Activity Report 2015

Project-Team GEOMETRICA

Geometric Computing

RESEARCH CENTERS
Sophia Antipolis - Méditerranée
Saclay - Île-de-France

THEME
Algorithmics, Computer Algebra and Cryptology

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Project-Team GEOMETRICA

Creation of the Project-Team: 2003 July 01, end of the Project-Team: 2015 December 31

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2. Overall Objectives

2.1. Overall Objectives

Research carried out by the Geometrica project team is dedicated to Computational Geometry and Topology and follows three major directions: (a). mesh generation and geometry processing; (b). topological and geometric inference; (c). data structures and robust geometric computation. The overall objective of the project-team is to give effective computational geometry and topology solid mathematical and algorithmic foundations, to provide solutions to key problems as well as to validate theoretical advances through extensive experimental research and the development of software packages that may serve as steps toward a standard for reliable and effective geometric computing. Most notably, Geometrica, together with several partners in Europe, plays a prominent role in the development of CGAL, a large library of computational geometry algorithms.

3. Research Program

3.1. Mesh Generation and Geometry Processing

Meshes are becoming commonplace in a number of applications ranging from engineering to multimedia through biomedecine and geology. For rendering, the quality of a mesh refers to its approximation properties. For numerical simulation, a mesh is not only required to faithfully approximate the domain of simulation, but also to satisfy size as well as shape constraints. The elaboration of algorithms for automatic mesh generation is a notoriously difficult task as it involves numerous geometric components: Complex data structures and algorithms, surface approximation, robustness as well as scalability issues. The recent trend to reconstruct domain boundaries from measurements adds even further hurdles. Armed with our experience on triangulations and algorithms, and with components from the CGAL library, we aim at devising robust algorithms for 2D, surface, 3D mesh generation as well as anisotropic meshes. Our research in mesh generation primarily focuses on the generation of simplicial meshes, i.e. triangular and tetrahedral meshes. We investigate both greedy approaches based upon Delaunay refinement and filtering, and variational approaches based upon energy functionals and associated minimizers.

The search for new methods and tools to process digital geometry is motivated by the fact that previous attempts to adapt common signal processing methods have led to limited success: Shapes are not just another signal but a new challenge to face due to distinctive properties of complex shapes such as topology, metric, lack of global parameterization, non-uniform sampling and irregular discretization. Our research in geometry processing ranges from surface reconstruction to surface remeshing through curvature estimation, principal component analysis, surface approximation and surface mesh parameterization. Another focus is on the robustness of the algorithms to defect-laden data. This focus stems from the fact that acquired geometric data obtained through measurements or designs are rarely usable directly by downstream applications. This generates bottlenecks, i.e., parts of the processing pipeline which are too labor-intensive or too brittle for practitioners. Beyond reliability and theoretical foundations, our goal is to design methods which are also robust to raw, unprocessed inputs.

3.2. Topological and Geometric Inference

Due to the fast evolution of data acquisition devices and computational power, scientists in many areas are asking for efficient algorithmic tools for analyzing, manipulating and visualizing more and more complex shapes or complex systems from approximative data. Many of the existing algorithmic solutions which come with little theoretical guarantee provide unsatisfactory and/or unpredictable results. Since these algorithms take as input discrete geometric data, it is mandatory to develop concepts that are rich enough to robustly and correctly approximate continuous shapes and their geometric properties by discrete models. Ensuring the correctness of geometric estimations and approximations on discrete data is a sensitive problem in many applications.

Data sets being often represented as point sets in high dimensional spaces, there is a considerable interest in analyzing and processing data in such spaces. Although these point sets usually live in high dimensional spaces, one often expects them to be located around unknown, possibly non linear, low dimensional shapes. These shapes are usually assumed to be smooth submanifolds or more generally compact subsets of the ambient space. It is then desirable to infer topological (dimension, Betti numbers,...) and geometric characteristics (singularities, volume, curvature,...) of these shapes from the data. The hope is that this information will help to better understand the underlying complex systems from which the data are generated. In spite of recent promising results, many problems still remain open and to be addressed, need a tight collaboration between mathematicians and computer scientists. In this context, our goal is to contribute to the development of new mathematically well founded and algorithmically efficient geometric tools for data analysis and processing of complex geometric objects. Our main targeted areas of application include machine learning, data mining, statistical analysis, and sensor networks.

3.3. Data Structures and Robust Geometric Computation

GEOMETRICA has a large expertise of algorithms and data structures for geometric problems.We are pursuing efforts to design efficient algorithms from a theoretical point of view, but we also put efforts in the effective implementation of these results.

In the past years, we made significant contributions to algorithms for computing Delaunay triangulations (which are used by meshes in the above paragraph). We are still working on the practical efficiency of existing algorithms to compute or to exploit classical Euclidean triangulations in 2 and 3 dimensions, but the current focus of our research is more aimed towards extending the triangulation efforts in several new directions of research.

