



Activity Report 2013

Project-Team **GEOMETRICA**

Geometric computing

RESEARCH CENTERS
Sophia Antipolis - Méditerranée
Saclay - Île-de-France

THEME
Algorithmics, Computer Algebra and
Cryptology

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Project-Team GEOMETRICA

Keywords: Machine Learning, Computational Geometry, Geometry Processing, Complexity, Algorithmic Geometry

Creation of the Project-Team: 2003 July 01.

1. Members

Research Scientists

Jean-Daniel Boissonnat [Team leader, Inria, Senior Researcher, Sophia Antipolis - Méditerranée, HdR]
Frédéric Chazal [Vice leader, Inria, Senior Researcher, Saclay - Île-de-France, HdR]
David Cohen-Steiner [Inria, Researcher, Sophia Antipolis - Méditerranée]
Olivier Devillers [Inria, Senior Researcher, Sophia Antipolis - Méditerranée, HdR]
Marc Glisse [Inria, Researcher, Saclay - Île-de-France]
Bertrand Michel [Univ. Paris VI, Associate Professor, from Sep., Saclay - Île-de-France]
Steve Oudot [Inria, Researcher, Saclay - Île-de-France]
Maks Ovsjanikov [Ecole Polytechnique, Associate Professor, Saclay - Île-de-France]
Monique Teillaud [Inria, Senior Researcher, Sophia Antipolis - Méditerranée, HdR]
Marianne Yvinec [Vice leader, Inria, Researcher, Sophia Antipolis - Méditerranée, HdR]

Engineers

Sonali Digambar Patil [Inria, Sophia Antipolis - Méditerranée]
Aymeric Pellé [Inria, from Sep 16, Sophia Antipolis - Méditerranée]

PhD Students

Mikhail Bogdanov [UNS until Sept, then Inria, Sophia Antipolis - Méditerranée]
Thomas Bonis [Inria, Intern, from Dec, Saclay - Île-de-France]
Mickaël Buchet [Univ. Paris XI, Saclay - Île-de-France]
Ross Hemsley [Inria, CORDI-S, Sophia Antipolis - Méditerranée]
Ruqi Huang [Inria, CORDI-S, from Dec, Saclay - Île-de-France]
Clément Maria [UNS, Sophia Antipolis - Méditerranée]
Mael Rouxel-Labbe [Geometry Factory, CIFRE, from October, Sophia Antipolis - Méditerranée]
Rémy Thomasse [Inria, CORDI-C granted by ANR Presage and Conseil Régional PACA, Sophia Antipolis - Méditerranée]

Post-Doctoral Fellows

Ramsay Dyer [Inria, FP7 CG Learning project, Jan., Sophia Antipolis - Méditerranée]
Viorica Patraucean [Inria, from Oct, Saclay - Île-de-France]
Donald Sheehy [Inria, FP7 CG Learning project, until July, Saclay - Île-de-France]
Régis Straubhaar [Fonds National Suisse, from Oct., Saclay - Île-de-France]
Anaïs Vergne [Inria, from Dec, Saclay - Île-de-France]

Visiting Scientists

Mirela Ben-Chen [Technion-Israel Institute of Technology Mar, Saclay - Île-de-France]
Fengtao Fan [Ohio State University- Columbus, from Jan to Feb, Saclay - Île-de-France]
Arijit Ghosh [Indian Statistical Institute, from May to Jul, Sophia Antipolis - Méditerranée]
Kyle Heath [Stanford University, from Oct to Nov, Saclay - Île-de-France]
Pedro Machado Manhães de Castro [Universidade Federal de Pernambuco, from Apr to May, Sophia Antipolis - Méditerranée]
Pooran Memari [CNRS, from Jun to Jul, Sophia Antipolis - Méditerranée]
Dmitriy Morozov [Lawrence Berkeley National Laboratory, Jun, Saclay - Île-de-France]
Jian Sun [Tsinghua University, China, from Jan to Feb, Saclay - Île-de-France]

Jane Tournois [Geometry Factory, to Dec, Sophia Antipolis - Méditerranée]

Yusu Wang [Ohio State University, Jun, Saclay - Île-de-France]

Yuan Yao [School of Mathematical Sciences, Beijing University, from Jan to Feb, Saclay - Île-de-France]

Administrative Assistants

Florence Barbara [Inria, from Jul, Sophia Antipolis - Méditerranée]

Christine Biard [Inria, Saclay - Île-de-France]

Others

Thomas Bonis [Inria, Intern, from Apr to Nov, Saclay - Île-de-France]

David Cozmar [Inria, Intern, from Jan to Mar, Sophia Antipolis - Méditerranée]

Siargey Kachanovich [Inria, Intern, FP7 CG Learning project, from May to Aug, Sophia Antipolis - Méditerranée]

Chunyuan Li [Inria, Intern, Concordia University, from Jun to Jul, Saclay - Île-de-France]

Arnaud Poinas [ENS Cachan, Intern, from May to Jul, Sophia Antipolis - Méditerranée]

Claudia Werner [Inria, Intern, until Feb, Sophia Antipolis - Méditerranée]

Venkata Yamajala [Inria, Intern, FP7 CG Learning project, from Sep, Sophia Antipolis - Méditerranée]

2. Overall Objectives

2.1. Overall Objectives

Research carried out by the Geometrica project team is dedicated to Computational Geometry and Topology and follows three major directions: (a). mesh generation and geometry processing; (b). topological and geometric inference; (c). data structures and robust geometric computation. The overall objective of the project-team is to give effective computational geometry and topology solid mathematical and algorithmic foundations, to provide solutions to key problems as well as to validate theoretical advances through extensive experimental research and the development of software packages that may serve as steps toward a standard for reliable and effective geometric computing. Most notably, Geometrica, together with several partners in Europe, plays a prominent role in the development of CGAL, a large library of computational geometry algorithms.

2.2. Highlights of the Year

Jean-Daniel Boissonnat has obtained an "advanced" grant from the ERC (European Research Council) for his project Gudhi : Geometry Understanding in Higher Dimensions.

3. Research Program

3.1. Mesh Generation and Geometry Processing

Meshes are becoming commonplace in a number of applications ranging from engineering to multimedia through biomedecine and geology. For rendering, the quality of a mesh refers to its approximation properties. For numerical simulation, a mesh is not only required to faithfully approximate the domain of simulation, but also to satisfy size as well as shape constraints. The elaboration of algorithms for automatic mesh generation is a notoriously difficult task as it involves numerous geometric components: Complex data structures and algorithms, surface approximation, robustness as well as scalability issues. The recent trend to reconstruct domain boundaries from measurements adds even further hurdles. Armed with our experience on triangulations and algorithms, and with components from the CGAL library, we aim at devising robust algorithms for 2D, surface, 3D mesh generation as well as anisotropic meshes. Our research in mesh generation primarily focuses on the generation of simplicial meshes, i.e. triangular and tetrahedral meshes. We investigate both greedy approaches based upon Delaunay refinement and filtering, and variational approaches based upon energy functionals and associated minimizers.

The search for new methods and tools to process digital geometry is motivated by the fact that previous attempts to adapt common signal processing methods have led to limited success: Shapes are not just another signal but a new challenge to face due to distinctive properties of complex shapes such as topology, metric, lack of global parameterization, non-uniform sampling and irregular discretization. Our research in geometry processing ranges from surface reconstruction to surface remeshing through curvature estimation, principal component analysis, surface approximation and surface mesh parameterization. Another focus is on the robustness of the algorithms to defect-laden data. This focus stems from the fact that acquired geometric data obtained through measurements or designs are rarely usable directly by downstream applications. This generates bottlenecks, i.e., parts of the processing pipeline which are too labor-intensive or too brittle for practitioners. Beyond reliability and theoretical foundations, our goal is to design methods which are also robust to raw, unprocessed inputs.

