



Activity Report 2014

Project-Team **GEOMETRICA**

Geometric computing

RESEARCH CENTERS
Sophia Antipolis - Méditerranée
Saclay - Île-de-France

THEME
Algorithmics, Computer Algebra and
Cryptology

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Project-Team GEOMETRICA

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Creation of the Project-Team: 2003 July 01.

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2. Overall Objectives

2.1. Overall Objectives

Research carried out by the Geometrica project team is dedicated to Computational Geometry and Topology and follows three major directions: (a). mesh generation and geometry processing; (b). topological and geometric inference; (c). data structures and robust geometric computation. The overall objective of the project-team is to give effective computational geometry and topology solid mathematical and algorithmic foundations, to provide solutions to key problems as well as to validate theoretical advances through extensive experimental research and the development of software packages that may serve as steps toward a standard for reliable and effective geometric computing. Most notably, Geometrica, together with several partners in Europe, plays a prominent role in the development of CGAL, a large library of computational geometry algorithms.

3. Research Program

3.1. Mesh Generation and Geometry Processing

Meshes are becoming commonplace in a number of applications ranging from engineering to multimedia through biomedecine and geology. For rendering, the quality of a mesh refers to its approximation properties. For numerical simulation, a mesh is not only required to faithfully approximate the domain of simulation, but also to satisfy size as well as shape constraints. The elaboration of algorithms for automatic mesh generation is a notoriously difficult task as it involves numerous geometric components: Complex data structures and algorithms, surface approximation, robustness as well as scalability issues. The recent trend

to reconstruct domain boundaries from measurements adds even further hurdles. Armed with our experience on triangulations and algorithms, and with components from the CGAL library, we aim at devising robust algorithms for 2D, surface, 3D mesh generation as well as anisotropic meshes. Our research in mesh generation primarily focuses on the generation of simplicial meshes, i.e. triangular and tetrahedral meshes. We investigate both greedy approaches based upon Delaunay refinement and filtering, and variational approaches based upon energy functionals and associated minimizers.

The search for new methods and tools to process digital geometry is motivated by the fact that previous attempts to adapt common signal processing methods have led to limited success: Shapes are not just another signal but a new challenge to face due to distinctive properties of complex shapes such as topology, metric, lack of global parameterization, non-uniform sampling and irregular discretization. Our research in geometry processing ranges from surface reconstruction to surface remeshing through curvature estimation, principal component analysis, surface approximation and surface mesh parameterization. Another focus is on the robustness of the algorithms to defect-laden data. This focus stems from the fact that acquired geometric data obtained through measurements or designs are rarely usable directly by downstream applications. This generates bottlenecks, i.e., parts of the processing pipeline which are too labor-intensive or too brittle for practitioners. Beyond reliability and theoretical foundations, our goal is to design methods which are also robust to raw, unprocessed inputs.

3.2. Topological and Geometric Inference

Due to the fast evolution of data acquisition devices and computational power, scientists in many areas are asking for efficient algorithmic tools for analyzing, manipulating and visualizing more and more complex shapes or complex systems from approximative data. Many of the existing algorithmic solutions which come with little theoretical guarantee provide unsatisfactory and/or unpredictable results. Since these algorithms take as input discrete geometric data, it is mandatory to develop concepts that are rich enough to robustly and correctly approximate continuous shapes and their geometric properties by discrete models. Ensuring the correctness of geometric estimations and approximations on discrete data is a sensitive problem in many applications.

Data sets being often represented as point sets in high dimensional spaces, there is a considerable interest in analyzing and processing data in such spaces. Although these point sets usually live in high dimensional spaces, one often expects them to be located around unknown, possibly non linear, low dimensional shapes. These shapes are usually assumed to be smooth submanifolds or more generally compact subsets of the ambient space. It is then desirable to infer topological (dimension, Betti numbers,...) and geometric characteristics (singularities, volume, curvature,...) of these shapes from the data. The hope is that this information will help to better understand the underlying complex systems from which the data are generated. In spite of recent promising results, many problems still remain open and to be addressed, need a tight collaboration between mathematicians and computer scientists. In this context, our goal is to contribute to the development of new mathematically well founded and algorithmically efficient geometric tools for data analysis and processing of complex geometric objects. Our main targeted areas of application include machine learning, data mining, statistical analysis, and sensor networks.

3.3. Data Structures and Robust Geometric Computation

GEOMETRICA has a large expertise of algorithms and data structures for geometric problems. We are pursuing efforts to design efficient algorithms from a theoretical point of view, but we also put efforts in the effective implementation of these results.

In the past years, we made significant contributions to algorithms for computing Delaunay triangulations (which are used by meshes in the above paragraph). We are still working on the practical efficiency of existing algorithms to compute or to exploit classical Euclidean triangulations in 2 and 3 dimensions, but the current focus of our research is more aimed towards extending the triangulation efforts in several new directions of research.

One of these directions is the triangulation of non Euclidean spaces such as periodic or projective spaces, with various potential applications ranging from astronomy to granular material simulation.

Another direction is the triangulation of moving points, with potential applications to fluid dynamics where the points represent some particles of some evolving physical material, and to variational methods devised to optimize point placement for meshing a domain with a high quality elements.

Increasing the dimension of space is also a stimulating direction of research, as triangulating points in medium dimension (say 4 to 15) has potential applications and raises new challenges to trade exponential complexity of the problem in the dimension for the possibility to reach effective and practical results in reasonably small dimensions.

On the complexity analysis side, we pursue efforts to obtain complexity analysis in some practical situations involving randomized or stochastic hypotheses. On the algorithm design side, we are looking for new paradigms to exploit parallelism on modern multicore hardware architectures.

Finally, all this work is done while keeping in mind concerns related to effective implementation of our work, practical efficiency and robustness issues which have become a background task of all different works made by GEOMETRICA.