One of these directions is the triangulation of non Euclidean spaces such as periodic or projective spaces, with various potential applications ranging from astronomy to granular material simulation.

Another direction is the triangulation of moving points, with potential applications to fluid dynamics where the points represent some particles of some evolving physical material, and to variational methods devised to optimize point placement for meshing a domain with a high quality elements.

Increasing the dimension of space is also a stimulating direction of research, as triangulating points in medium dimension (say 4 to 15) has potential applications and raises new challenges to trade exponential complexity of the problem in the dimension for the possibility to reach effective and practical results in reasonably small dimensions.

On the complexity analysis side, we pursue efforts to obtain complexity analysis in some practical situations involving randomized or stochastic hypotheses. On the algorithm design side, we are looking for new paradigms to exploit parallelism on modern multicore hardware architectures.

Finally, all this work is done while keeping in mind concerns related to effective implementation of our work, practical efficiency and robustness issues which have become a background task of all different works made by GEOMETRICA.

4. Application Domains

4.1. Main Application Domains

Our work is mostly of a fundamental nature but finds applications in a variety of application domains. Transfer is mostly conducted via GeometryFactory, the startup company that commercializes CGAL (see Section 8.1.2).

- Medical Imaging
- Numerical simulation
- Geometric modeling
- Visualization
- Data analysis

4.2. Secondary Application Domains

- Geographic information systems
- Geophysics
- Astrophysics
- Material physics

5. Highlights of the Year

5.1. Highlights of the Year

5.1.1. Awards

Clément Maria has been awarded the Prix de thèse Gilles Kahn - Académie des Sciences.

5.1.2. Books

Steve Oudot published a book on persistence theory in the AMS series *Mathematical Surveys and Monographs* [35].

6. New Software and Platforms

6.1. GUDHI

Geometric Understanding in Higher Dimensions

SCIENTIFIC DESCRIPTION

The GUDHI open source library will provide the central data structures and algorithms that underly applications in geometry understanding in higher dimensions. It is intended to both help the development of new algorithmic solutions inside and outside the project, and to facilitate the transfer of results in applied fields.

FUNCTIONAL DESCRIPTION

The current release of the GUDHI library includes: – Data structures to represent, construct and manipulate simplicial complexes. – Algorithms to compute persistent homology and multi-field persistent homology. – Simplification methods via implicit representations. - A graphical user interface and several examples and datasets.
It also has improved performance, portability and documentation.

- Participants: Jean-Daniel Boissonnat, Marc Glisse, Anatole Moreau, Vincent Rouvreau and David Salinas
- Contact: Jean-Daniel Boissonnat
- URL: https://project.inria.fr/gudhi/software/

6.2. CGAL dD Triangulations

CGAL module: Triangulations in any dimension

KEYWORDS: Triangulation - Delaunay triangulation
FUNCTIONAL DESCRIPTION

This package of CGAL (Computational Geometry Algorithms Library, http://www.cgal.org) allows to compute triangulations and Delaunay triangulations in any dimension. Those triangulations are built incrementally and can be modified by insertion or removal of vertices.

- Participants: Samuel Hornus, Olivier Devillers and Clément Jamin
- Contact: Clément Jamin
- URL: http://doc.cgal.org/4.6/Triangulation/

6.3. CGAL Kernel_d

CGAL module: High-dimensional kernel Epick_d

FUNCTIONAL DESCRIPTION

Several functions were added in release 4.7 in preparation for a future alpha-complex implementation.

- Participants: Marc Glisse
- Contact: Marc Glisse
- URL: http://doc.cgal.org/4.7/Kernel_d/

6.4. R package TDA

Topological Data Analysis package for the R software

FUNCTIONAL DESCRIPTION

the R package TDA provides some tools for Topological Data Analysis. In particular, it includes implementations of functions that, given some data, provide topological information about the underlying space, such as the distance function, the distance to a measure, the kNN density estimator, the kernel density estimator, and the kernel distance.

- Participants: Clément Maria, Vincent Rouvreau
- Contact: Vincent Rouvreau
- URL: https://cran.r-project.org/web/packages/TDA/index.html

6.5. cgal Periodic Triangulations and Meshes

The CGAL library offers a package to compute the 3D periodic Delaunay triangulation of a point set in $\mathbb{R}^3$, more precisely the Delaunay triangulation of a point set in the 3-dimensional flat torus with cubic domain [49]. The package has been used in various fields. [1]

We have been extending this package in three directions:

First, a few new small functions have been added to the Delaunay triangulation class and integrated in CGAL 4.7.

We have developed and documented some new classes allowing to compute *weighted* periodic Delaunay triangulations. They have been submitted to the CGAL editorial board and accepted for inclusion in CGAL. The code still needs some polishing, and the testsuite must be completed, before a public distribution in CGAL.