3.2. Topological and Geometric Inference

Due to the fast evolution of data acquisition devices and computational power, scientists in many areas are asking for efficient algorithmic tools for analyzing, manipulating and visualizing more and more complex shapes or complex systems from approximative data. Many of the existing algorithmic solutions which come with little theoretical guarantee provide unsatisfactory and/or unpredictable results. Since these algorithms take as input discrete geometric data, it is mandatory to develop concepts that are rich enough to robustly and correctly approximate continuous shapes and their geometric properties by discrete models. Ensuring the correctness of geometric estimations and approximations on discrete data is a sensitive problem in many applications.

Data sets being often represented as point sets in high dimensional spaces, there is a considerable interest in analyzing and processing data in such spaces. Although these point sets usually live in high dimensional spaces, one often expects them to be located around unknown, possibly non linear, low dimensional shapes. These shapes are usually assumed to be smooth submanifolds or more generally compact subsets of the ambient space. It is then desirable to infer topological (dimension, Betti numbers,...) and geometric characteristics (singularities, volume, curvature,...) of these shapes from the data. The hope is that this information will help to better understand the underlying complex systems from which the data are generated. In spite of recent promising results, many problems still remain open and to be addressed, need a tight collaboration between mathematicians and computer scientists. In this context, our goal is to contribute to the development of new mathematically well founded and algorithmically efficient geometric tools for data analysis and processing of complex geometric objects. Our main targeted areas of application include machine learning, data mining, statistical analysis, and sensor networks.

3.3. Data Structures and Robust Geometric Computation

GEOMETRICA has a large expertise of algorithms and data structures for geometric problems. We are pursuing efforts to design efficient algorithms from a theoretical point of view, but we also put efforts in the effective implementation of these results.

In the past years, we made significant contributions to algorithms for computing Delaunay triangulations (which are used by meshes in the above paragraph). We are still working on the practical efficiency of existing algorithms to compute or to exploit classical Euclidean triangulations in 2 and 3 dimensions, but the current focus of our research is more aimed towards extending the triangulation efforts in several new directions of research.

One of these directions is the triangulation of non Euclidean spaces such as periodic or projective spaces, with various potential applications ranging from astronomy to granular material simulation.

Another direction is the triangulation of moving points, with potential applications to fluid dynamics where the points represent some particles of some evolving physical material, and to variational methods devised to optimize point placement for meshing a domain with a high quality elements.

Increasing the dimension of space is also a stimulating direction of research, as triangulating points in medium dimension (say 4 to 15) has potential applications and raises new challenges to trade exponential complexity of the problem in the dimension for the possibility to reach effective and practical results in reasonably small dimensions.

On the complexity analysis side, we pursue efforts to obtain complexity analysis in some practical situations involving randomized or stochastic hypotheses. On the algorithm design side, we are looking for new paradigms to exploit parallelism on modern multicore hardware architectures.

Finally, all this work is done while keeping in mind concerns related to effective implementation of our work, practical efficiency and robustness issues which have become a background task of all different works made by GEOMETRICA.

4. Application Domains

4.1. Application Domains

- Medical Imaging
- Numerical simulation
- Geometric modeling
- Geographic information systems
- Visualization
- Data analysis
- Astrophysics
- Material physics

5. Software and Platforms

5.1. CGAL, the Computational Geometry Algorithms Library

Participants: Jean-Daniel Boissonnat, Olivier Devillers, Monique Teillaud, Mariette Yvinec.

With the collaboration of Pierre Alliez, Hervé Brönnimann, Manuel Caroli, Pedro Machado Manhães de Castro, Frédéric Cazals, Frank Da, Christophe Delage, Andreas Fabri, Julia Flötotto, Philippe Guigue, Michael Hemmer, Samuel Hornus, Clément Jamin, Menelaos Karavelas, Sébastien Loriot, Abdelkrim Mebarki, Naceur Meskini, Andreas Meyer, Sylvain Pion, Marc Pouget, François Rebufat, Laurent Rineau, Laurent Saboret, Stéphane Tayeb, Jane Tournois, Radu Ursu, and Camille Wormser <http://www.cgal.org>

CGAL is a C++ library of geometric algorithms and data structures. Its development has been initially funded and further supported by several European projects (CGAL, GALIA, ECG, ACS, AIM@SHAPE) since 1996. The long term partners of the project are research teams from the following institutes: Inria Sophia Antipolis - Méditerranée, Max-Planck Institut Saarbrücken, ETH Zürich, Tel Aviv University, together with several others. In 2003, CGAL became an Open Source project (under the LGPL and QPL licenses).

The transfer and diffusion of CGAL in industry is achieved through the company GEOMETRY FACTORY (<http://www.geometryfactory.com>). GEOMETRY FACTORY is a *Born of Inria* company, founded by Andreas Fabri in January 2003. The goal of this company is to pursue the development of the library and to offer services in connection with CGAL (maintenance, support, teaching, advice). GEOMETRY FACTORY is a link between the researchers from the computational geometry community and the industrial users.

The aim of the CGAL project is to create a platform for geometric computing supporting usage in both industry and academia. The main design goals are genericity, numerical robustness, efficiency and ease of use. These goals are enforced by a review of all submissions managed by an editorial board. As the focus is on fundamental geometric algorithms and data structures, the target application domains are numerous: from geological modeling to medical images, from antenna placement to geographic information systems, etc.

The CGAL library consists of a kernel, a list of algorithmic packages, and a support library. The kernel is made of classes that represent elementary geometric objects (points, vectors, lines, segments, planes, simplices, isothetic boxes, circles, spheres, circular arcs...), as well as affine transformations and a number of predicates and geometric constructions over these objects. These classes exist in dimensions 2 and 3 (static dimension) and d (dynamic dimension). Using the template mechanism, each class can be instantiated following several representation modes: one can choose between Cartesian or homogeneous coordinates, use different number types to store the coordinates, and use reference counting or not. The kernel also provides some robustness features using some specifically-devised arithmetic (interval arithmetic, multi-precision arithmetic, static filters...).

A number of packages provide geometric data structures as well as algorithms. The data structures are polygons, polyhedra, triangulations, planar maps, arrangements and various search structures (segment trees, d -dimensional trees...). Algorithms are provided to compute convex hulls, Voronoi diagrams, Boolean operations on polygons, solve certain optimization problems (linear, quadratic, generalized of linear type). Through class and function templates, these algorithms can be used either with the kernel objects or with user-defined geometric classes provided they match a documented interface.

Finally, the support library provides random generators, and interfacing code with other libraries, tools, or file formats (ASCII files, QT or LEDA Windows, OpenGL, Open Inventor, Postscript, Geomview...). Partial interfaces with Python, SCILAB and the Ipe drawing editor are now also available.

GEOMETRICA is particularly involved in general maintenance, in the arithmetic issues that arise in the treatment of robustness issues, in the kernel, in triangulation packages and their close applications such as alpha shapes, in mesh generation and related packages. Two researchers of GEOMETRICA are members of the CGAL Editorial Board, whose main responsibilities are the control of the quality of CGAL, making decisions about technical matters, coordinating communication and promotion of CGAL.

CGAL is about 700,000 lines of code and supports various platforms: GCC (Linux, Mac OS X, Cygwin...), Visual C++ (Windows), Intel C++. A new version of CGAL is released twice a year, and it is downloaded about 10000 times a year. Moreover, CGAL is directly available as packages for the Debian, Ubuntu and Fedora Linux distributions.

More numbers about CGAL: there are now 12 editors in the editorial board, with approximately 20 additional developers. The user discussion mailing-list has more than 1000 subscribers with a relatively high traffic of 5-10 mails a day. The announcement mailing-list has more than 3000 subscribers.

6. New Results

6.1. Mesh Generation and Geometry Processing

6.1.1. Splat-based Surface Reconstruction from Defect-Laden Point Sets.

Participant: Mariette Yvinec.

