INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team cqfd

Quality control and dynamic reliability

Bordeaux - Sud-Ouest

Theme : Stochastic Methods and Models

Activity Report

2010

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1. Team

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2. Overall Objectives

2.1. Presentation

The core component of our scientific agenda focuses on the development of statistical and probabilistic methods for the modeling and the optimization of complex systems. These systems require mathematical representations which are in essence dynamic and stochastic with discrete and/or continuous variables. This increasing complexity poses genuine scientific challenges that can be addressed through complementary approaches and methodologies:

- Modeling: design and analysis of realistic and tractable models for such complex real-life systems and various probabilistic phenomena;
- Estimation: developing theoretical and computational procedures in order to estimate and evaluate the parameters and the performance of the system;
- Optimization: developing theoretical and numerical control tools to optimize the performance and/or to maintain the system function in operating state.

2.2. Highlights

F. Dufour is scientific advisor for the C.E.A, (French council for the atomic energy).

F. Dufour has been invited to present a paper at the conference Stochastic Processes and their Applications (SPA) in Osaka, Japan (sept. 2010).

J. Saracco has been invited to present a paper at International workshop on Time Series, Quantile Regression and Model Choice, in Dortmund, Germany (sept. 2010).

The team CQFD has been very involved in the organization of the "Journées MAS 2010". This biennial meeting is the most important event for French probabilists and statisticians. It took place in Bordeaux University in september 2010. The main topic of this meeting was stochastic algorithms and combinatorics. 13 plenary sessions, about 100 contributed talks and the topics of the team were represented in particular in the sessions "classification", "degradation models and maintenance politics" and "stochastics models for crack propagation". More details are available via the link http://www.math.u-bordeaux1.fr/MAS10/.

3. Scientific Foundations

3.1. Introduction

The scientific objectives of the team are to provide mathematical tools for modeling and optimization of complex systems. These systems require mathematical representations which are in essence dynamic, multi-model and stochastic. This increasing complexity poses genuine scientific challenges in the domain of modeling and optimization. More precisely, our research activities are focused on stochastic optimization and (parametric, semi-parametric, multidimensional) statistics which are complementary and interlinked topics. It is essential to develop simultaneously statistical methods for the estimation and control methods for the optimization of the models.

3.2. Main research topics

- Stochastic modeling: Markov chain, Piecewise Deterministic Markov Processes (PDMP), Markov Decision Processes (MDP).
  The mathematical representation of complex systems is a preliminary step to our final goal corresponding to the optimization of its performance. For example, in order to optimize the predictive maintenance of a system, it is necessary to choose the adequate model for its representation. The step of modeling is crucial before any estimation or computation of quantities related to its optimization. For this we have to represent all the different states of the system and the behavior of the physical variables under each of these states. Moreover, we must also select the dynamic variables which have a potential effect on the physical variable and the quantities of interest. The team CQFD works on the theory of Piecewise Deterministic Markov Processes (PDMP's) and on Markov Decision Processes (MDP's). These two classes of systems form general families of controlled stochastic processes suitable for the modeling of sequential decision-making problems in the continuous-time (PDMPs) and discrete-time (MDP's) context. They appear in many fields such as engineering, computer science, economics, operations research and constitute powerful class of processes for the modeling of complex system.
- Estimation methods: estimation for PDMP; estimation in non- and semi parametric regression modeling.
  To the best of our knowledge, there does not exist any general theory for the problems of estimating parameters of PDMPs although there already exist a large number of tools for sub-classes of PDMPs such as point processes and marked point processes. However, to fill the gap between these specific models and the general class of PDMPs, new theoretical and mathematical developments will be on the agenda of the whole team. In the framework of non-parametric regression or quantile regression, we focus on kernel estimators or kernel local linear estimators for complete data or censored data. New strategies for estimating semi-parametric models via recursive estimation procedures have also received an increasing interest recently. The advantage of the recursive estimation approach is to take into account the successive arrivals of the information and to refine, step after step, the implemented estimation algorithms. These recursive methods do require restarting calculation of parameter estimation from scratch when new data are added to the base. The idea is to use only the previous estimations and the new data to refresh the estimation. The gain in time could be very interesting and there are many applications of such approaches.