We have continued our work to use this package together with the 3D mesh generation package of CGAL [48], in order to propose a construction of meshes of periodic volumes. Although last year's preliminary results were already convincing [50], [51], the work is not ready yet for being submitted to CGAL: the code requires to be completed, documented, and extensively tested.

- Participant : Aymeric Pellé
- Contact: Monique Teillaud (Vegas project-team)
- This work was done in the framework of the Inria ADT *(Action de Développement Technologique)* OrbiCGAL http://www.loria.fr/~teillaud/ADT-OrbiCGAL/

7. New Results

7.1. Mesh Generation and Geometry processing

7.1.1. Discrete Derivatives of Vector Fields on Surfaces An Operator Approach

Participants: Frédéric Chazal, Maksim Ovsjanikov.

In collaboration with O. Azencot, M. Ben Chen (Technion, Israel Institute of Technology).

Vector fields on surfaces are fundamental in various applications in computer graphics and geometry processing. In many cases, in addition to representing vector fields, the need arises to compute their derivatives, for example, for solving partial differential equations on surfaces or for designing vector fields with prescribed smoothness properties. In this work, we consider the problem of computing the Levi-Civita covariant derivative, that is, the tangential component of the standard directional derivative, on triangle meshes. This problem is challenging since, formally, tangent vector fields on polygonal meshes are often viewed as being discontinuous, hence it is not obvious what a good derivative formulation would be. We leverage the relationship between the Levi-Civita covariant derivative of a vector field and the directional derivative of its component functions to provide a simple, easy-to-implement discretization for which we demonstrate experimental convergence. In addition, we introduce two linear operators which provide access to additional constructs in Riemannian geometry that are not easy to discretize otherwise, including the parallel transport operator which can be seen simply as a certain matrix exponential. Finally, we show the applicability of our operator to various tasks, such as fluid simulation on curved surfaces and vector field design, by posing algebraic constraints on the covariant derivative operator.

[1] see http://www.cgal.org/projects.html

7.1.2. Isotopic Meshing within a Tolerance Volume

Participant: David Cohen-Steiner.

In collaboration with M. Mandad, P. Alliez (Titane Project-team).

We give an algorithm [22] that generates from an input tolerance volume a surface triangle mesh guaranteed to be within the tolerance, intersection free and topologically correct. A pliant meshing algorithm is used to capture the topology and discover the anisotropy in the input tolerance volume in order to generate a concise output. We first refine a 3D Delaunay triangulation over the tolerance volume while maintaining a piecewise-linear function on this triangulation, until an isosurface of this function matches the topology sought after. We then embed the isosurface into the 3D triangulation via mutual tessellation, and simplify it while preserving the topology. Our approach extends to surfaces with boundaries and to non-manifold surfaces. We demonstrate the versatility and efficacy of our approach on a variety of data sets and tolerance volumes.

7.1.3. CGALmesh: A Generic Framework for Delaunay Mesh Generation

Participants: Jean-Daniel Boissonnat, Clément Jamin, Mariette Yvinec.

In collaboration with P. Alliez (Titane Project-team).

CGALmesh [21] is the mesh generation software package of the Computational Geometry Algorithm Library (CGAL). It generates isotropic simplicial meshes—surface triangular meshes or volume tetrahedral meshes—from input surfaces, 3D domains, and 3D multidomains, with or without sharp features. The underlying meshing algorithm relies on restricted Delaunay triangulations to approximate domains and surfaces and on Delaunay refinement to ensure both approximation accuracy and mesh quality. CGALmesh provides guarantees on approximation quality and on the size and shape of the mesh elements. It provides four optional mesh optimization algorithms to further improve the mesh quality. A distinctive property of CGALmesh is its high flexibility with respect to the input domain representation. Such a flexibility is achieved through a careful software design, gathering into a single abstract concept, denoted by the oracle, all required interface features between the meshing engine and the input domain. We already provide oracles for domains defined by polyhedral and implicit surfaces.

7.2. Topological and Geometric Inference

7.2.1. Subsampling Methods for Persistent Homology

Participants: Frédéric Chazal, Bertrand Michel.

In collaboration with B.T. Fasy, F. Lecci, A. Rinaldo and L. Wasserman (Carnegie Mellon University).

Persistent homology is a multiscale method for analyzing the shape of sets and functions from point cloud data arising from an unknown distribution supported on those sets. When the size of the sample is large, direct computation of the persistent homology is prohibitive due to the combinatorial nature of the existing algorithms. We propose to compute the persistent homology of several subsamples of the data and then combine the resulting estimates. We study the risk of two estimators and we prove that the subsampling approach carries stable topological information while achieving a great reduction in computational complexity.