4. Application Domains

4.1. Application Domains

- Medical Imaging
- Numerical simulation
- Geometric modeling
- Geographic information systems
- Visualization
- Data analysis
- Astrophysics
- Material physics

5. New Software and Platforms

5.1. CGAL, the Computational Geometry Algorithms Library

Participants: Jean-Daniel Boissonnat, Olivier Devillers, Marc Glisse, Aymeric Pellé, Monique Teillaud, Mariette Yvinec.

With the collaboration of Pierre Alliez, Hervé Brönnimann, Manuel Caroli, Pedro Machado Manhães de Castro, Frédéric Cazals, Frank Da, Christophe Delage, Andreas Fabri, Julia Flötotto, Philippe Guigue, Michael Hemmer, Samuel Hornus, Clément Jamin, Menelaos Karavelas, Sébastien Lorient, Abdelkrim Mebarki, Naceur Meskini, Andreas Meyer, Sylvain Pion, Marc Pouget, François Rebufat, Laurent Rineau, Laurent Saboret, Stéphane Tayeb, Jane Tournois, Radu Ursu, and Camille Wormser <http://www.cgal.org>

CGAL is a C++ library of geometric algorithms and data structures. Its development has been initially funded and further supported by several European projects (CGAL, GALIA, ECG, ACS, AIM@SHAPE) since 1996. The long term partners of the project are research teams from the following institutes: Inria Sophia Antipolis - Méditerranée, Max-Planck Institut Saarbrücken, ETH Zürich, Tel Aviv University, together with several others. In 2003, CGAL became an Open Source project (under the LGPL and QPL licenses).

The transfer and diffusion of CGAL in industry is achieved through the company GEOMETRY FACTORY (<http://www.geometryfactory.com>). GEOMETRY FACTORY is a *Born of Inria* company, founded by Andreas Fabri in January 2003. The goal of this company is to pursue the development of the library and to offer services in connection with CGAL (maintenance, support, teaching, advice). GEOMETRY FACTORY is a link between the researchers from the computational geometry community and the industrial users.

The aim of the CGAL project is to create a platform for geometric computing supporting usage in both industry and academia. The main design goals are genericity, numerical robustness, efficiency and ease of use. These goals are enforced by a review of all submissions managed by an editorial board. As the focus is on fundamental geometric algorithms and data structures, the target application domains are numerous: from geological modeling to medical images, from antenna placement to geographic information systems, etc.

The CGAL library consists of a kernel, a list of algorithmic packages, and a support library. The kernel is made of classes that represent elementary geometric objects (points, vectors, lines, segments, planes, simplices, isothetic boxes, circles, spheres, circular arcs...), as well as affine transformations and a number of predicates and geometric constructions over these objects. These classes exist in dimensions 2 and 3 (static dimension) and d (dynamic dimension). Using the template mechanism, each class can be instantiated following several representation modes: one can choose between Cartesian or homogeneous coordinates, use different number types to store the coordinates, and use reference counting or not. The kernel also provides some robustness features using some specifically-devised arithmetic (interval arithmetic, multi-precision arithmetic, static filters...).

A number of packages provide geometric data structures as well as algorithms. The data structures are polygons, polyhedra, triangulations, planar maps, arrangements and various search structures (segment trees, d -dimensional trees...). Algorithms are provided to compute convex hulls, Voronoi diagrams, Boolean operations on polygons, solve certain optimization problems (linear, quadratic, generalized of linear type). Through class and function templates, these algorithms can be used either with the kernel objects or with user-defined geometric classes provided they match a documented interface.

Finally, the support library provides random generators, and interfacing code with other libraries, tools, or file formats (ASCII files, QT or LEDA Windows, OpenGL, Open Inventor, Postscript, Geomview...). Partial interfaces with Python, SCILAB and the Ipe drawing editor are now also available.

GEOMETRICA is particularly involved in general maintenance, in the arithmetic issues that arise in the treatment of robustness issues, in the kernel, in triangulation packages and their close applications such as alpha shapes, in mesh generation and related packages. Two researchers of GEOMETRICA are members of the CGAL Editorial Board, whose main responsibilities are the control of the quality of CGAL, making decisions about technical matters, coordinating communication and promotion of CGAL.

CGAL is about 700,000 lines of code and supports various platforms: GCC (Linux, Mac OS X, Cygwin...), Visual C++ (Windows), Intel C++. A new version of CGAL is released twice a year, and it is downloaded about 10000 times a year. Moreover, CGAL is directly available as packages for the Debian, Ubuntu and Fedora Linux distributions.

More numbers about CGAL: there are now 12 editors in the editorial board, with approximately 20 additional developers. The user discussion mailing-list has more than 1000 subscribers with a relatively high traffic of 5-10 mails a day. The announcement mailing-list has more than 3000 subscribers.

5.1.1. *High-dimensional kernel Epick_d*

Participant: Marc Glisse.

We implemented a new high-dimensional kernel taking advantage of the progress that was made in dimensions 2 and 3. It is meant to be used with a reimplementations of high-dimensional triangulations (in progress).

5.1.2. *Number type Mpzf*

Participant: Marc Glisse.

We added a new exact ring number type that can represent all finite double floating-point numbers. It makes building a Delaunay triangulation 8 times faster than with earlier CGAL releases in some degenerate cases.

5.1.3. CGALmesh: a Generic Framework for Delaunay Mesh Generation

Participants: Jean-Daniel Boissonnat, Mariette Yvinec.

In collaboration with Pierre Alliez (EPI Titane), Clément Jamin (EPI Titane)

CGALmesh is the mesh generation software package of the Computational Geometry Algorithm Library (CGAL). It generates isotropic simplicial meshes – surface triangular meshes or volume tetrahedral meshes – from input surfaces, 3D domains as well as 3D multi-domains, with or without sharp features. The underlying meshing algorithm relies on restricted Delaunay triangulations to approximate domains and surfaces, and on Delaunay refinement to ensure both approximation accuracy and mesh quality. CGALmesh provides guarantees on approximation quality as well as on the size and shape of the mesh elements. It provides four optional mesh optimization algorithms to further improve the mesh quality. A distinctive property of CGALmesh is its high flexibility with respect to the input domain representation. Such a flexibility is achieved through a careful software design, gathering into a single abstract concept, denoted by the oracle, all required interface features between the meshing engine and the input domain. We already provide oracles for domains defined by polyhedral and implicit surfaces. [27] [53]

5.1.4. Periodic Meshes

Participants: Aymeric Pellé, Monique Teillaud.

