houston, Texas, United States, March 2015, <https://hal.inria.fr/hal-01207886>
- [50] J. DIAZ, V. PÉRON. *Equivalent Robin Boundary Conditions for Acoustic and Elastic Media*, in "First Inria-Mexico Workshop in Applied Mathematics and Computer Science", Mexico City, Mexico, June 2015, <https://hal.inria.fr/hal-01254193>
- [51] A. ERDOZAIN, V. PERON. *Impedance Transmission Conditions for the Electric Potential across a Highly Conductive Casing*, in "Waves 2015", Karlsruhe, Germany, July 2015, <https://hal.inria.fr/hal-01196181>
- [52] F. FAUCHER, H. BARUCQ, H. CALANDRA, M. V. DE HOOP, J. SHI. *Elastic isotropic full waveform inversion via quantitative stability estimates*, in "PANACM (Pan-American Congress On Computational Mechanics) 2015", Buenos Aires, Argentina, April 2015, <https://hal.archives-ouvertes.fr/hal-01207064>
- [53] F. FAUCHER, H. BARUCQ, M. V. DE HOOP, H. CALANDRA, J. SHI. *Elastic isotropic full waveform inversion via quantitative stability estimates*, in "GEAGAM Workshop On Advanced Subsurface Visualization Methods", Pau, France, May 2015, <https://hal.archives-ouvertes.fr/hal-01207070>
- [54] V. MATTESI. *Equivalent point source modelling of small heterogeneities in the context of 3D time-domain wave propagation equation*, in "SMAI 2015", Les Karelis, France, June 2015, <https://hal.archives-ouvertes.fr/hal-01213998>

Research Reports

- [55] H. BARUCQ, A. BENDALI, M. FARES, V. MATTESI, S. TORDEUX. *A Symmetric Trefftz-DG Formulation based on a Local Boundary Element Method for the Solution of the Helmholtz Equation*, Inria Bordeaux, October 2015, n^o RR-8800, 31 p. , <https://hal.inria.fr/hal-01218784>
- [56] H. BARUCQ, J. DIAZ, V. MATTESI. *Polynomial speeds in a Discontinuous Galerkin code*, Inria Bordeaux, July 2015, n^o RR-8756, 57 p. , <https://hal.inria.fr/hal-01176854>
- [57] H. BARUCQ, V. MATTESI, S. TORDEUX. *Asymptotic modelling of an acoustic scattering problem involving very small obstacles: mathematical justification*, Inria Bordeaux, December 2015, n^o RR-8829, <https://hal.archives-ouvertes.fr/hal-01250654>
- [58] H. BARUCQ, V. MATTESI, S. TORDEUX. *The Mellin Transform*, Inria Bordeaux ; Inria, June 2015, n^o RR-8743, 16 p. , <https://hal.inria.fr/hal-01165453>
- [59] L. KRÄHENBÜHL, V. PÉRON, R. PERRUSSEL, C. POIGNARD. *On the asymptotic expansion of the magnetic potential in eddy current problem: a practical use of asymptotics for numerical purposes*, Inria Bordeaux ; Inria, June 2015, n^o RR-8749, <https://hal.inria.fr/hal-01174009>

Other Publications

- [60] L. BOILLOT. *Portable task-based programming for Seismic Imaging*, May 2015, PRACEdays, Poster, <https://hal.inria.fr/hal-01158967>
- [61] J. DIAZ, V. PÉRON. *Equivalent Robin Boundary Conditions for Acoustic and Elastic Media*, 2015, working paper or preprint, <https://hal.inria.fr/hal-01254194>
- [62] M. DURUFLÉ, V. PÉRON, K. SCHMIDT. *Equivalent transmission conditions for the time-harmonic Maxwell equations in 3D for a medium with a highly conductive thin sheet*, 2015, working paper or preprint, <https://hal.archives-ouvertes.fr/hal-01260111>
- [63] V. PÉRON. *Asymptotic expansion for the magnetic potential in the eddy current problem : the ferromagnetic case*, 2015, working paper or preprint, <https://hal.inria.fr/hal-01253971>