7.2.2. Efficient and Robust Persistent Homology for Measures

Participants: Frédéric Chazal, Steve Oudot.

In collaboration with M. Buchet (Ohio State University) and Donald Sheehy (University of Connecticut).

A new paradigm for point cloud data analysis has emerged recently, where point clouds are no longer treated as mere compact sets but rather as empirical measures. A notion of distance to such measures has been defined and shown to be stable with respect to perturbations of the measure. This distance can eas- ily be computed pointwise in the case of a point cloud, but its sublevel-sets, which carry the geometric infor- mation about the measure, remain hard to compute or approximate. This makes it challenging to adapt many powerful techniques based on the Euclidean distance to a point cloud to the more general setting of the distance to a measure on a metric space. We propose [28] an efficient and reliable scheme to approximate the topological structure of the family of sublevel-sets of the distance to a measure. We obtain an algorithm for approximating the persistent homology of the distance to an empirical measure that works in arbitrary metric spaces. Precise quality and complexity guarantees are given with a discussion on the behavior of our approach in practice.

7.2.3. *Topological analysis of scalar fields with outliers*

Participants: Frédéric Chazal, Steve Oudot.

In collaboration with M. Buchet, T.K. Dey, F. Fan, Y. Wang (Ohio State University).

Given a real-valued function f defined over a manifold M embedded in Euclidean space, we are interested in recovering structural information about f from the sole information of its values on a finite sample P [27]. Existing methods provide approximation to the persistence diagram of f when the noise is bounded in both the functional and geometric domains. However, they fail in the presence of aberrant values, also called outliers, both in theory and practice. We propose a new algorithm that deals with outliers. We handle aberrant functional values with a method inspired from the k-nearest neighbors regression and the local median filtering, while the geometric outliers are handled using the distance to a measure. Combined with topological results on nested filtrations, our algorithm performs robust topological analysis of scalar fields in a wider range of noise models than handled by current methods. We provide theoretical guarantees on the quality of our approximation and some experimental results illustrating its behavior.

7.2.4. *Zigzag Persistence via Reflections and Transpositions*

Participants: Clément Maria, Steve Oudot.

We introduce [33] a simple algorithm for computing zigzag persistence, designed in the same spirit as the standard persistence algorithm. Our algorithm reduces a single matrix, maintains an explicit set of chains encoding the persistent homology of the current zigzag, and updates it under simplex insertions and removals. The total worst-case running time matches the usual cubic bound.

A noticeable difference with the standard persistence algorithm is that we do not insert or remove new simplices "at the end" of the zigzag, but rather "in the middle". To do so, we use arrow reflections and transpositions, in the same spirit as reflection functors in quiver theory. Our analysis introduces a new kind of reflection called the "weak-diamond", for which we are able to predict the changes in the interval decomposition and associated compatible bases. Arrow transpositions have been studied previously in the context of standard persistent homology, and we extend the study to the context of zigzag persistence. For both types of transformations, we provide simple procedures to update the interval decomposition and associated compatible homology basis.

7.2.5. *Stable Topological Signatures for Points on 3D Shapes*

Participants: Mathieu Carrière, Steve Oudot, Maksims Ovsjanikovs.

Comparing points on 3D shapes is among the fundamental operations in shape analysis. To facilitate this task, a great number of local point signatures or descriptors have been proposed in the past decades. However, the vast majority of these descriptors concentrate on the local geometry of the shape around the point, and thus are insensitive to its connectivity structure. By contrast, several *global* signatures have been proposed that successfully capture the overall topology of the shape and thus characterize the shape as a whole. We propose [29], [43] the first point descriptor that captures the topology structure of the shape as ‘seen’ from a single point, in a multiscale and provably stable way. We also demonstrate how a large class of topological signatures, including ours, can be mapped to vectors, opening the door to many classical analysis and learning methods. We illustrate the performance of this approach on the problems of supervised shape labeling and shape matching. We show that our signatures provide complementary information to existing ones and allow to achieve better performance with less training data in both applications.

7.2.6. Structure and Stability of the 1-Dimensional Mapper

Participants: Mathieu Carrière, Steve Oudot.

Given a continuous function $f : X \to \mathbb{R}$ and a cover I of its image by intervals, the Mapper is the nerve of a refinement of the pullback cover $f^{-1}(I)$. Despite its success in applications, little is known about the structure and stability of this construction from a theoretical point of view. As a pixelized version of the Reeb graph of f, it is expected to capture a subset of its features (branches, holes), depending on how the interval cover is positioned with respect to the critical values of the function. Its stability should also depend on this positioning. We propose [44] a theoretical framework that relates the structure of the Mapper to the one of the Reeb graph, making it possible to predict which features will be present and which will be absent in the Mapper given the function and the cover, and for each feature, to quantify its degree of unstability. Using this framework, we can derive guarantees on the structure of the Mapper, on its stability, and on its convergence to the Reeb graph as the granularity of the cover I goes to zero.