There is a growing need for a 3D periodic mesh generator for various fields, such as material engineering or modeling of nano-structures. We are writing a software package answering this need, and which will be made publicly available in the open source library CGAL. The software is based on the CGAL 3D volume mesh generator package and the CGAL 3D periodic triangulations package. [42] [63]

5.2. Gudhi library

Participants: Jean-Daniel Boissonnat, Marc Glisse, Clément Maria, Mariette Yvinec.

With the collaboration of David Salinas

<https://project.inria.fr/gudhi/software/>

The GUDHI open source library will provide the central data structures and algorithms that underly applications in geometry understanding in higher dimensions. It is intended to both help the development of new algorithmic solutions inside and outside the project, and to facilitate the transfert of results in applied fields. The first release of the GUDHI library includes: – Data structures to represent, construct and manipulate simplicial complexes; – Algorithms to compute persistent homology and multi-field persistent homology.

6. New Results

6.1. Highlights of the Year

[10] was elected among the notable articles of 2013 by ACM and Computing Reviews (see http://computingreviews.com/recommend/bestof/notableitems_2013.cfm).

6.2. Mesh Generation and Geometry Processing

6.2.1. A Surface Reconstruction Method for In-Detail Underwater 3D Optical Mapping

Participant: Mariette Yvinec.

In collaboration with Pierre Alliez (EPI Titane), Ricard Campos (University of Girona), Raphael Garcia (University of Girona)

Underwater range scanning techniques are starting to gain interest in underwater exploration, providing new tools to represent the seafloor. These scans (often) acquired by underwater robots usually result in an unstructured point cloud, but given the common downward-looking or forward-looking configuration of these sensors with respect to the scene, the problem of recovering a piecewise linear approximation representing the scene is normally solved by approximating these 3D points using a heightmap (2.5D). Nevertheless, this representation is not able to correctly represent complex structures, especially those presenting arbitrary concavities normally exhibited in underwater objects. We present a method devoted to full 3D surface reconstruction that does not assume any specific sensor configuration. The method presented is robust to common defects in raw scanned data such as outliers and noise often present in extreme environments such as underwater, both for sonar and optical surveys. Moreover, the proposed method does not need a manual preprocessing step. It is also generic as it does not need any information other than the points themselves to work. This property leads to its wide application to any kind of range scanning technologies and we demonstrate its versatility by using it on synthetic data, controlled laser-scans, and multibeam sonar surveys. Finally, and given the unbeatable level of detail that optical methods can provide, we analyze the application of this method on optical datasets related to biology, geology and archeology. [23]

6.2.2. A Transfer Principle and Applications to Eigenvalue Estimates for Graphs

Participant: David Cohen-Steiner.

In collaboration with Omid Amini (ENS),

In this paper, we prove a variant of the Burger-Brooks transfer principle which, combined with recent eigenvalue bounds for surfaces, allows to obtain upper bounds on the eigenvalues of graphs as a function of their genus. More precisely, we show the existence of a universal constants C such that the k -th eigenvalue λ_k of the normalized Laplacian of a graph G of (geometric) genus g on n vertices satisfies $\lambda_k \leq C d_{max}(g + k)/n$ where d_{max} denotes the maximum valence of vertices of the graph. This result is tight up to a change in the value of the constant C . We also use our transfer theorem to relate eigenvalues of the Laplacian on a metric graph to the eigenvalues of its simple graph models, and discuss an application to the mesh partitioning problem. [44]

6.3. Topological and Geometric Inference

6.3.1. Only distances are required to reconstruct submanifolds

Participants: Jean-Daniel Boissonnat, Steve Oudot.

In collaboration with Ramsay Dyer (Johann Bernoulli Institute, University of Groningen, Pays Bas) and Arijit Ghosh (Max-Planck-Institut für Informatik, Saarbrücken, Germany).

In [45], we give the first algorithm that outputs a faithful reconstruction of a submanifold of Euclidean space without maintaining or even constructing complicated data structures such as Voronoi diagrams or Delaunay complexes. Our algorithm uses the witness complex and relies on the stability of *power protection*, a notion introduced in this paper. The complexity of the algorithm depends exponentially on the intrinsic dimension of the manifold, rather than the dimension of ambient space, and linearly on the dimension of the ambient space. Another interesting feature of this work is that no explicit coordinates of the points in the point sample is needed. The algorithm only needs the *distance matrix* as input, i.e., only distance between points in the point sample as input.

6.3.2. Computing Persistent Homology with Various Coefficient Fields in a Single Pass

Participants: Jean-Daniel Boissonnat, Clément Maria.

In [32], we introduce an algorithm to compute the persistent homology of a filtered complex with various coefficient fields in a single matrix reduction. The algorithm is output-sensitive in the total number of distinct persistent homological features in the diagrams for the different coefficient fields. This computation allows us to infer the prime divisors of the torsion coefficients of the integral homology groups of the topological space at any scale, hence furnishing a more informative description of topology than persistence in a single coefficient field. We provide theoretical complexity analysis as well as detailed experimental results.

6.3.3. Recognizing shrinkable complexes is NP-complete

Participants: Olivier Devillers, Marc Glisse.

In collaboration with Dominique Attali (Gipsa-lab, Grenoble), Sylvain Lazard (Inria Nancy - Grand Est)

We say that a simplicial complex is shrinkable if there exists a sequence of admissible edge contractions that reduces the complex to a single vertex. We prove [31] that it is NP-complete to decide whether a (three-dimensional) simplicial complex is shrinkable. Along the way, we describe examples of contractible complexes which are not shrinkable.

6.3.4. Zigzag Zoology: Rips Zigzags for Homology Inference

Participant: Steve Oudot.