References in notes

- [64] M. AINSWORTH, P. MONK, W. MUNIZ. *Dispersive and dissipative properties of discontinuous Galerkin finite element methods for the second-order wave equation*, in "Journal of Scientific Computing", 2006, vol. 27
- [65] J. ALVAREZ-ARAMBERRI, D. PARDO, H. BARUCQ. *Inversion of Magnetotelluric Measurements Using Multigoal Oriented hp-adaptivity*, in "ICCS 2013-International Conference on Computational Science", Barcelona, Spain, V. ALEXANDROV, M. LEES, V. KRZHIZHANOVSKAYA, J. DONGARRA, P. M. SLOOT (editors), Procedia Computer Science, Elsevier, June 2013, vol. 18, pp. 1564 - 1573 [DOI : 10.1016/J.PROCS.2013.05.324], <https://hal.inria.fr/hal-00944838>
- [66] M. AMARA, H. CALANDRA, R. DJELLOULI, M. GRIGOROSCUA-STRUGARU. *A modified discontinuous Galerkin method for solving efficiently Helmholtz problems*, in "Communications in Computational Physics", 2012, vol. 11, n^o 2, pp. 335–350, <https://hal.inria.fr/hal-00768457>
- [67] M. AMARA, H. CALANDRA, R. DJELLOULI, M. GRIGOROSCUA-STRUGARU. *A stable discontinuous Galerkin-type method for solving efficiently Helmholtz problems.*, in "Computers and Structures", 2012, vol. 106-107, pp. 258-272, <https://hal.inria.fr/hal-00768455>
- [68] X. ANTOINE. *Fast approximate computation of a time-harmonic scattered field using the on-surface radiation condition method*, in "IMA J. Appl. Math", 2001, vol. 66, pp. 83–110
- [69] L. BEIRÃO DA VEIGA, F. BREZZI, A. CANGIANI, G. MANZINI, L. MARINI, A. RUSSO. *Basic principles of virtual element methods*, in "Mathematical Models and Methods in Applied Sciences", 2013, vol. 23, n^o 01, pp. 199–214
- [70] J. BARRIÈRE, C. BORDES, D. BRITO, P. SÉNÉCHAL, H. PERROUD. *Laboratory monitoring of P waves in partially saturated sand*, in "Geophysical Journal International", 2012, vol. 191, n^o 3, pp. 1152–1170
- [71] H. BARUCQ, J. CHABASSIER, J. DIAZ, E. ESTECAHANDY. *Numerical Analysis of a reduced formulation of an elasto-acoustic scattering problem*, in "WAVES 13 : 11th International Conference on Mathematical and Numerical Aspects of Waves", Gammarth, Tunisia, June 2013, <https://hal.inria.fr/hal-00873633>

- [72] H. BARUCQ, R. DJELLOULI, E. ESTECAHANDY. *Efficient DG-like formulation equipped with curved boundary edges for solving elasto-acoustic scattering problems*, in "International Journal for Numerical Methods in Engineering", 2014, To appear, <https://hal.inria.fr/hal-00931852>
- [73] L. BEIRAO DA VEIGA, F. BREZZI, L. D. MARINI. *Virtual Elements for linear elasticity problems*, in "SIAM Journal on Numerical Analysis", 2013, vol. 51, n^o 2, pp. 794–812
- [74] L. BEIRAO DA VEIGA, F. BREZZI, L. MARINI, A. RUSSO. *The hitchhiker's guide to the virtual element method*, in "Mathematical Models and Methods in Applied Sciences", 2014, vol. 24, n^o 08, pp. 1541–1573
- [75] L. BEIRÃO DA VEIGA, G. MANZINI. *A virtual element method with arbitrary regularity*, in "IMA Journal of Numerical Analysis", 2013, drt018 p.
- [76] F. BREZZI, L. DONATELLA MARINI. *Virtual Element Method for plate bending problems*, in "Computer Methods in Applied Mechanics and Engineering", 2012
- [77] F. BREZZI, L. MARINI. *Virtual Element and Discontinuous Galerkin Methods*, in "Recent Developments in Discontinuous Galerkin Finite Element Methods for Partial Differential Equations", Springer, 2014, pp. 209–221
- [78] E. BÉCACHE, P. JOLY, J. RODRÍGUEZ. *Space-time mesh refinement for elastodynamics. Numerical results*, in "Comput. Methods Appl. Mech. Engrg.", 2005, vol. 194, n^o 2-5, pp. 355–366
- [79] J. CHABASSIER, S. IMPERIALE. *Introduction and study of fourth order theta schemes for linear wave equations*, in "Journal of Computational and Applied Mathematics", January 2013, vol. 245, pp. 194–212 [DOI : 10.1016/J.CAM.2012.12.023], <https://hal.inria.fr/hal-00873048>
- [80] X. CLAEYS, F. COLLINO. *Asymptotic and numerical analysis for Holland and Simpson's thin wire formalism*, in "Journal of computational and applied mathematics", 2011, vol. 235, n^o 15, pp. 4418–4438
- [81] F. COLLINO, T. FOUQUET, P. JOLY. *Conservative space-time mesh refinement methods for the FDTD solution of Maxwell's equations*, in "J. Comput. Phys.", 2006, vol. 211, n^o 1, pp. 9–35
- [82] J. DIAZ, M. J. GROTE. *Energy Conserving Explicit Local Time-Stepping for Second-Order Wave Equations*, in "SIAM Journal on Scientific Computing", 2009, vol. 31, n^o 3, pp. 1985–2014, <http://hal.inria.fr/inria-00409233/en/>
- [83] V. DOLEAN, H. FAHS, L. FEZOU, S. LANTERI. *Locally implicit discontinuous Galerkin method for time domain electromagnetics*, in "Journal of Computational Physics", 2010, vol. 229, n^o 2, pp. 512–526
- [84] M. DUMBSER, M. KÄSER, E. F. TORO. *An arbitrary high-order Discontinuous Galerkin method for elastic waves on unstructured meshes - V. Local time stepping and p-adaptivity*, in "Geophysical Journal International", 2007, vol. 171, n^o 2, pp. 695–717, <http://dx.doi.org/10.1111/j.1365-246X.2007.03427.x>
- [85] S. GARAMBOIS, M. DIETRICH. *Seismoelectric wave conversions in porous media: Field measurements and transfer function analysis*, in "Geophysics", 2001, vol. 66, n^o 5, pp. 1417–1430