7.2.7. Persistence Theory: From Quiver Representations to Data Analysis

Participant: Steve Oudot.

Persistence theory emerged in the early 2000s as a new theory in the area of applied and computational topology. This book [35] provides a broad and modern view of the subject, including its algebraic, topological, and algorithmic aspects. It also elaborates on applications in data analysis. The level of detail of the exposition has been set so as to keep a survey style, while providing sufficient insights into the proofs so the reader can understand the mechanisms at work.

7.3. Data Structures and Robust Geometric Computation

7.3.1. A probabilistic approach to reducing the algebraic complexity of computing Delaunay triangulations

Participant: Jean-Daniel Boissonnat.

In collaboration with Ramsay Dyer (Johann Bernoulli Institute, University of Groningen, Netherlands) and Arijit Ghosh (Max-Planck-Institut für Informatik, Saarbrücken, Germany).

Computing Delaunay triangulations in $\mathbb{R}^d$ involves evaluating the so-called in_sphere predicate that determines if a point x lies inside, on or outside the sphere circumscribing $d+1$ points $p_0, ..., p_d$. This predicate reduces to evaluating the sign of a multivariate polynomial of degree $d+2$ in the coordinates of the points $x, p_0, ..., p_d$. Despite much progress on exact geometric computing, the fact that the degree of the polynomial increases with d makes the evaluation of the sign of such a polynomial problematic except in very low dimensions. In this paper, we propose a new approach that is based on the witness complex, a weak form of the Delaunay complex introduced by Carlsson and de Silva. The witness complex $\mathrm{Wit}(L, W)$ is defined from two sets L and W in some metric space X: a finite set of points L on which the complex is built, and a set W of witnesses that serves as an approximation of X. A fundamental result of de Silva states that $\mathrm{Wit}(L, W) = \mathrm{Del}(L)$ if $W = X = \mathbb{R}^d$. In [25], [41], we give conditions on L that ensure that the witness complex and the Delaunay triangulation coincide when W is a finite set, and we introduce a new perturbation scheme to compute a perturbed set L' close to L such that $\mathrm{Del}(L') = \mathrm{Wit}(L', W)$. Our perturbation algorithm is a geometric application of the Moser-Tardos constructive proof of the Lovász local lemma. The only numerical operations we use are (squared) distance comparisons (i.e., predicates of degree 2). The time-complexity of the algorithm is sublinear in $|W|$. Interestingly, although the algorithm does not compute any measure of simplex quality, a lower bound on the thickness of the output simplices can be guaranteed.

7.3.2. Smoothed complexity of convex hulls

Participants: Marc Glisse, Rémy Thomasse.

In collaboration with O. Devillers (VEGAS Project-team) and X. Goaoc (Université Marne-la-Vallée)

We establish an upper bound on the smoothed complexity of convex hulls in $\mathbb{R}^d$ under uniform Euclidean (ℓ^2) noise. Specifically, let $\{p_1^*, p_2^*, ..., p_n^*\}$ be an arbitrary set of n points in the unit ball in $\mathbb{R}^d$ and let $p_i = p_i^* + x_i$, where $x_1, x_2, ..., x_n$ are chosen independently from the unit ball of radius δ. We show that the expected complexity, measured as the number of faces of all dimensions, of the convex hull of $\{p_1, p_2, ..., p_n\}$ is $O\left(n^{2-\frac{4}{d+1}}(1+1/\delta)^{d-1}\right)$; the magnitude δ of the noise may vary with n. For $d=2$ this bound improves to $O\left(n^{\frac{2}{3}}(1+\delta^{-\frac{2}{3}})\right)$.

We also analyze the expected complexity of the convex hull of ℓ^2 and Gaussian perturbations of a nice sample of a sphere, giving a lower-bound for the smoothed complexity. We identify the different regimes in terms of the scale, as a function of n, and show that as the magnitude of the noise increases, that complexity varies monotonically for Gaussian noise but non-monotonically for ℓ^2 noise [31], [38].

7.3.3. Realization Spaces of Arrangements of Convex Bodies

Participant: Alfredo Hubard.

In collaboration with M. Dobbins (PosTech, South Korea) and A. Holmsen (KAIST, South Korea)

In [23], we introduce combinatorial types of arrangements of convex bodies, extending order types of point sets to arrangements of convex bodies, and study their realization spaces. Our main results witness a trade-off between the combinatorial complexity of the bodies and the topological complexity of their realization space. On one hand, we show that every combinatorial type can be realized by an arrangement of convex bodies and (under mild assumptions) its realization space is contractible. On the other hand, we prove a universality theorem that says that the restriction of the realization space to arrangements of convex polygons with a bounded number of vertices can have the homotopy type of any primary semialgebraic set.