In collaboration with Donald Sheehy (University of Connecticut)

For points sampled near a compact set X , the persistence barcode of the Rips filtration built from the sample contains information about the homology of X as long as X satisfies some geometric assumptions. The Rips filtration is prohibitively large, however zigzag persistence can be used to keep the size linear. We present [28] several species of Rips-like zigzags and compare them with respect to the signal-to-noise ratio, a measure of how well the underlying homology is represented in the persistence barcode relative to the noise in the barcode at the relevant scales. Some of these Rips-like zigzags have been available as part of the Dionysus library for several years while others are new. Interestingly, we show that some species of Rips zigzags will exhibit less noise than the (non-zigzag) Rips filtration itself. Thus, the Rips zigzag can offer improvements in both size complexity and signal-to-noise ratio. Along the way, we develop new techniques for manipulating and comparing persistence barcodes from zigzag modules. We give methods for reversing arrows and removing spaces from a zigzag. We also discuss factoring zigzags and a kind of interleaving of two zigzags that allows their barcodes to be compared. These techniques were developed to provide our theoretical analysis of the signal-to-noise ratio of Rips-like zigzags, but they are of independent interest as they apply to zigzag modules generally.

6.3.5. Zigzag Persistence via Reflections and Transpositions

Participants: Clément Maria, Steve Oudot.

We introduce [40] a simple algorithm for computing zigzag persistence, designed in the same spirit as the standard persistence algorithm. Our algorithm reduces a single matrix, maintains an explicit set of chains encoding the persistent homology of the current zigzag, and updates it under simplex insertions and removals. The total worst-case running time matches the usual cubic bound. A noticeable difference with the standard persistence algorithm is that we do not insert or remove new simplices "at the end" of the zigzag, but rather "in the middle". To do so, we use arrow reflections and transpositions, in the same spirit as reflection functors in quiver theory. Our analysis introduces a new kind of reflection called the "weak-diamond", for which we are able to predict the changes in the interval decomposition and associated compatible bases. Arrow transpositions have been studied previously in the context of standard persistent homology, and we extend the study to the context of zigzag persistence. For both types of transformations, we provide simple procedures to update the interval decomposition and associated compatible homology basis.

6.3.6. Topological analysis of scalar fields with outliers

Participants: Mickaël Buchet, Frédéric Chazal, Steve Oudot.

In collaboration with Tamal K. Dey (University of Ohio) Fengtao Fan (University of Ohio) Yusu Wang (University of Ohio)

We extend [57] the notion of the distance to a measure from Euclidean space to probability measures on general metric spaces as a way to do topological data analysis in a way that is robust to noise and outliers. We then give an efficient way to approximate the sub-level sets of this function by a union of metric balls and extend previous results on sparse Rips filtrations to this setting. This robust and efficient approach to topological data analysis is illustrated with several examples from an implementation.

6.3.7. *Efficient and Robust Persistent Homology for Measures.*

Participants: Mickaël Buchet, Frédéric Chazal, Steve Oudot.

In collaboration with Donald Sheehy (University of Connecticut)

In [34], we extend the notion of the distance to a measure from Euclidean space to probability measures on general metric spaces as a way to do topological data analysis in a way that is robust to noise and outliers. We then give an efficient way to approximate the sub-level sets of this function by a union of metric balls and extend previous results on sparse Rips filtrations to this setting. This robust and efficient approach to topological data analysis is illustrated with several examples from an implementation.

6.3.8. *Persistence-based Structural Recognition*

Participants: Frédéric Chazal, Maksims Ovsjanikovs.

In collaboration with Chunyuan Li (former intern in Saclay in 2013)

In [39] we present a framework for object recognition using topological persistence. In particular, we show that the so-called persistence diagrams built from functions defined on the objects can serve as compact and informative descriptors for images and shapes. Complementary to the bag-of-features representation, which captures the distribution of values of a given function, persistence diagrams can be used to characterize its structural properties, reflecting spatial information in an invariant way. In practice, the choice of function is simple: each dimension of the feature vector can be viewed as a function. The proposed method is general: it can work on various multimedia data, including 2D shapes, textures and triangle meshes. Extensive experiments on 3D shape retrieval, hand gesture recognition and texture classification demonstrate the performance of the proposed method in comparison with state-of-the-art methods. Additionally, our approach yields higher recognition accuracy when used in conjunction with the bag-of-features.

6.3.9. *Convergence rates for persistence diagram estimation in Topological Data Analysis*

Participants: Frédéric Chazal, Marc Glisse, Bertrand Michel.

In collaboration with Catherine Labruère (University of Burgundy)

Computational topology has recently known an important development toward data analysis, giving birth to the field of topological data analysis. Topological persistence, or persistent homology, appears as a fundamental tool in this field. In [36], we study topological persistence in general metric spaces, with a statistical approach. We show that the use of persistent homology can be naturally considered in general statistical frameworks and persistence diagrams can be used as statistics with interesting convergence properties. Some numerical experiments are performed in various contexts to illustrate our results.

6.3.10. *Stochastic Convergence of Persistence Landscapes and Silhouettes*

Participant: Frédéric Chazal.

In collaboration with Brittany Fasy (Tulane University) Fabrizio Lecci (Carnegie Mellon University) Alessandro Rinaldo (Carnegie Mellon University) Larry Wasserman (Carnegie Mellon University)

Persistent homology is a widely used tool in Topological Data Analysis that encodes multiscale topological information as a multi-set of points in the plane called a persistence diagram. It is difficult to apply statistical theory directly to a random sample of diagrams. Instead, we can summarize the persistent homology with the persistence landscape, introduced by Bubenik, which converts a diagram into a well-behaved real-valued function. In [35], we investigate the statistical properties of landscapes, such as weak convergence of the average landscapes and convergence of the bootstrap. In addition, we introduce an alternate functional summary of persistent homology, which we call the silhouette, and derive an analogous statistical theory.

6.3.11. *Subsampling Methods for Persistent Homology*

Participants: Frédéric Chazal, Bertrand Michel.