In collaboration with Pierre Alliez (EPI Titane), Ricard Campos (University of Girona), Raphael Garcia (University of Girona)

We introduce a method for surface reconstruction from point sets that is able to cope with noise and outliers. First, a splat-based representation is computed from the point set. A robust local 3D RANSAC-based procedure is used to filter the point set for outliers, then a local jet surface – a low-degree surface approximation – is fitted to the inliers. Second, we extract the reconstructed surface in the form of a surface triangle mesh through Delaunay refinement. The Delaunay refinement meshing approach requires computing intersections between line segment queries and the surface to be meshed. In the present case, intersection queries are solved from the set of splats through a 1D RANSAC procedure. [14].

6.1.2. *Constructing Intrinsic Delaunay Triangulations of Submanifolds*

Participants: Jean-Daniel Boissonnat, Ramsay Dyer.

In collaboration with Arijit Ghosh (Indian Statistical Institute)

We describe an algorithm to construct an intrinsic Delaunay triangulation of a smooth closed submanifold of Euclidean space [42]. Using results established in a companion paper on the stability of Delaunay triangulations on δ -generic point sets, we establish sampling criteria which ensure that the intrinsic Delaunay complex coincides with the restricted Delaunay complex and also with the recently introduced tangential Delaunay complex. The algorithm generates a point set that meets the required criteria while the tangential complex is being constructed. In this way the computation of geodesic distances is avoided, the runtime is only linearly dependent on the ambient dimension, and the Delaunay complexes are guaranteed to be triangulations of the manifold.

6.1.3. *Delaunay Triangulation of Manifolds*

Participants: Jean-Daniel Boissonnat, Ramsay Dyer.

In collaboration with Arijit Ghosh (Indian Statistical Institute)

We present an algorithmic framework for producing Delaunay triangulations of manifolds [44]. The input to the algorithm is a set of sample points together with coordinate patches indexed by those points. The transition functions between nearby coordinate patches are required to be bi-Lipschitz with a constant close to 1. The primary novelty of the framework is that it can accommodate abstract manifolds that are not presented as submanifolds of Euclidean space. The output is a manifold simplicial complex that is the Delaunay complex of a perturbed set of points on the manifold. The guarantee of a manifold output complex demands no smoothness requirement on the transition functions, beyond the bi-Lipschitz constraint. In the smooth setting, when the transition functions are defined by common coordinate charts, such as the exponential map on a Riemannian manifold, the output manifold is homeomorphic to the original manifold, when the sampling is sufficiently dense.

6.1.4. *Anisotropic Delaunay Meshes of Surfaces*

Participants: Jean-Daniel Boissonnat, Mariette Yvinec.

In collaboration with Jane Tournois (GeometryFactory) and Kan-Le Shi (Tsing Hua University)

Anisotropic simplicial meshes are triangulations with elements elongated along prescribed directions. Anisotropic meshes have been shown to be well suited for interpolation of functions or solving PDEs. They can also significantly enhance the accuracy of a surface representation. Given a surface S endowed with a metric tensor field, we propose a new approach to generate an anisotropic mesh that approximates S with elements shaped according to the metric field [13], [47]. The algorithm relies on the well-established concepts of restricted Delaunay triangulation and Delaunay refinement and comes with theoretical guarantees. The star of each vertex in the output mesh is Delaunay for the metric attached to this vertex. Each facet has a good aspect ratio with respect to the metric specified at any of its vertices. The algorithm is easy to implement. It can mesh various types of surfaces like implicit surfaces, polyhedra or isosurfaces in 3D images. It can handle complicated geometries and topologies, and very anisotropic metric fields.

6.2. Topological and Geometric Inference

6.2.1. An Efficient Data Structure for Computing Persistent Cohomology

Participants: Jean-Daniel Boissonnat, Clément Maria.

In collaboration with Tamal Dey (Ohio State University)

Persistent homology with coefficients in a field F coincides with the same for cohomology because of duality. We propose an implementation of a recently introduced algorithm for persistent cohomology that attaches annotation vectors with the simplices. We separate the representation of the simplicial complex from the representation of the cohomology groups, and introduce a new data structure for maintaining the annotation matrix, which is more compact and reduces substantially the amount of matrix operations. In addition, we propose a heuristic to further simplify the representation of the cohomology groups and improve both time and space complexities. The paper provides a theoretical analysis, as well as a detailed experimental study of our implementation and comparison with state-of-the-art software for persistent homology and cohomology [41], [29].

6.2.2. Multi-Field Persistent Homology

Participants: Jean-Daniel Boissonnat, Clément Maria.

In [46], we introduce the *multi-field persistence diagram* for the persistence homology of a filtered complex. It encodes compactly the *superimposition* of the persistence diagrams of the complex with several field coefficients, and provides a substantially more precise description of the topology of the filtered complex. Specifically, the multi-field persistence diagram encodes the Betti numbers of integral homology and the prime divisors of the torsion coefficients of the underlying shape. Moreover, it enjoys similar stability properties as the ones of standard persistence diagrams, with the appropriate notion of distance. These properties make the multi-field persistence diagram a useful tool in computational topology. The multi-field algorithms are, in practice, as fast as algorithms that compute persistent homology in a single field.

6.2.3. Zigzag Zoology: Rips Zigzags for Homology Inference

Participants: Steve Oudot, Donald Sheehy.

For points sampled near a compact set X , the persistence barcode of the Rips filtration built from the sample contains information about the homology of X as long as X satisfies some geometric assumptions. The Rips filtration is prohibitively large, however zigzag persistence can be used to keep the size linear. We present several species of Rips-like zigzags and compare them with respect to the signal-to-noise ratio, a measure of how well the underlying homology is represented in the persistence barcode relative to the noise in the barcode at the relevant scales. Some of these Rips-like zigzags have been available as part of the Dionysus library for several years while others are new. Interestingly, we show that some species of Rips zigzags will exhibit less noise than the (non-zigzag) Rips filtration itself. Thus, Rips zigzags can offer improvements in both size complexity and signal-to-noise ratio. Along the way, we develop new techniques for manipulating and comparing persistence barcodes from zigzag modules. In particular, we give methods for reversing arrows and removing spaces from a zigzag while controlling the changes occurring in its barcode. We also discuss factoring zigzags and a kind of interleaving of two zigzags that allows their barcodes to be compared. These techniques were developed to provide our theoretical analysis of the signal-to-noise ratio of Rips-like zigzags, but they are of independent interest as they apply to zigzag modules generally [33].

6.2.4. Efficient and Robust Topological Data Analysis on Metric Spaces

Participants: Mickaël Buchet, Frédéric Chazal, Steve Oudot, Donald Sheehy.

We extend the notion of the distance to a measure from Euclidean space to probability measures on general metric spaces as a way to perform topological data analysis in a way that is robust to noise and outliers. We then give an efficient way to approximate the sub-level sets of this function by a union of metric balls and extend previous results on sparse Rips filtrations to this setting. This robust and efficient approach to topological data analysis is illustrated with several examples from an implementation [54].

6.2.5. *Noise-Adaptive Shape Reconstruction from Raw Point Sets*

Participant: David Cohen-Steiner.

In collaboration with Pierre Alliez (EPI Titane), Simon Giraudot (EPI Titane)

We propose a noise-adaptive shape reconstruction method specialized to smooth, closed hypersurfaces. Our algorithm takes as input a defect-laden point set with variable noise and outliers, and comprises three main steps. First, we compute a novel type of robust distance function to the data. As a robust distance function, its sublevel-sets have the correct homotopy type when the data is a sufficiently good sample of a regular shape. The new feature is a built-in scale selection mechanism that adapts to the local noise level, under the assumption that the inferred shape is a smooth submanifold of known dimension. Second, we estimate the sign and confidence of the function at a set of seed points, based on estimated crossing parities along the edges of a uniform random graph. That component is inspired by the classical MAXCUT relaxation, except that we only require a linear solve as opposed to an eigenvector computation. Third, we compute a signed implicit function through a random walker approach with soft constraints chosen as the most confident seed points computed in previous step. The resulting pipeline is scalable and offers excellent behavior for data exhibiting variable noise levels [19].