7.3.4. Limits of order types

Participant: Alfredo Hubard.

In collaboration with X. Goaoc (Institut G. Monge), R. de Joannis de Verclos (CNRS-INPG), J-S. Sereni (LORIA), and J. Volec (ETH)

The notion of limits of dense graphs was invented, among other reasons, to attack problems in extremal graph theory. It is straightforward to define limits of order types in analogy with limits of graphs, and in [24] we examine how to adapt to this setting two approaches developed to study limits of dense graphs. We first consider flag algebras, which were used to open various questions on graphs to mechanical solving via semidefinite programming. We define flag algebras of order types, and use them to obtain, via the semidefinite method, new lower bounds on the density of 5- or 6-tuples in convex position in arbitrary point sets, as well as some inequalities expressing the difficulty of sampling order types uniformly. We next consider graphons, a representation of limits of dense graphs that enable their study by continuous probabilistic or analytic methods. We investigate how planar measures fare as a candidate analogue of graphons for limits of order types. We show that the map sending a measure to its associated limit is continuous and, if restricted to uniform measures on compact convex sets, a homeomorphism. We prove, however, that this map is not surjective. Finally, we examine a limit of order types similar to classical constructions in combinatorial geometry (Erdös-Szekeres, Horton...) and show that it cannot be represented by any somewhere regular measure; we analyze this example via an analogue of Sylvester's problem on the probability that k random points are in convex position.

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

8.1.1. Cifre Contract with Geometry Factory

Mael Rouxel-Labbé's PhD thesis is supported by a Cifre contract with GEOMETRY FACTORY (http://www.geometryfactory.com). The subject is the generation of anisotropic meshes.

8.1.2. Commercialization of cgal packages through Geometry Factory

In 2015, GEOMETRY FACTORY (http://www.geometryfactory.com) had the following new customers for CGAL packages developed by GEOMETRICA:

CSM3D (UK, Cad chaussures): surface parametrization

Silvaco (USA, simulation) : 3d mesh generation

Cimmi (Canada): Approximation of Ridges and Umbilics on Triangulated Surface Meshes, Estimation of Local Differential Properties, AABB Tree, Principal Compoment Analysis, Point Set Processing

Varel (France, forage): 2D triangulations

Powel (Norway, GIS): point set processing, surface reconstruction

ExxonMobil (USA) : 2D triangulations, surface parametrization

Metrologic (France, metrology): point set processing

Geomage (Israel, oil&gas): 2D and 3D triangulations

Corvid (USA, simulation) : 3D triangulations

Medicim (Belgium, medical imaging): 3D mesh generation

Huntsman (Belgium), Pasco (Japan), Qualcomm (USA), Facebook (USA): industrial research licenses

9. Partnerships and Cooperations

9.1. National Initiatives

9.1.1. ANR Présage

Participants: Marc Glisse, Rémy Thomasse.

- Acronym: Presage.

- Type: ANR blanc.

- Title: *méthodes PRobabilistes pour l'Éfficacité des Structures et Algorithmes GÉométriques*.

- Coordinator: Xavier Goaoc.

- Duration: 31 december 2011 - 31 december 2015.

- Other partners: Inria VEGAS team, University of Rouen.

- Abstract: This project brings together computational and probabilistic geometers to tackle new probabilistic geometry problems arising from the design and analysis of geometric algorithms and data structures. We focus on properties of discrete structures induced by or underlying random continuous geometric objects. This raises questions such as:

- What does a random geometric structure (convex hulls, tessellations, visibility regions...) look like?
- How to analyze and optimize the behavior of classical geometric algorithms on *usual* inputs?
- How can we generate randomly *interesting* discrete geometric structures?

9.1.2. ANR TOPDATA

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Mariette Yvinec, Steve Oudot, Marc Glisse, Clément Levrard.

- Acronym : TopData.

- Type : ANR blanc.

- Title : Topological Data Analysis: Statistical Methods and Inference.

- Coordinator : Frédéric Chazal (GEOMETRICA).

- Duration : 4 years starting October 2013.

- Others Partners: Département de Mathématiques (Université Paris Sud), Institut de Mathématiques (Université de Bourgogne), LPMA (Université Paris Diderot), LSTA (Université Pierre et Marie Curie).

- Abstract: TopData aims at designing new mathematical frameworks, models and algorithmic tools to infer and analyze the topological and geometric structure of data in different statistical settings. Its goal is to set up the mathematical and algorithmic foundations of Statistical Topological and Geometric Data Analysis and to provide robust and efficient tools to explore, infer and exploit the underlying geometric structure of various data.