In collaboration with Brittany Fasy (Tulane University) Fabrizio Lecci (Carnegie Mellon University) Alessandro Rinaldo (Carnegie Mellon University) Larry Wasserman (Carnegie Mellon University)

Persistent homology is a multiscale method for analyzing the shape of sets and functions from point cloud data arising from an unknown distribution supported on those sets. When the size of the sample is large, direct computation of the persistent homology is prohibitive due to the combinatorial nature of the existing algorithms. We propose to compute the persistent homology of several subsamples of the data and then combine the resulting estimates. We study the risk of two estimators and we prove that the subsampling approach carries stable topological information while achieving a great reduction in computational complexity.

6.3.12. *The observable structure of persistence modules*

Participant: Frédéric Chazal.

In collaboration with Vin de Silva (Pomona College) William Crawley-Boevey (University of Leeds)

In persistent topology, q-tame modules appear as a natural and large class of persistence modules indexed over the real line for which a persistence diagram is de- finable. However, unlike persistence modules indexed over a totally ordered finite set or the natural numbers, such diagrams do not provide a complete invariant of q-tame modules. The purpose of [59] is to show that the category of persistence modules can be adjusted to overcome this issue. We introduce the observable category of persis- tence modules: a localization of the usual category, in which the classical properties of q-tame modules still hold but where the persistence diagram is a complete isomorphism invariant and all q-tame modules admit an interval decomposition.

6.4. Data Structures and Robust Geometric Computation

6.4.1. *Efficiently Navigating a Random Delaunay Triangulation*

Participants: Olivier Devillers, Ross Hemsley.

In collaboration with Nicolas Broutin (EPI RAP)

Planar graph navigation is an important problem with significant implications to both point location in geometric data structures and routing in networks. Whilst many algorithms have been proposed, very little theoretical analysis is available for the properties of the paths generated or the computational resources required to generate them. In this work, we propose and analyse a new planar navigation algorithm for the Delaunay triangulation. We then demonstrate a number of strong theoretical guarantees for the algorithm when it is applied to a random set of points in a convex region [33]. In a side result, we give a new polylogarithmic bound on the maximum degree of a random Delaunay triangulation in a smooth convex, that holds with probability one as the number of points goes to infinity. In particular, our new bound holds even for points arbitrarily close to the boundary of the domain. [56]

6.4.2. *A chaotic random convex hull*

Participants: Olivier Devillers, Marc Glisse, Rémy Thomasse.

The asymptotic behavior of the expected size of the convex hull of uniformly random points in a convex body in \mathcal{R}^d is polynomial for a smooth body and polylogarithmic for a polytope. We construct a body whose expected size of the convex hull oscillates between these two behaviors when the number of points increases [62]

6.4.3. *A generator of random convex polygons in a disc*

Participants: Olivier Devillers, Rémy Thomasse.

In collaboration with Philippe Duchon (LABRI)

Let \mathcal{D} a disc in \mathbb{R}^2 with radius 1 centered at \mathbf{o} , and (x_1, \dots, x_n) a sample of n points uniformly and independently distributed in \mathcal{D} . Let's define the polygon P_n as the convex hull of (x_1, \dots, x_n) , and $f_0(P_n)$ its number of vertices. This kind of polygon has been well studied, and it is known, see [65], that

$$\mathbb{E}f_0(P_n) = c n^{\frac{1}{3}} + o(n^{\frac{1}{3}})$$

where $c > 0$ is constant. To generate such a polygon, one can explicitly generate n points uniformly in \mathcal{D} and compute the convex hull. For a very large quantity of points, it could be interesting to generate less points to get the same polygon, for example to have some estimations on asymptotic properties, such as the distribution of the size of the edges. We propose an algorithm that generate far less points at random in order to get P_n , so that the time and the memory needed is reduced for n large. Namely [61], we generate a number of points of the same order of magnitude than the final hull, up to a polylogarithmic factor

6.4.4. On the complexity of the representation of simplicial complexes by trees

Participants: Jean-Daniel Boissonnat, Dorian Mazauric.

In [46], we investigate the problem of the representation of simplicial complexes by trees. We introduce and analyze local and global tree representations. We prove that the global tree representation is more efficient in terms of time complexity for searching a given simplex and we show that the local tree representation is more efficient in terms of size of the structure. The simplicial complexes are modeled by hypergraphs. We then prove that the associated combinatorial optimization problems are very difficult to solve and to approximate even if the set of maximal simplices induces a cubic graph, a planar graph, or a bounded degree hypergraph. However, we prove polynomial time algorithms that compute constant factor approximations and optimal solutions for some classes of instances.

6.4.5. Building Efficient and Compact Data Structures for Simplicial Complexes

Participant: Jean-Daniel Boissonnat.

In collaboration with Karthik C.S (Weizmann Institute of Science, Israël) and Sébastien Tavenas (Max-Planck-Institut für Informatik, Saarbrücken, Germany).

The Simplex Tree is a recently introduced data structure that can represent abstract simplicial complexes of any dimension and allows to efficiently implement a large range of basic operations on simplicial complexes. In this paper, we show how to optimally compress the simplex tree while retaining its functionalities. In addition, we propose two new data structures called Maximal Simplex Tree and Compact Simplex Tree. We analyze the Compressed Simplex Tree, the Maximal Simplex Tree and the Compact Simplex Tree under various settings.

6.4.6. Delaunay triangulations over finite universes

Participant: Jean-Daniel Boissonnat.

In collaboration with Ramsay Dyer (Johann Bernoulli Institute, University of Groningen, Pays Bas) and Arijit Ghosh (Max-Planck-Institut für Informatik, Saarbrücken, Germany).