6.2.6. *Optimal Rates of Convergence for Persistence Diagrams in Topological Data Analysis*

Participants: Frédéric Chazal, Marc Glisse, Bertrand Michel.

In collaboration with Catherine Labruère (Université de Bourgogne).

Computational topology has recently known an important development toward data analysis, giving birth to the field of topological data analysis. Topological persistence, or persistent homology, appears as a fundamental tool in this field. In this paper [57] (to appear in proc. ICML 2014), we study topological persistence in general metric spaces, with a statistical approach. We show that the use of persistent homology can be naturally considered in general statistical frameworks and persistence diagrams can be used as statistics with interesting convergence properties. Some numerical experiments are performed in various contexts to illustrate our results.

6.2.7. *Bootstrap and Stochastic Convergence for Persistence Diagrams and Landscapes*

Participant: Frédéric Chazal.

In collaboration with B. Fasy (Tulane University), F. Lecci, A. Rinaldo, A. Singh, L. Wasserman (Carnegie Mellon University).

Persistent homology probes topological properties from point clouds and functions. By looking at multiple scales simultaneously, one can record the births and deaths of topological features as the scale varies. We can summarize the persistent homology with the persistence landscape, introduced by Bubenik, which converts a diagram into a well-behaved real-valued function. We investigate the statistical properties of landscapes, such as weak convergence of the average landscapes and convergence of the bootstrap. In addition, we introduce an alternate functional summary of persistent homology, which we call the silhouette, and derive an analogous statistical theory [55].

6.2.8. *Gromov-Hausdorff Approximation of Metric Spaces with Linear Structure*

Participant: Frédéric Chazal.

In collaboration with S. Jian (Tsinghua University).

In many real-world applications data come as discrete metric spaces sampled around 1-dimensional filamentary structures that can be seen as metric graphs. In this paper [58] we address the metric reconstruction problem of such filamentary structures from data sampled around them. We prove that they can be approximated, with respect to the Gromov-Hausdorff distance by well-chosen Reeb graphs (and some of their variants) and we provide an efficient and easy to implement algorithm to compute such approximations in almost linear time. We illustrate the performances of our algorithm on a few synthetic and real data sets.

6.2.9. Analysis and Visualization of Maps Between Shapes

Participants: Frédéric Chazal, Maks Ovsjanikov.

In collaboration with L. Guibas (Stanford University), M. Ben Chen (Technion).

In this work we propose a method for analyzing and visualizing individual maps between shapes, or collections of such maps [23]. Our method is based on isolating and highlighting areas where the maps induce significant distortion of a given measure in a multi-scale way. Unlike the majority of prior work which focuses on discovering maps in the context of shape matching, our main focus is on evaluating, analyzing and visualizing a given map, and the distortion(s) it introduces, in an efficient and intuitive way. We are motivated primarily by the fact that most existing metrics for map evaluation are quadratic and expensive to compute in practice, and that current map visualization techniques are suitable primarily for global map understanding, and typically do not highlight areas where the map fails to meet certain quality criteria in a multi-scale way. We propose to address these challenges in a unified way by considering the functional representation of a map, and performing spectral analysis on this representation. In particular, we propose a simple multi-scale method for map evaluation and visualization, which provides detailed multi-scale information about the distortion induced by a map, which can be used alongside existing global visualization techniques.

6.2.10. Map-Based Exploration of Intrinsic Shape Differences and Variability

Participants: Frédéric Chazal, Maks Ovsjanikov.

In collaboration with L. Guibas and Raif Rustamov (Stanford University), M. Ben Chen and O. Azencot (Technion).

We develop a novel formulation for the notion of shape differences, aimed at providing detailed information about the location and nature of the differences or distortions between the two shapes being compared [27]. Our difference operator, derived from a shape map, is much more informative than just a scalar global shape similarity score, rendering it useful in a variety of applications where more refined shape comparisons are necessary. The approach is intrinsic and is based on a linear algebraic framework, allowing the use of many common linear algebra tools (e.g. SVD, PCA) for studying a matrix representation of the operator. Remarkably, the formulation allows us not only to localize shape differences on the shapes involved, but also to compare shape differences across pairs of shapes, and to analyze the variability in entire shape collections based on the differences between the shapes. Moreover, while we use a map or correspondence to define each shape difference, consistent correspondences between the shapes are not necessary for comparing shape differences, although they can be exploited if available. We give a number of applications of shape differences, including parameterizing the intrinsic variability in a shape collection, exploring shape collections using local variability at different scales, performing shape analogies, and aligning shape collections.

6.2.11. An operator Approach to Tangent Vector Field Processing

Participants: Frédéric Chazal, Maks Ovsjanikov.

In collaboration with M. Ben Chen and O. Azencot (Technion).

In this work [34], we introduce a novel coordinate-free method for manipulating and analyzing vector fields on discrete surfaces. Unlike the commonly used representations of a vector field as an assignment of vectors to the faces of the mesh, or as real values on edges, we argue that vector fields can also be naturally viewed as operators whose domain and range are functions defined on the mesh. Although this point of view is common in differential geometry it has so far not been adopted in geometry processing applications. We recall the theoretical properties of vector fields represented as operators, and show that composition of vector fields with other functional operators is natural in this setup. This leads to the characterization of vector field properties through commutativity with other operators such as the Laplace-Beltrami and symmetry operators, as well as to a straight-forward definition of differential properties such as the Lie derivative. Finally, we demonstrate a range of applications, such as Killing vector field design, symmetric vector field estimation and joint design on multiple surfaces.

6.3. Data Structures and Robust Geometric Computation

6.3.1. *The Stability of Delaunay triangulations*

Participants: Jean-Daniel Boissonnat, Ramsay Dyer.

In collaboration with Arijit Ghosh (Indian Statistical Institute)

We introduce a parametrized notion of genericity for Delaunay triangulations which, in particular, implies that the Delaunay simplices of δ -generic point sets are thick [45]. Equipped with this notion, we study the stability of Delaunay triangulations under perturbations of the metric and of the vertex positions. We quantify the magnitude of the perturbations under which the Delaunay triangulation remains unchanged.

6.3.2. *Delaunay Stability via Perturbations*

Participants: Jean-Daniel Boissonnat, Ramsay Dyer.

In collaboration with Arijit Ghosh (Indian Statistical Institute)

We present an algorithm that takes as input a finite point set in Euclidean space, and performs a perturbation that guarantees that the Delaunay triangulation of the resulting perturbed point set has quantifiable stability with respect to the metric and the point positions [43]. There is also a guarantee on the quality of the simplices: they cannot be too flat. The algorithm provides an alternative tool to the weighting or refinement methods to remove poorly shaped simplices in Delaunay triangulations of arbitrary dimension, but in addition it provides a guarantee of stability for the resulting triangulation.

6.3.3. *Deletions in 3D Delaunay Triangulation*

Participant: Olivier Devillers.

In collaboration with Kevin Buchin (Technical University Eindhoven, The Netherlands), Wolfgang Mulzer (Freie Universität Berlin, Germany), Okke Schrijvers, (Stanford University, USA) and Jonathan Shewchuk (University of California at Berkeley, USA)

Deleting a vertex in a Delaunay triangulation is much more difficult than inserting a new vertex because the information present in the triangulation before the deletion is difficult to exploit to speed up the computation of the new triangulation.

The removal of the tetrahedra incident to the deleted vertex creates a hole in the triangulation that need to be retriangulated. First we propose a technically sound framework to compute incrementally a triangulation of the hole vertices: *the conflict Delaunay triangulation*. The conflict Delaunay triangulation matches the hole boundary and avoid to compute extra tetrahedra outside the hole. Second, we propose a method that uses *guided randomized reinsertion* to speed up the point location during the computation of the conflict triangulation. The hole boundary is a polyhedron, this polyhedron is simplified by deleting its vertices one by one in a random order maintaining a polyhedron called *link Delaunay triangulation*, then the points are inserted in reverse order into the conflict Delaunay triangulation using the information from the link Delaunay triangulation to avoid point location [30].