Our conviction, at the root of this project, is that there is a real need to combine statistical and topological/geometric approaches in a common framework, in order to face the challenges raised by the inference and the study of topological and geometric properties of the wide variety of larger and larger available data. We are also convinced that these challenges need to be addressed both from the mathematical side and the algorithmic and application sides. Our project brings together in a unique way experts in Statistics, Geometric Inference and Computational Topology and Geometry. Our common objective is to design new theoretical frameworks and algorithmic tools and thus to contribute to the emergence of a new field at the crossroads of these domains. Beyond the purely scientific aspects we hope this project will help to give birth to an active interdisciplinary community. With these goals in mind we intend to promote, disseminate and make our tools available and useful for a broad audience, including people from other fields.

- See also: http://geometrica.saclay.inria.fr/collaborations/TopData/Home.html

9.2. European Initiatives

9.2.1. FP7 & H2020 Projects

9.2.1.1. ERC GUDHI

Title: Algorithmic Foundations of Geometry Understanding in Higher Dimensions.

Program: FP7.

Type: ERC.

Duration: February 2014 - January 2019.

Coordinator: Inria.

PI: Jean-Daniel Boissonnat.

'The central goal of this proposal is to settle the algorithmic foundations of geometry understanding in dimensions higher than 3. We coin the term geometry understanding to encompass a collection of tasks including the computer representation and the approximation of geometric structures, and the inference of geometric or topological properties of sampled shapes. The need to understand geometric structures is ubiquitous in science and has become an essential part of scientific computing and data analysis. Geometry understanding is by no means limited to three dimensions. Many applications in physics, biology, and engineering require a keen understanding of the geometry of a variety of higher dimensional spaces to capture concise information from the underlying often highly nonlinear structure of data. Our approach is complementary to manifold learning techniques and aims at developing an effective theory for geometric and topological data analysis. To reach these objectives, the guiding principle will be to foster a symbiotic relationship between theory and practice, and to address fundamental research issues along three parallel advancing fronts. We will simultaneously develop mathematical approaches providing theoretical guarantees, effective algorithms that are amenable to theoretical analysis and rigorous experimental validation, and perennial software development. We will undertake the development of a high-quality open source software platform to implement the most important geometric data structures and algorithms at the heart of geometry understanding in higher dimensions. The platform will be a unique vehicle towards researchers from other fields and will serve as a basis for groundbreaking advances in scientific computing and data analysis.'

9.3. International Initiatives

9.3.1. CATS

Title: Computations And Topological Statistics.

International Partner (Institution - Laboratory - Researcher):

Carnegie Mellon University (United States) - Department of Statistics - Larry Wasserman

Start year: 2015.

See also: http://geometrica.saclay.inria.fr/collaborations/CATS/CATS.html

Topological Data Analysis (TDA) is an emergent field attracting interest from various communities, that has recently known academic and industrial successes. Its aim is to identify and infer geometric and topological features of data to develop new methods and tools for data exploration and data analysis. TDA results mostly rely on deterministic assumptions which are not satisfactory from a statistical viewpoint and which lead to a heuristic use of TDA tools in practice. Bringing together the strong expertise of two groups in Statistics (L. Wasserman's group at CMU) and Computational Topology and Geometry (Inria Geometrica), the main objective of CATS is to set-up the mathematical foundations of Statistical TDA, to design new TDA methods and to develop efficient and easy-to-use software tools for TDA.

9.4. International Research Visitors

9.4.1. Visits of International Scientists

Ramsay Dyer (University of Groningen), May
Arijit Ghosh (MPII, Saarbrucken), June-July
Clément Maria (Queen's College, Brisbane), June
Omer Brobowski (Duke University), May
Jessica Cisewski (Carnegie Mellon), October
Jisu Kim (Carnegie Mellon), May-July
Yanir Kleiman (Tel Aviv University), October
Bertrand Michel (Paris 6), 2015
Jan Felix Senge (Bremen), October
Primoz Skraba (Jozef Stefan Institute), May
Kelly Spendlove (Rutgers), May-July
Jian Sun (Tsinghua), February
Justin Solomon (Stanford), February

9.4.1.1. Internships

Sivaprasad Sudhir (IIT Bombay), June-July
Stéphane Lundy (Supélec), July-August
Siargey Kachanovich (ENS Rennes), March-August
Anatole Moreau (EPITA), May-August
Tullia Padellini (Roma University), May-September
Yuping Ren (Erasmus), January-July

9.4.2. Visits to International Teams

9.4.2.1. Research stays abroad

+ Steve Oudot spent 1 month in July-August in the group of Benjamin Burton at the Pure Maths Department of University of Queensland, Australia.

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific events organisation

10.1.1.1. General chair, scientific chair

- Jean-Daniel Boissonnat and Frédéric Chazal co-organized the joint GUDHI-TOPDATA workshop in Porquerolles, October 20-24.
- Frédéric Chazal co-organised the Focused program on “Functoriality in Geometric Data” at the Institute for Advanced Study, HKUST Hong-Kong, April 2015.