The witness complex was introduced by Carlsson and de Silva as a weak form of the Delaunay complex that is suitable for finite metric spaces and is computed using only distance comparisons. The witness complex $\text{Wit}(L, W)$ is defined from two sets L and W in some metric space X : a finite set of points L on which the complex is built, and a set W of witnesses that serves as an approximation of X . A fundamental result of de Silva states that $\text{Wit}(L, W) = \text{Del}(L)$ if $W = X = \mathbb{R}^d$. In this paper we give conditions on L that ensure that the witness complex and the Delaunay triangulation coincide when $W \subset \mathbb{R}^d$ is a finite set, and we introduce a new perturbation scheme to compute a perturbed set L' close to L such that $\text{Del}(L') = \text{Wit}(L', W)$. The algorithm constructs $\text{Wit}(L', W)$ in time sublinear in $|W|$.

The only numerical operations used by our algorithms are (squared) distance comparisons (i.e., predicates of degree 2). In particular, we do not use orientation or in-sphere predicates, whose degree depends on the dimension d , and are difficult to implement robustly in higher dimensions. Although the algorithm does not compute any measure of simplex quality, a lower bound on the thickness of the output simplices can be guaranteed. Another novelty in the analysis is the use of the Moser-Tardos constructive proof of the general Lovász local lemma.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

7.1.1. Cifre Contract with Geometry Factory

Mael Rouxel-Labbé's PhD thesis is supported by a Cifre contract with GEOMETRY FACTORY (<http://www.geometryfactory.com>). The subject is the generation of anisotropic meshes.

7.1.2. Commercialization of cgal packages through Geometry Factory

In 2014, GEOMETRY FACTORY (<http://www.geometryfactory.com>) had the following new customers for CGAL packages developed by GEOMETRICA:

- LMI Technologies (Canada, GIS): 2D triangulations
- Rio Tinto (Australie, mining): 2D triangulations
- Geovariances (France, oil and gas): 3D triangulations and meshes
- Elektrobit (Allemagne, GIS): 2D triangulations
- First Light Fusion (UK, energie): 2D triangulations

8. Partnerships and Cooperations

8.1. Technological Development Actions

8.1.1. ADT PH

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Sonali Digambar Patil, Marc Glisse, Steve Oudot, Clément Maria, Mariette Yvinec.

- Title: Persistent Homology
- Coordinator: Mariette Yvinec (GEOMETRICA)
- Duration: 1 year renewable once, starting date December 2012. Renewed for 1 year from January 1st 2014 to December 31st 2014
- Others Partners: Inria team ABS, Gipsa Lab (UMR 5216, Grenoble, <http://www.gipsa-lab.inpg.fr/>)
- Abstract: Geometric Inference is a rapidly emerging field that aims to analyse the structural, geometric and topological, properties of point cloud data in high dimensional spaces. The goal of the ADT PH is to make available, a robust and comprehensive set of algorithmic tools resulting from recent advances in Geometric Inference. The software will include:

- tools to extract from the data sets, families of simplicial complexes,
- data structures to handle those simplicial complexes,
- algorithmic modules to compute the persistent homology of those complexes,
- applications to clustering, segmentation and analysis of scalar fields such as the energy landscape of macromolecular systems.

8.1.2. ADT OrbiCGAL

Participants: Aymeric Pellé, Monique Teillaud.

- Title: OrbiCGAL
- Coordinator: Monique Teillaud (GEOMETRICA)
- Duration: 1 year renewable once, starting date September 2013.

- Abstract: OrbiCGAL is a software project supported by Inria as a Technological Development Action (ADT). It is motivated by applications ranging from infinitely small (nano-structures) to infinitely large (astronomy), through material engineering, physics of condensed matter, solid chemistry, etc

The project consists in developing or improving software packages to compute triangulations and meshes in several types of non-Euclidean spaces: sphere, 3D closed flat manifolds, hyperbolic plane.

8.2. Regional Initiatives

8.2.1. Digiteo project TOPERA

Participants: Frédéric Chazal, Marc Glisse, Anaïs Vergne.

TOPERA is a project that aims at developing methods from Topological Data Analysis to study covering properties and quality of cellular networks. It also involves L. Decreusefond and P. Martins from Telecom Paris.

- Starting date: December 2013

- Duration: 18 months

8.3. National Initiatives

8.3.1. ANR Présage

Participants: Olivier Devillers, Marc Glisse, Ross Hemsley, Monique Teillaud, Rémy Thomasse.

- Acronym: Presage.

- Type: ANR blanc.

- Title: *méthodes PRobabilistes pour l'Éfficacité des Structures et Algorithmes GÉométriques.*

- Coordinator: Xavier Goaoc.

- Duration: 31 december 2011 - 31 december 2015.

- Other partners: Inria VEGAS team, University of Rouen.

- Abstract: This project brings together computational and probabilistic geometers to tackle new probabilistic geometry problems arising from the design and analysis of geometric algorithms and data structures. We focus on properties of discrete structures induced by or underlying random continuous geometric objects. This raises questions such as:

- What does a random geometric structure (convex hulls, tessellations, visibility regions...) look like?
- How to analyze and optimize the behavior of classical geometric algorithms on *usual* inputs?
- How can we generate randomly *interesting* discrete geometric structures?

- Year publications: [56], [33], [48], [52], [62], [61], [12]

8.3.2. ANR TOPDATA

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Mariette Yvinec, Steve Oudot, Marc Glisse, Clément Levrard.

- Acronym : TopData.

- Title : Topological Data Analysis: Statistical Methods and Inference.

- Type : ANR blanc

- Coordinator : Frédéric Chazal (GEOMETRICA)

- Duration : 4 years starting October 2013.

- Others Partners: Département de Mathématiques (Université Paris Sud), Institut de Mathématiques (Université de Bourgogne), LPMA (Université Paris Diderot), LSTA (Université Pierre et Marie Curie)

- Abstract: TopData aims at designing new mathematical frameworks, models and algorithmic tools to infer and analyze the topological and geometric structure of data in different statistical settings. Its goal is to set up the mathematical and algorithmic foundations of Statistical Topological and Geometric Data Analysis and to provide robust and efficient tools to explore, infer and exploit the underlying geometric structure of various data.