6.3.4. *A Convex Body with a Chaotic Random Polytope*

Participants: Olivier Devillers, Marc Glisse, Rémy Thomasse.

Consider a sequence of points in a convex body in dimension d whose convex hull is dynamically maintained when the points are inserted one by one, the convex hull size may increase, decrease, or being constant when a new point is added. Studying the expected size of the convex hull when the points are evenly distributed in the convex is a classical problem of probabilistic geometry that yields to some surprising facts. For example, although it seems quite natural to think that the expected size of the convex hull is increasing with n the number of points, this fact is only formally proven for n big enough [16]. The asymptotic behavior of the expected size is known to be logarithmic for a polyhedral body and polynomial for a smooth one. If for a polyhedral or a smooth body, the asymptotic behavior is *somehow* “nice” it is possible to construct strange convex objects that have no such nice behaviors and we exhibit a convex body, such that the behavior of the expected size of a random polytope oscillates between the polyhedral and smooth behaviors when n increases [51].

6.3.5. Delaunay Triangulations and Cycles on Closed Hyperbolic surfaces

Participants: Mikhail Bogdanov, Monique Teillaud.

This work [40] is motivated by applications of *periodic* Delaunay triangulations in the Poincaré disk conformal model of the hyperbolic plane \mathbb{H}^2 . A periodic triangulation is defined by an infinite point set that is the image of a finite point set by a (non commutative) discrete group G generated by hyperbolic translations, such that the hyperbolic area of a Dirichlet region is finite (i.e., a cocompact Fuchsian group acting on \mathbb{H}^2 without fixed points).

We consider the projection of such a Delaunay triangulation onto the closed orientable hyperbolic surface $M = \mathbb{H}^2/G$. The graph of its edges may have cycles of length one or two. We prove that there always exists a finite-sheeted covering space of M in which there is no cycle of length ≤ 2 . We then focus on the group defining the Bolza surface (homeomorphic to a torus having two handles), and we explicitly construct a sequence of subgroups of finite index allowing us to exhibit a covering space of the Bolza surface in which, for any input point set, there is no cycle of length one, and another covering space in which there is no cycle of length two. We also exhibit a small point set such that the projection of the Delaunay triangulation on the Bolza surface for any superset has no cycle of length ≤ 2 .

The work uses mathematical proofs, algorithmic constructions, and implementation.

6.3.6. Universal Point Sets for Planar Graph Drawings with Circular Arcs

Participant: Monique Teillaud.

In collaboration with Patrizio Angelini (Roma Tre University), David Eppstein (University of California, Irvine), Fabrizio Frati (The University of Sydney), Michael Kaufmann (MPI, Tübingen), Sylvain Lazard (EPI VEGAS), Tamara Mchedlidze (Karlsruhe Institute of Technology), and Alexander Wolff (Universität Würzburg).

We prove that there exists a set S of n points in the plane such that every n -vertex planar graph G admits a plane drawing in which every vertex of G is placed on a distinct point of S and every edge of G is drawn as a circular arc. [25]

6.3.7. A Generic Implementation of dD Combinatorial Maps in CGAL

Participant: Monique Teillaud.

In collaboration with Guillaume Damiand (Université de Lyon, LIRIS, UMR 5205 CNRS)

We present a generic implementation of dD combinatorial maps and linear cell complexes in CGAL. A combinatorial map describes an object subdivided into cells; a linear cell complex describes the linear geometry embedding of such a subdivision. In this paper [49], we show how generic programming and new techniques recently introduced in the C++11 standard allow a fully generic and customizable implementation of these two data structures, while maintaining optimal memory footprint and direct access to all information. To the best of our knowledge, the CGAL software packages presented here [59], [60] offer the only available generic implementation of combinatorial maps in any dimension.

6.3.8. Silhouette of a Random Polytope

Participant: Marc Glisse.

In collaboration with Sylvain Lazard and Marc Pouget (EPI VEGAS) and Julien Michel (LMA-Poitiers).

We consider random polytopes defined as the convex hull of a Poisson point process on a sphere in \mathbb{R}^3 such that its average number of points is n . We show [52] that the expectation over all such random polytopes of the maximum size of their silhouettes viewed from infinity is $\Theta(\sqrt{n})$.

6.3.9. A New Approach to Output-Sensitive Voronoi Diagrams and Delaunay Triangulations

Participant: Donald Sheehy.

In collaboration with Gary Miller (Carnegie Mellon University)

We describe [35] a new algorithm for computing the Voronoi diagram of a set of n points in constant-dimensional Euclidean space. The running time of our algorithm is $O(f \log n \log \Delta)$ where f is the output complexity of the Voronoi diagram and Δ is the spread of the input, the ratio of largest to smallest pairwise distances. Despite the simplicity of the algorithm and its analysis, it improves on the state of the art for all inputs with polynomial spread and near-linear output size. The key idea is to first build the Voronoi diagram of a superset of the input points using ideas from Voronoi refinement mesh generation. Then, the extra points are removed in a straightforward way that allows the total work to be bounded in terms of the output complexity, yielding the output sensitive bound. The removal only involves local flips and is inspired by kinetic data structures.

6.3.10. A Fast Algorithm for Well-Spaced Points and Approximate Delaunay Graphs

Participant: Donald Sheehy.

In collaboration with Gary Miller and Ameya Velingker (Carnegie Mellon University)

We present [32] a new algorithm that produces a well-spaced superset of points conforming to a given input set in any dimension with guaranteed optimal output size. We also provide an approximate Delaunay graph on the output points. Our algorithm runs in expected time $O(2^{O(d)}(n \log n + m))$, where n is the input size, m is the output point set size, and d is the ambient dimension. The constants only depend on the desired element quality bounds.

To gain this new efficiency, the algorithm approximately maintains the Voronoi diagram of the current set of points by storing a superset of the Delaunay neighbors of each point. By retaining quality of the Voronoi diagram and avoiding the storage of the full Voronoi diagram, a simple exponential dependence on d is obtained in the running time. Thus, if one only wants the approximate neighbors structure of a refined Delaunay mesh conforming to a set of input points, the algorithm will return a size $2^{O(d)}m$ graph in $2^{O(d)}(n \log n + m)$ expected time. If m is superlinear in n , then we can produce a hierarchically well-spaced superset of size $2^{O(d)}n$ in $2^{O(d)}n \log n$ expected time.

6.3.11. Geometric Separators and the Parabolic Lift

Participant: Donald Sheehy.

A geometric separator for a set U of n geometric objects (usually balls) is a small (sublinear in n) subset whose removal disconnects the intersection graph of U into roughly equal sized parts. These separators provide a natural way to do divide and conquer in geometric settings. A particularly nice geometric separator algorithm originally introduced by Miller and Thurston has three steps: compute a centerpoint in a space of one dimension higher than the input, compute a conformal transformation that “centers” the centerpoint, and finally, use the computed transformation to sample a sphere in the original space. The output separator is the subset of S intersecting this sphere. It is both simple and elegant. We show [36] that a change of perspective (literally) can make this algorithm even simpler by eliminating the entire middle step. By computing the centerpoint of the points lifted onto a paraboloid rather than using the stereographic map as in the original method, one can sample the desired sphere directly, without computing the conformal transformation.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

7.1.1. Contrat Cifre with Geometry Factory

Mael Rouxel-Labbé’s PhD thesis is supported by a Cifre contract with GEOMETRY FACTORY (<http://www.geometryfactory.com>). The subject is the generation of anisotropic meshes.