10.1.2. Scientific events selection

10.1.2.1. Member of the conference program committees

Frédéric Chazal: Geometric Science of Information 2015 (GSI 2015).

10.1.3. Journal

10.1.3.1. Member of the editorial boards

Jean-Daniel Boissonnat is a member of the Editorial Board of *Journal of the ACM*, *Discrete and Computational Geometry*, *Algorithmica*, *International Journal on Computational Geometry and Applications*.

Frédéric Chazal is a member of the Editorial Board of *SIAM Journal on Imaging Sciences*, *Discrete and Computational Geometry*, *Graphical Models*.

Steve Oudot is a member of the Editorial Board of *Journal of Computational Geometry*.

10.1.4. Invited talks

Frédéric Chazal, Focused program on “Functoriality in Geometric Data”, Institute for Advanced Study, HKUST Hong-Kong, April 2015.

Frédéric Chazal, Geometry and Data Analysis conference, Stevanovich Center for financial mathematics, University of Chicago, June 2015.

Frédéric Chazal, Mini-conference on Topological Data Analysis, Oxford University, June 2015.

Frédéric Chazal, European Meeting of Statisticians (EMS 2015), Amsterdam, July 2015.

Frédéric Chazal, 2nda Escuela/Conferencia de Análisis Topológico de Datos, Queretaro, Mexico, December 2015

Steve Oudot, *GETCO* (*Applications of Algebraic Topology in Computer Science and Data Analysis*), Aalborg, April 2015.

10.1.5. Scientific expertise

Frédéric Chazal was a member of the ANR committee, CES 40 (Mathematics and Computer Science).

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Master : S. Oudot, *Computational Geometry: from Theory to Applications*, 18h, École polytechnique.

Master : S. Oudot, *Topological Data Analysis*, 36h, École polytechnique.

Master : S. Oudot, *Geometric Methods for Data Analysis*, 12h, École Centrale Paris.

Master : J.D. Boissonnat, *Winter School : “Algorithmic Geometry of Triangulations”*, January 2015.

Master : S. Oudot, *Summer School BioMed : "Topological Descriptors for Geometric Data Comparison"*, July 2015.

Master : S. Oudot, *Young Researchers School at the* Journées de l'AFIG*: "Topological Methods for 3D Processing"*, November 2015.

Doctorat : Frédéric Chazal, Topology and Geometry of Data Analysis, 9h eq-TD, nd EATCS Young Researchers School and TopDrim School "Understanding Complexity and Concurrency through Topology and Data", Camerino, Italy, July 2015.

10.2.2. Supervision

PhD in progress : Mael Rouxel-Labbé, Anisotropic Mesh Generation, started October 1st, 2013, Jean-Daniel Boissonnat and Mariette Yvinec.

PhD in progress : Alba Chiara de Vitis, Computational Geometry in High Dimensions, started February 2014. Jean-Daniel Boissonnat and David Cohen-Steiner.

PhD in progress : Siargey Kachanovich : Manifold Learning, started October 2015. Jean-Daniel Boissonnat

PhD in progress: Thomas Bonis, Statistical Learning Algorithms for Geometric and Topological Data Analysis, started December 1st, 2013, Frédéric Chazal.

PhD in progress: Ruqi Huang, Algorithms for topological inference in metric spaces, started December 1st, 2013, Frédéric Chazal.

PhD in progress: Eddie Aamari, A Statistical Approach of Topological Data Analysis, started September 1st, 2014, Frédéric Chazal (co-advised by Pascal Massart).

PhD in progress: Claire Brécheteau, Statistical aspects of distance-like functions , started September 1st, 2015, Frédéric Chazal (co-advised by Pascal Massart).

PhD in progress: Jérémy Cochoy, Zigzag persistence: stability and applications to Topological Data Analysis, started September 1st, 2015, Steve Oudot (co-advised by F. Chazal).

PhD in progress: Mathieu Carrière, Signatures for Geometric Shapes, started September 1st, 2014, Steve Oudot.

10.2.3. Juries

Jean-Daniel Boissonnat was a member of the HDR defense committee of Quentin Mérigot (Université de Grenoble).

Frédéric Chazal was a member of the HDR defense committee of Bertrand Michel (Université Pierre et Marie Curie).

Frédéric Chazal was a member (and reviewer) of the PhD defense committee of Julien André (Université Grenoble).

Frédéric Chazal was a member of the PhD defense committee of Stéphane Calderon (Telecom Paris).

Frédéric Chazal was a member of the PhD defense committee of Fabrizio Lecci (Carnegie Mellon University).

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Major publications by the team in recent years

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[11] R. Thomasse. *Complexity analysis of random convex hulls*, Univeristé Nice Sophia Antipolis, December 2015, https://hal.inria.fr/tel-01252937

Articles in International Peer-Reviewed Journals

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