Our conviction, at the root of this project, is that there is a real need to combine statistical and topological/geometric approaches in a common framework, in order to face the challenges raised by the inference and the study of topological and geometric properties of the wide variety of larger and larger available data. We are also convinced that these challenges need to be addressed both from the mathematical side and the algorithmic and application sides. Our project brings together in a unique way experts in Statistics, Geometric Inference and Computational Topology and Geometry. Our common objective is to design new theoretical frameworks and algorithmic tools and thus to contribute to the emergence of a new field at the crossroads of these domains. Beyond the purely scientific aspects we hope this project will help to give birth to an active interdisciplinary community. With these goals in mind we intend to promote, disseminate and make our tools available and useful for a broad audience, including people from other fields.

- See also: <http://geometrica.saclay.inria.fr/collaborations/TopData/Home.html>

8.4. European Initiatives

8.4.1. FP7 & H2020 Projects

8.4.1.1. GUDHI

Type: FP7

Instrument: ERC Advanced Grant

Duration: February 2014 - January 2019

Coordinator: Jean-Daniel Boissonnat

Inria contact: Jean-Daniel Boissonnat

Abstract: The central goal of this project is to settle the algorithmic foundations of geometry understanding in dimensions higher than 3. Geometry understanding encompasses a collection of tasks including the approximation and computer representation of geometric structures, and the inference of geometric or topological properties of sampled shapes.

See also <https://project.inria.fr/gudhi/>

8.5. International Research Visitors

8.5.1. Visits of International Scientists

Pedro Machado Manhães de Castro (Universidade Federal de Pernambuco)

Arijit Ghosh (MPII, Saarbrücken), april, november-december

Antoine Vigneron (KAUST), may

Ramsay Dyer (Johann Bernoulli Institute, University of Groningen), octobre

Kira Vyatkina (Saint Petersburg Academic University), octobre

Vissarion Fisikopoulos (Université Libre de Bruxelles), november

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific events organisation

9.1.1.1. general chair, scientific chair

Monique Teillaud co-organized the “Workshop on Geometric Structures with Symmetry and Periodicity”, part of the Computational Geometry Week 2014, with Ileana Streinu (Smith College, Northampton, MA), in Kyoto University, Japan, June 8-9. http://www-sop.inria.fr/geometrica/events/wocg14-symmetry_and_periodicity/

Jean-Daniel Boissonnat organized the first GUDHI workshop in Sophia Antipolis, October 27-29. <https://project.inria.fr/gudhi/first-gudhi-workshop/>

9.1.2. Scientific events selection

9.1.2.1. responsable of the conference program committee

Olivier Devillers was program co-chair of 2014 Symposium on Computational Geometry.

9.1.2.2. member of the conference program committee

Monique Teillaud, European Symposium on Algorithms, ESA, <http://algo2014.ii.uni.wroc.pl/esa/>

9.1.2.3. member of the steering committee

Jean-Daniel Boissonnat is a member of the steering committee of the international conference on Curves and Surfaces.

Monique Teillaud is a member of the Steering Committee of the European Symposium on Algorithms (ESA).

9.1.3. Journal

9.1.3.1. member of the editorial board

Jean-Daniel Boissonnat is a member of the Editorial Board of *Journal of the ACM*, *Discrete and Computational Geometry*, *Algorithmica*, *Journal on Computational Geometry*, *International Journal on Computational Geometry and Applications*.

Frédéric Chazal is a member of the Editorial Board of *SIAM Journal on Imaging Sciences*, *Graphical Models* and *Discrete and Computational Geometry*.

Olivier Devillers is a member of the Editorial Board of *Graphical Models*.

Monique Teillaud is a member of the Editorial Boards of *CGTA*, *Computational Geometry: Theory and Applications*, and of *IJCGA*, *International Journal of Computational Geometry and Applications*.

M. Yvinec is a member of the editorial board of *Journal of Discrete Algorithms*.

Monique Teillaud and Mariette Yvinec are members of the CGAL editorial board.

9.1.4. Inria committees

Frédéric Chazal was a member of the recruitment committee of Inria Saclay (vice-chair).

Monique Teillaud was a member of the Inria Evaluation Committee until August.

She was a member of the national Inria DR2 interview committee and the local CR2 interview committees in Nancy and Lille.

She was also a member of the Advanced and Starting "Research Positions" interview committees.

Monique Teillaud was a member of the local Committee for Technologic Development until her move to Nancy.

She was also a member of the local committee for transversal masters.

9.1.5. National committees

Jean-Daniel Boissonnat was a member of the Conseil de l'AERES (Agence d'Evaluation de la Recherche et de l'Enseignement Supérieur).

Frédéric Chazal was a member of the Comités d'évaluation scientifique "Mathématiques et Informatique Théorique" of the ANR.

9.1.6. Web site

M. Teillaud is maintaining the Computational Geometry Web Pages <http://www.computational-geometry.org/>, hosted by Inria Sophia Antipolis until November. This site offers general interest information for the computational geometry community, in particular the Web proceedings of the Video Review of Computational Geometry, part of the Annual Symposium on Computational Geometry.

9.1.7. Geometrica seminar

The seminar featured presentations by the following scientists:

- Omri Azencot (Technion) : An Operator Approach to Tangent Vector Field Processing
- Mirela Ben Chen (Technion - Israel Institute of Technology) : Can Mean-Curvature Flow be Modified to be Non-singular?
- Benjamin Burton (University of Queensland) : Untangling knots using combinatorial optimisation
- A. Chiara de Vitis (Pavia) : Geometrical and Topological Descriptors for Protein Structures
- C. Couprie (Courant Institute) : Graph-based Variational Optimization and Applications in Computer Vision
- J. Demantke (IGN) : Geometric Feature Extraction from LIDAR Point Clouds and Photorealistic 3D Facade Model Reconstruction from Terrestrial LIDAR and Image Data
- Kyle Heath (Stanford University) : Image Webs: Discovering and using object-manifold structure in large-scale image collections
- N. Mitra (UCL London) : Computational Design Tools for Smart Models Synthesis
- P. Machado Manhães de Castro (UFPE, Brasil) : Invariance for Single Curved Manifolds
- Natan Rubin (Freie Universität Berlin) : On Kinetic Delaunay Triangulations: A Near Quadratic Bound for Unit Speed Motions
- D. Salinas (Gipsa Lab, Grenoble) : Using the Rips complex for Topologically Certified Manifold Learning
- Régis Straubhaar (Université de Neuchâtel) : Numerical optimization of an eigenvalue of the Laplacian on a domain in surfaces
- Jian Sun (Mathematical Sciences Center, Tsinghua University) : Rigidity of Infinite Hexagonal Triangulation of Plane
- Anaïs Vergne (Télécom ParisTech) : Algebraic topology and sensor networks
- Yuan Yao (School of Mathematical Sciences, Peking University) : The Landscape of Complex Networks
- H. Zimmer (Aachen) : Geometry Optimization for Dual-Layer Support Structures