7.1.2. Commercialization of cgal packages through Geometry Factory

In 2013, GEOMETRY FACTORY (<http://www.geometryfactory.com>) had the following new customers for CGAL packages developed by GEOMETRICA:

GeoSoft (oil and gas, USA) : 2D constrained triangulation, AABB tree

British Geological Survey (oil and gas, UK) : 2D Meshes, Interpolation

Hexagon Machine Control (GIS, Sweden) 3D triangulations, point set processing

Thales (GIS, France) 2D constrained triangulation

8. Partnerships and Cooperations

8.1. Technological Development Actions

8.1.1. ADT PH

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Sonali Digambar Patil, Marc Glisse, Steve Oudot, Clément Maria, Mariette Yvinec.

- Title: Persistent Homology

- Coordinator: Mariette Yvinec (GEOMETRICA)

- Duration: 1 year renewable once, starting date December 2012.

- Others Partners: Inria team ABS, Gipsa Lab (UMR 5216, Grenoble, <http://www.gipsa-lab.inpg.fr/>)

- Abstract: Geometric Inference is a rapidly emerging field that aims to analyse the structural, geometric and topological, properties of point cloud data in high dimensional spaces. The goal of the ADT PH is to make available, a robust and comprehensive set of algorithmic tools resulting from recent advances in Geometric Inference. The software will include:

tools to extract from the data sets, families of simplicial complexes,

data structures to handle those simplicial complexes,

algorithmic modules to compute the persistent homology of those complexes,

applications to clustering, segmentation and analysis of scalar fields such as the energy landscape of macromolecular systems.

8.1.2. ADT OrbiCGAL

Participants: Mikhail Bogdanov, Aymeric Pellé, Monique Teillaud.

- Title: OrbiCGAL

- Coordinator: Monique Teillaud (GEOMETRICA)

- Duration: 1 year renewable once, starting date September 2013.

- Abstract: OrbiCGAL is a software project supported by Inria as a Technological Development Action (ADT). It is motivated by applications ranging from infinitely small (nano-structures) to infinitely large (astronomy), through material engineering, physics of condensed matter, solid chemistry, etc

The project consists in developing or improving software packages to compute triangulations and meshes in several types of non-Euclidean spaces: sphere, 3D closed flat manifolds, hyperbolic plane.

8.2. Regional Initiatives

8.2.1. Digiteo project TOPERA

Participants: Frédéric Chazal, Marc Glisse, Anaïs Vergne.

TOPERA is a project that aims at developing methods from Topological Data Analysis to study covering properties and quality of cellular networks. It also involves L. Decreasefond and P. Martins from Telecom Paris.

- Starting date: December 2013
- Duration: 18 months

8.3. National Initiatives

8.3.1. ANR Présage

Participants: Olivier Devillers, Marc Glisse, Ross Hemsley, Monique Teillaud, Rémy Thomasse.

- Acronym: Presage.
- Type: ANR blanc.
- Title: *méthodes PRobabilistes pour l'Éfficacité des Structures et Algorithmes GÉométriques.*
- Coordinator: Xavier Goaoc.
- Duration: 31 december 2011 - 31 december 2015.
- Other partners: Inria VEGAS team, University of Rouen.
- Abstract: This project brings together computational and probabilistic geometers to tackle new probabilistic geometry problems arising from the design and analysis of geometric algorithms and data structures. We focus on properties of discrete structures induced by or underlying random continuous geometric objects. This raises questions such as:
 - What does a random geometric structure (convex hulls, tessellations, visibility regions...) look like?
 - How to analyze and optimize the behavior of classical geometric algorithms on *usual* inputs?
 - How can we generate randomly *interesting* discrete geometric structures?
- Year publications: [16], [31], [51].

8.3.2. ANR GIGA

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Mariette Yvinec, Steve Oudot, Marc Glisse.

- Acronym : GIGA.
- Title : Geometric Inference and Geometric Approximation.
- Type: ANR blanc
- Coordinator: Frédéric Chazal (GEOMETRICA)
- Duration: 4 years starting October 2009.
- Others Partners: Inria team-project Titane, Inria team-project ABS, CNRS (Grenoble), Dassault Systèmes.
- Abstract: GIGA stands for Geometric Inference and Geometric Approximation. GIGA aims at designing mathematical models and algorithms for analyzing, representing and manipulating discretized versions of continuous shapes without losing their topological and geometric properties. By shapes, we mean sub-manifolds or compact subsets of, possibly high dimensional, Riemannian manifolds. This research project is divided into tasks which have Geometric Inference and Geometric Approximation as a common thread. Shapes can be represented in three ways: a physical representation (known only through measurements), a mathematical representation (abstract and continuous), and a computerized representation (inherently discrete). The GIGA project aims at studying the transitions from one type to the other, as well as the associated discrete data structures.

Some tasks are motivated by problems coming from data analysis, which can be found when studying data sets in high dimensional spaces. They are dedicated to the development of mathematically well-founded models and tools for the robust estimation of topological and geometric properties of data sets sampled around an unknown compact set in Euclidean spaces or around Riemannian manifolds.

Some tasks are motivated by problems coming from data generation, which can be found when studying data sets in lower dimensional spaces (Euclidean spaces of dimension 2 or 3). The proposed research activities aim at leveraging some concepts from computational geometry and harmonic forms to provide novel algorithms for generating discrete data structures either from mathematical representations (possibly deriving from an inference process) or from raw, unprocessed discrete data. We target both isotropic and anisotropic meshes, and simplicial as well as quadrangle and hexahedron meshes.

- See also: <http://www-sop.inria.fr/geometrica/collaborations/giga/>

8.3.3. ANR TOPDATA

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Mariette Yvinec, Steve Oudot, Marc Glisse.

- Acronym : TopData.
- Title : Topological Data Analysis: Statistical Methods and Inference.
- Type : ANR blanc
- Coordinator : Frédéric Chazal (GEOMETRICA)
- Duration : 4 years starting October 2013.
- Others Partners: Département de Mathématiques (Université Paris Sud), Institut de Mathématiques (Université de Bourgogne), LPMA (Université Paris Diderot), LSTA (Université Pierre et Marie Curie)
- Abstract: TopData aims at designing new mathematical frameworks, models and algorithmic tools to infer and analyze the topological and geometric structure of data in different statistical settings. Its goal is to set up the mathematical and algorithmic foundations of Statistical Topological and Geometric Data Analysis and to provide robust and efficient tools to explore, infer and exploit the underlying geometric structure of various data.

Our conviction, at the root of this project, is that there is a real need to combine statistical and topological/geometric approaches in a common framework, in order to face the challenges raised by the inference and the study of topological and geometric properties of the wide variety of larger and larger available data. We are also convinced that these challenges need to be addressed both from the mathematical side and the algorithmic and application sides. Our project brings together in a unique way experts in Statistics, Geometric Inference and Computational Topology and Geometry. Our common objective is to design new theoretical frameworks and algorithmic tools and thus to contribute to the emergence of a new field at the crossroads of these domains. Beyond the purely scientific aspects we hope this project will help to give birth to an active interdisciplinary community. With these goals in mind we intend to promote, disseminate and make our tools available and useful for a broad audience, including people from other fields.

8.4. European Initiatives

8.4.1. FP7 Projects

8.4.1.1. CG-Learning

Type: COOPERATION

Defi: FET Open

Instrument: Specific Targeted Research Project

Objectif: FET-Open: Challenging Current Thinking

Duration: November 2010 - October 2013

Coordinator: Friedrich-Schiller-Universität Jena (Germany)

Others partners: National and Kapodistrian University of Athens (Greece), Technische Universität Dortmund (Germany), Tel Aviv University (Israel), Eidgenössische Technische Hochschule Zürich (Switzerland), Rijksuniversiteit Groningen (Netherlands), Freie Universität Berlin (Germany)

Inria contact: Mariette Yvinec

See also: <http://cgl.uni-jena.de/>

Abstract: The Computational Geometric Learning project aims at extending the success story of geometric algorithms with guarantees to high-dimensions. This is not a straightforward task. For many problems, no efficient algorithm exist that compute the exact solution in high dimensions. This behavior is commonly called the curse of dimensionality. We try to address the curse of dimensionality by focusing on inherent structure in the data like sparsity or low intrinsic dimension, and by resorting to fast approximation algorithms.