- [86] S. GARAMBOIS, M. DIETRICH. *Full waveform numerical simulations of seismoelectromagnetic wave conversions in fluid-saturated stratified porous media*, in "Journal of Geophysical Research", 2002, vol. 107, n^o B7, 2148 p.
- [87] M. J. GROTE, T. MITKOVA. *High-order explicit local time-stepping methods for damped wave equations*, in "Journal of Computational and Applied Mathematics", 2013, vol. 239, n^o 0, pp. 270 - 289 [DOI : 10.1016/J.CAM.2012.09.046], <http://www.sciencedirect.com/science/article/pii/S0377042712004190>
- [88] S. IMPERIALE, P. JOLY. *Mathematical and numerical modelling of piezoelectric sensors*, in "ESAIM-Mathematical Modelling and Numerical Analysis", 2012, vol. 46, n^o 4, 875 p.
- [89] R. KIRBY, S. SHERWIN, B. COCKBURN. *To CG or to HDG: A Comparative Study*, in "Journal of Scientific Computing", 2012, vol. 51, n^o 1, pp. 183-212, <http://dx.doi.org/10.1007/s10915-011-9501-7>
- [90] G. KRIEGSMANN, A. TAFLOVE, K. UMASHANKAR. *A new formulation of electromagnetic wave scattering using an on-surface radiation boundary condition approach*, in "IEEE Trans. Antennas and Propagation", 1987, vol. 35, pp. 153-161
- [91] M. KÄSER, M. DUMBSEER. *An arbitrary high-order discontinuous Galerkin method for elastic waves on unstructured meshes - I. The two-dimensional isotropic case with external source terms*, in "Geophysical Journal International", 2006, vol. 166, n^o 2, pp. 855–877, <http://dx.doi.org/10.1111/j.1365-246X.2006.03051.x>
- [92] L. LI, S. LANTERI, R. PERRUSSEL. *Numerical investigation of a high order hybridizable discontinuous Galerkin method for 2d time-harmonic Maxwell's equations*, in "COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering", 2013, vol. 32, n^o 3, pp. 1112–1138
- [93] J. MELENK, S. SAUTER. *Convergence analysis for finite element discretizations of the Helmholtz equation with Dirichlet-to-Neumann boundary conditions*, in "Mathematics of Computation", 2010, vol. 79, n^o 272, pp. 1871–1914
- [94] N. C. NGUYEN, J. PERAIRE, B. COCKBURN. *High-order implicit hybridizable discontinuous Galerkin methods for acoustics and elastodynamics*, in "Journal of Computational Physics", 2011, vol. 230, n^o 10, pp. 3695–3718
- [95] D. PARDO, C. TORRES-VERDÍN, Z. ZHANG. *Sensitivity study of borehole-to-surface and crosswell electromagnetic measurements acquired with energized steel casing to water displacement in hydrocarbon-bearing layers*, in "Geophysics", 2008, vol. 73, n^o 6, pp. F261–F268
- [96] S. PIPERNO. *Symplectic local time-stepping in non-dissipative DGTD methods applied to wave propagation problems*, in "M2AN Math. Model. Numer. Anal.", 2006, vol. 40, n^o 5, pp. 815–841
- [97] S. R. PRIDE, S. GARAMBOIS. *Electroseismic wave theory of Frenkel and more recent developments*, in "Journal of Engineering Mechanics", 2005, vol. 131, n^o 9, pp. 898–907
- [98] S. WARDEN, S. GARAMBOIS, P. SAILHAC, L. JOUNIAUX, M. BANO. *Curvelet-based seismoelectric data processing*, in "Geophysical Journal International", 2012, vol. 190, n^o 3, pp. 1533–1550