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

- Master : J.D. Boissonnat, C. Maria, M. Yvinec, *Algorithmic Geometry of Triangulations*, 24h, International Master Sophia Antipolis.
- Master : S. Oudot, *Computational Geometry: from Theory to Applications*, 18h, École polytechnique.
- Master : S. Oudot, *Geometric Methods for Data Analysis*, 12h, École Centrale Paris.
- Master: Jean-Daniel Boissonnat, Frédéric Chazal *Computational Geometric Learning*, 24h, Master MPRI, Paris.
- M. Teillaud, *Génération de maillages dans la bibliothèque CGAL*, in the framework of a training course for young researchers (PhD students and master students on environmental sciences) on modelling, organized by ECCOREV, 3h, Aix-en-Provence.

9.2.2. Supervision

- HDR: Steve Oudot, Persistence Theory: From Quiver Representations to Data Analysis [14], Université Paris-Sud, November 26th 2014.

PhD : Clément Maria, Algorithms and Data Structures in Computational Topology [13], Université de Nice Sophia Antipolis, October 28, 2014, Jean-Daniel Boissonnat.

PhD: Ross Hemsley, Probabilistic methods for the analysis of algorithms on random tessellations [12], Université de Nice Sophia Antipolis, December 16, 2014, Olivier Devillers.

PhD: Mickaël Buchet, Topological and geometric inference from measures [11], Université Paris XI, December 2014, Frédéric Chazal and Steve Oudot.

PhD in progress: Thomas Bonis, Topological persistence for learning, started on December 2013, Frédéric Chazal.

PhD in progress: Mathieu Carrière, Topological signatures for geometric shapes, started in November 2014, Steve Oudot.

PhD in progress: Ruqi Huang, Algorithms for topological inference in metric spaces, started on December 2013, Frédéric Chazal.

PhD in progress: Eddie Amaari, A statistical approach of topological Data Analysis, started on Sept. 2014, Frédéric Chazal (co-advised with Pascal Massart).

PhD in progress : Rémy Thomasse, Smoothed complexity of geometric structures and algorithms, started December 1st 2012, Olivier Devillers.

PhD in progress : Mael Rouxel-Labbé, Anisotropic Mesh Generation, started October 1st, 2013, Jean-Daniel Boissonnat and Mariette Yvinec.

PhD in progress : Alba Chiri de Vitis, Computational Geometry in High Dimensions, started February 2014. Jean-Daniel Boissonnat and David Cohen-Steiner.

9.2.3. Juries

Monique Teillaud was a member of the HDR defense committee of Francis Lazarus (univ. Grenoble).

Monique Teillaud was a member of the PhD defense committee of Yacine Bouzidi (Univ. de Lorraine).

Monique Teillaud was a member of the PhD defense committee (member of the three-member advisory committee) of Vissarion Fisikopoulos (Univ. Athens, Grece).

Monique Teillaud was a member of the PhD defense committee of Arnaud de Mesmay (École Normale Supérieure).

Jean-Daniel Boissonnat was a member of the HDR defense committees of Q. Mérigot and S. Oudot.

Frédéric Chazal was a member (reviewer) of the PhD defense of Arnaud de Mesmay (Ecole Normale Supérieure de Paris).

Frédéric Chazal was a member of the PhD defense of Clément Levrard (Université Paris-Sud).

Steve Oudot was a member of the PhD defense of Clément Maria (Université Nice Sophia Antipolis).

Marc Glisse was a member of the PhD defense committee of Octavio Razafindramanana, Université François Rabelais de Tours.

9.2.4. Internships

Thomas Bonis, image and shape classification using persistent homology (F. Chazal)

François Godi, Computing the bottleneck distance for persistence diagrams (Jean-Daniel Boissonnat)

Venkata Yamajala, Equating the witness and the Delaunay complexes (Jean-Daniel Boissonnat)

Chunyuan Li, Persistence-based object recognition (F. Chazal and M. Ovsjanikov)

Eddie Aamari, topological Data Analysis (F. Chazal)

Mathieu Carrière, local topological signatures for shapes (S. Oudot)

9.3. Popularization

Thomas Bonis and Mathieu Carrière presented the *Photomaton 3d* at the *Fête de la Science* in October 2014. This animation consists in scanning a volunteering person in 3d using a Kinect and dedicated software, then illustrating the concepts of 3d shape comparison, retrieval and matching.

10. Bibliography

Major publications by the team in recent years

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- [9] F. DE GOES, D. COHEN-STEINER, P. ALLIEZ, M. DESBRUN. *An Optimal Transport Approach to Robust Reconstruction and Simplification of 2D Shapes*, in "Computer Graphics Forum", 2011, vol. 30, n^o 5, pp. 1593–1602, Special issue for EUROGRAPHICS Symposium on Geometry Processing, <http://hal.inria.fr/hal-00758019>
- [10] L. J. GUIBAS, S. Y. OUDOT, P. SKRABA, F. CHAZAL. *Persistence-Based Clustering in Riemannian Manifolds*, in "Journal of the ACM", November 2013, vol. 60, n^o 6, 38 p. , <http://hal.archives-ouvertes.fr/hal-00923563>

Publications of the year

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