8.5. International Initiatives

8.5.1. Inria Associate Teams

8.5.1.1. COMET

Title: Computational methods for the analysis of high-dimensional data

Inria principal investigator: Steve Y. Oudot

International Partner (Institution - Laboratory - Researcher):

Stanford University (United States) - Computer Science - Leonidas Guibas

Ohio State University (United States) - Computer Science and Engineering - Yusu Wang

Duration: 2011 - 2013

See also: <http://geometrica.saclay.inria.fr/collaborations/CoMeT/index.html>

CoMeT is an associate team between the Geometrica group at Inria, the Geometric Computing group at Stanford University, and the Computational Geometry group at the Ohio State University. Its focus is on the design of computational methods for the analysis of high-dimensional data, using tools from metric geometry and algebraic topology. Our goal is to extract enough structure from the data, so we can get a higher-level informative understanding of these data and of the spaces they originate from. The main challenge is to be able to go beyond mere dimensionality reduction and topology inference, without the need for a costly explicit reconstruction. To validate our approach, we intend to set our methods against real-life data sets coming from a variety of applications, including (but not restricted to) clustering, image or shape segmentation, sensor field monitoring, shape classification and matching. The three research groups involved in this project have been active contributors in the field of Computational Topology in the recent years, and some of their members have had long-standing collaborations. We believe this associate team can help create new synergies between these groups.

8.6. International Research Visitors

Mirel Ben Chen (Technion - Israel Institute of Technology)

Benjamin Burton (University of Queensland)

Pedro Machado Manhães de Castro (Universidade Federal de Pernambuco)

Arijit Ghosh (Indian Statistical Institute)

Michael Hemmer (University of Technology Braunschweig)

Dmitriy Morozov (Berkeley)

Yusu Wang (Ohio State University)

Jian Sun (Tsinghua University - China)

Yuan Yao (Peiking University - China)

9. Dissemination

9.1. Scientific Animation

9.1.1. Editorial boards of scientific journals

Jean-Daniel Boissonnat is a member of the Editorial Board of *Journal of the ACM*, *Discrete and Computational Geometry*, *Algorithmica*, *International Journal on Computational Geometry and Applications*.

Frédéric Chazal is a member of the Editorial Board of *SIAM Journal on Imaging Sciences*, *Graphical Models* and *Discrete and Computational Geometry* (start in Jan. 2014).

Olivier Devillers is a member of the Editorial Board of *Graphical Models*.

Monique Teillaud is a member of the Editorial Boards of CGTA, *Computational Geometry: Theory and Applications*, and of IJCGA, *International Journal of Computational Geometry and Applications*.

M. Yvinec is a member of the editorial board of *Journal of Discrete Algorithms*.

Monique Teillaud and Mariette Yvinec are members of the CGAL editorial board.

9.1.2. Conference program committees

Jean-Daniel Boissonnat was a member of the PC of the Symposium on Geometry Processing SGP 2013.

Jean-Daniel Boissonnat chaired WoCG (Workshops in Computational Geometry) and was a member of the program committee of the Young Researchers Forum, two satellite events of the ACM Symposium on Computational Geometry SoCG 2013.

Frédéric Chazal was a member of the PC of the ACM Symposium on Computational Geometry 2013, of the Scientific committee of the SMAI 2013 conference, and of Geometric Science of Information (GSI 2013).

Monique Teillaud was a member of the PC of EuroCG, the European workshop on computational geometry.

9.1.3. Steering committees

Jean-Daniel Boissonnat is a member of the steering committee of the international conference on Curves and Surfaces.

Monique Teillaud was a member of the Computational Geometry Steering Committee until April.

Monique Teillaud has been a member of the Steering Committee of the European Symposium on Algorithms (ESA) since September.

9.1.4. Inria committees

Jean-Daniel Boissonnat was a member of the recruitment committee of Inria Rhône-Alpes.

Frédéric Chazal was a member of the recruitment committee of Inria Saclay (vice-chair).

Monique Teillaud is a member of the Inria Evaluation Committee.

She was a member of the national Inria DR2 interview committee and the local CR2 interview committee.

Monique Teillaud is a member of the local Committee for Technologic Development.

She was also a member of the local committee for transversal masters.

9.1.5. Other committees

Jean-Daniel Boissonnat is a member of the Conseil de l'AERES (Agence d'Evaluation de la Recherche et de l'Enseignement Supérieur).

9.1.6. Conference organization

- O. Devillers and M. Teillaud coorganized the workshop on Geometric Computing (<http://www.acmac.uoc.gr/GC2013/> Heraklion, Greece, January) with Menelaos Karavelas (University of Crete) and Elias Tsigaridas (EPI POLSYS).
- M. Teillaud coorganized the Dagstuhl Seminar on Computational Geometry [38] (<http://www.dagstuhl.de/en/program/calendar/semhp/?semnr=13101>, March 4-8) with Otfried Cheong (KAIST) and Kurt Mehlhorn (MPI Saarbrücken)
- M. Teillaud coorganized the Dagstuhl Seminar on Drawing Graphs and Maps with Curves [39] (<http://www.dagstuhl.de/en/program/calendar/semhp/?semnr=13151>, April 8-12) with Stephen Kobourov (University of Arizona) and Martin Nöllenburg (Karlsruhe Institute of Technology)
- O. Devillers organized the workshop of ANR Presage (Valberg, France, June).
- M. Teillaud (chair) and O. Devillers coorganized ALGO 2013, in cooperation with members of EPI ABS and COATI (<http://algo2013.inria.fr/>. Sophia Antipolis, France, September 2-6). ALGO 2013 combined the European Symposium on Algorithms (ESA) and six more specialized conferences and workshops. There were about three hundred attendees.
- D. Cohen-Steiner organized the 2013 edition of the “Journées de Géométrie Algorithmique” held at the CIRM, Luminy in December <http://quentin.mrgt.fr/events/jga2013.html>.

9.1.7. Web site

M. Teillaud is maintaining the Computational Geometry Web Pages <http://www.computational-geometry.org/>, hosted by Inria. This site offers general interest information for the computational geometry community, in particular the Web proceedings of the Video Review of Computational Geometry, part of the Annual Symposium on Computational Geometry.

9.1.8. Geometrica seminar

The seminar featured presentations by the following scientists:

Omri Azencot (Technion) : An Operator Approach to Tangent Vector Field Processing

Mirela Ben Chen (Technion - Israel Institute of Technology) : Can Mean-Curvature Flow be Modified to be Non-singular?

Benjamin Burton (University of Queensland) : Untangling knots using combinatorial optimisation

A. Chiara de Vitis (Pavia) : Geometrical and Topological Descriptors for Protein Structures

C. Couprie (Courant Institute) : Graph-based Variational Optimization and Applications in Computer Vision

J. Demantke (IGN) : Geometric Feature Extraction from LIDAR Point Clouds and Photorealistic 3D Facade Model Reconstruction from Terrestrial LIDAR and Image Data

Kyle Heath (Stanford University) : Image Webs: Discovering and using object-manifold structure in large-scale image collections

N. Mitra (UCL London) : Computational Design Tools for Smart Models Synthesis

P. Machado Manhães de Castro (UFPE, Brasil) : Invariance for Single Curved Manifolds

Natan Rubin (Freie Universität Berlin) : On Kinetic Delaunay Triangulations: A Near Quadratic Bound for Unit Speed Motions

D. Salinas (Gipsa Lab, Grenoble) : Using the Rips complex for Topologically Certified Manifold Learning

Régis Straubhaar (Université de Neuchâtel) : Numerical optimization of an eigenvalue of the Laplacian on a domain in surfaces

Jian Sun (Mathematical Sciences Center, Tsinghua University) : Rigidity of Infinite Hexagonal Triangulation of Plane

Anaïs Vergne (Télécom ParisTech) : Algebraic topology and sensor networks

Yuan Yao (School of Mathematical Sciences, Peking University) : The Landscape of Complex Networks

H. Zimmer (Aachen) : Geometry Optimization for Dual-Layer Support Structures

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Graduate level: Olivier Devillers, *Delaunay triangulation*, 6h, Universidade Federal de Pernambuco, Brasil.

Master : J.D. Boissonnat, D. Cohen-Steiner, M. Yvinec, *Computational Geometric Learning* , 24h, International Master Sophia-Antipolis.

Master : S. Oudot, *Computational Geometry: from Theory to Applications*, 36h, École polytechnique.

Master: Jean-Daniel Boissonnat, Frédéric Chazal, Mariette Yvinec, *Computational Geometric Learning*, 24h, Master MPRI, Paris.

Master: Olivier Devillers and Monique Teillaud, *Algorithmes géométriques, théorie et pratique*, 16h, Master SSTIM-VIM, Université Nice Sophia Antipolis.

Doctorat : Jean-Daniel Boissonnat, Frédéric Chazal, Mariette Yvinec, *Analyse géométrique des données* , 7h, Ecole Jeunes Chercheurs GDR Informatique Mathématique, Perpignan.

9.2.2. Supervision

PhD: Mikhail Bogdanov, Delaunay triangulations of spaces of constant negative curvature, Université de Nice - Sophia Antipolis, December 9, Monique Teillaud.

PhD in progress: Thomas Bonis, Topological persistence for learning, started on December 2013, Frédéric Chazal.

PhD in progress: Mickaël Buchet, Topological and geometric inference from measures, Université Paris XI, started October 2011, Frédéric Chazal and Steve Oudot.

PhD in progress : Ross Hemsley, Probabilistic methods for the efficiency of geometric structures and algorithms, started October 1st 2011, Olivier Devillers.

PhD in progress: Ruqi Huang, Algorithms for topological inference in metric spaces, started on December 2013, Frédéric Chazal.

PhD in progress : Clément Maria, Data structures and Algorithms in Computational Topology, started October 1st, 2011, Jean-Daniel Boissonnat.

PhD in progress : Rémy Thomasse, Smoothed complexity of geometric structures and algorithms, started December 1st 2012, Olivier Devillers.

PhD in progress : Mael Rouxel-Labbé, Anisotropic Mesh Generation, started October 1st, 2013, Jean-Daniel Boissonnat and Mariette Yvinec.

9.2.3. Juries

Jean-Daniel Boissonnat was a member (reviewer) of the HDR defense of Dominique Attali (Univ. Grenoble).

Jean-Daniel Boissonnat was a member (reviewer) of the PhD defense committee of David Salinas (Univ. Grenoble). Steve Oudot was also part of that defense committee.

Jean-Daniel Boissonnat was a member (reviewer) of the PhD defense of Jérémy Espinas (Univ. Lyon).

Frédéric Chazal was a member (reviewer) of the PhD defense of Lucie Druoton (Univ. de Bourgogne).

Frédéric Chazal was a member (reviewer) of the PhD defense of Anaïs Vergne (Telecom Paris).

Olivier Devillers was a member of the HDR defense committee of Nicolas Bonichon (Univ. Bordeaux).

Monique Teillaud was a member of the PhD defense committee of Marcel Roeloffzen, TU Eindhoven, October.

9.2.4. Internships

Thomas Bonis, image and shape classification using persistent homology (F. Chazal)

Claudia Werner, Triangulations on the sphere, Hochschule für Technik Stuttgart (Monique Teillaud)

Arnaud Poinas, Statistical manifold reconstruction (Jean-Daniel Boissonnat)

Sergei Kachanovich, Graph-induced simplicial complex (Jean-Daniel Boissonnat)

Chunyuan Li, Persistence-based object recognition (F. Chazal and M. Ovsjanikov)

Venkata Yamajala, Implementing (and simplifying) the tangential complex (Jean-Daniel Boissonnat)

9.3. Popularization

Jean-Daniel Boissonnat, Au delà de la dimension 3, Caféin Inria Sophia Antipolis.

Jean-Daniel Boissonnat, Geometry Understanding in Higher Dimensions. Conference for the students of ENS Lyon (Inria Sophia Antipolis)

Monique Teillaud, “à quoi sert un triangle ?”, 2x2h, Collège Le Prés des Roures, Le Rouret, in the framework of the national Week of Mathematics.

Steve Oudot was coordinator of the *Photomaton 3d* booth at the *Nuit des chercheurs* event at École polytechnique in September 2013. Marc Glisse, Maks Ovsjanikov, Mickaël Buchet, and Thomas Bonis also participated.

9.4. Participation to conferences, seminars, invitations

9.4.1. Invited Talks

Jean-Daniel Boissonnat gave an invited lecture at the International Symposium on Voronoi Diagrams (St Petersburg) : on the empty sphere on manifolds. July.

Jean-Daniel Boissonnat gave an invited talk at the Jean-Paul Laumond’s day (on the occasion of his 60th anniversary) : Comprendre la géométrie des données.

Frédéric Chazal gave invited talks at the Workshop on Topological Data analysis, Institute for Mathematics and its Applications, Minneapolis, October 2013; at the Workshop on Topological Methods in Complexity Science, European Conference on Complex Systems satellite conference, Barcelona, September 2013.

9.4.2. Seminars

Members of the project have presented their published articles at conferences. The reader can refer to the bibliography to obtain the corresponding list. We list below all other talks given in seminars, summer schools and other workshops.

Frédéric Chazal, Filtrations et entrelacements : théorie et applications en Analyse Topologique des Données, Séminaire Brillouin, Paris, Dec. 2013.

Frédéric Chazal, Transport de mesures et inférence géométrique, Journées de Contrôle et Transport Optimal, Dijon, February 2013.

Frédéric Chazal, Computer Science and Machine Learning Seminar at Carnegie Mellon University, Pittsburg, September 2013.

Frédéric Chazal, Convergence rates for persistence diagrams in Topological Data Analysis, Workshop on Applied and Computational Topology, Bremen, July 2013.

Frédéric Chazal, Inférence géométrique et analyse topologique des données à l'aide de fonctions distance, colloquium de mathématiques, Univ. Paris 6, May 2013.

Olivier Devillers. Qualitative Symbolic Perturbations. Workshop on Geometric Computing. Heraklion. January.

Olivier Devillers. Hyperbolic Delaunay Triangulation, Universidade Federal de Pernambuco. June.

Monique Teillaud. Delaunay Triangulations of Point Sets in Closed Euclidean d -orbifolds. Workshop on Geometric Computing. Heraklion. January.

Monique Teillaud. Delaunay Triangulations of Point Sets in Closed Euclidean d -orbifolds. Meeting of ANR Presage. January.

Monique Teillaud. Curves in CGAL (with Michael Hemmer, University of Technology Braunschweig). Dagstuhl Seminar on Drawing graphs and maps with curves. April.

Monique Teillaud. 3D meshes in CGAL. Mathematics for Industry and Society, French Embassy Berlin. July.

9.4.3. Short scientific visits

Frédéric Chazal, Carnegie Mellon University, Sept. 2013.

Frédéric Chazal, Stanford University, May 2013.

Olivier Devillers and Monique Teillaud, University of Athens, January.

Monique Teillaud, Zuse Institut Berlin, July.

Monique Teillaud, TU Eindhoven, October.

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Articles in International Peer-Reviewed Journals

- [11] O. AMINI, J.-D. BOISSONNAT, P. MEMARI. *Geometric Tomography with Topological Guarantees*, in "Discrete and Computational Geometry", December 2013, vol. 50, n^o 4, pp. 821-856 [DOI : 10.1007/s00454-013-9531-z], <http://hal.archives-ouvertes.fr/inria-00440322/>, <http://hal.inria.fr/hal-00921910>
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