- Dimension reduction: dimension-reduction via SIR and related methods, dimension-reduction via multidimensional and classification methods.
  Most of the dimension reduction approaches seek for lower dimensional subspaces minimizing the loss of some statistical information. This can be achieved in modeling framework or in exploratory data analysis context.
  In modeling framework we focus our attention on semi-parametric models in order to conjugate the advantages of parametric and nonparametric modeling. On the one hand, the parametric part of the model allows a suitable interpretation for the user. On the other hand, the functional part of the model offers a lot of flexibility. In this project, we are especially interested in the semi-parametric regression model $Y = f(X'\theta) + \varepsilon$, the unknown parameter θ belongs to $\mathbb{R}^p$ for a single index model, or is such that $\theta = [\theta_1, \cdots, \theta_d]$ (where each θ_k belongs to $\mathbb{R}^p$ and $d \leq p$ for a multiple indices model), the noise ε is a random error with unknown distribution, and the link function f is an unknown real valued function. Another way to see this model is the following: the variables X and Y are independent given $X'\theta$. In our semi-parametric framework, the main objectives are to estimate the parametric part θ as well as the nonparametric part which can be the link function f, the conditional distribution function of Y given X or the conditional quantile q_α. In order to estimate the dimension reduction parameter θ we focus on the Sliced Inverse Regression (SIR) method which has been introduced by Li [58] and Duan and Li [49]
  Methods of dimension reduction are also important tools in the field of data analysis, data mining and machine learning.They provide a way to understand and visualize the structure of complex data sets.Traditional methods among others are principal component analysis for quantitative variables or multiple component analysis for qualitative variables. New techniques have also been proposed to address these challenging tasks involving many irrelevant and redundant variables and often comparably few observation units. In this context, we focus on the problem of synthetic variables construction, whose goals include increasing the predictor performance and building more compact variables subsets. Clustering of variables is used for feature construction. The idea is to replace a group of "similar" variables by a cluster centroid, which becomes a feature. The most popular algorithms include K-means and hierarchical clustering. For a review, see, e.g., the textbook of Duda [50]
- Stochastic optimal control: optimal stopping, impulse control, continuous control, linear programming, singular perturbation, martingale problem.
  The first objective is to focus on the development of computational methods.
  - In the continuous-time context, stochastic control theory has from the numerical point of view, been mainly concerned with Stochastic Differential Equations (SDEs in short). From the practical and theoretical point of view, the numerical developments for this class of processes are extensive and largely complete. It capitalizes on the connection between SDEs and second order partial differential equations (PDEs in short) and the fact that the properties of the latter equations are very well understood. It is, however, hard to deny that the development of computational methods for the control of PDMPs has received little attention. One of the main reasons is that the role played by the familiar PDEs in the diffusion models is here played by certain systems of integro-differential equations for which there is not (and cannot be) a unified theory such as for PDEs as emphasized by M.H.A. Davis in his book. To the best knowledge of the team, there is only one attempt to tackle this difficult problem by O.L.V. Costa and M.H.A. Davis. The originality of our project consists in studying this unexplored area. It is very important to stress the fact that these numerical developments will give rise to a lot of theoretical issues such as type of approximations, convergence results, rates of convergence,....
  - Theory for MDP's has reached a rather high degree of maturity, although the classical tools such as value iteration, policy iteration and linear programming, and their various extensions, are not applicable in practice. We believe that the theoretical progress of MDP's must be in parallel with the corresponding numerical developments. Therefore, solving

MDP's numerically is an awkward and important problem both from the theoretical and practical point of view. In order to meet this challenge, the fields of neural networks, neuro-dynamic programming and approximate dynamic programming became recently an active area of research. Such methods found their roots in heuristic approaches, but theoretical results for convergence results are mainly obtained in the context of finite MDP's. Hence, an ambitious challenge is to investigate such numerical problems but for models with general state and action spaces. Our motivation is to develop theoretically consistent computational approaches for approximating optimal value functions and finding optimal policies.

Analysis of various problems arising in MDPs leads to a large variety of interesting mathematical problems. The second objective of the team is to study some theoretical aspects related to MDPs such as convex analytical methods and singular perturbation.

4. Application Domains

4.1. Application Domains

Our abilities in probability and statistics apply naturally to industry in particular in studies of dependability and safety.

An illustrative example which gathers all the topics of team is a collaboration started in May 2010 with Thales Optronique on the subject of *optimization of the maintenance of a digital camera equipped with HUMS* (Health Unit Monitoring Systems). This subject is very interesting for us because it combines many aspects of our project. Classification tools will be used to select significant variables as the first step in the modeling of a digital camera. The model will then be analysed and estimated in order to optimize the maintenance.

A second example concerns the optimization of the maintenance date for an aluminum metallic structure subject to corrosion. It is a structure of strategic ballistic missile that is stored in a nuclear submarine missile launcher in peace-time and inspected with a given periodicity. The requirement for security on this structure is very strong. The mechanical stress exerted on the structure depends on its thickness. It is thus crucial to control the evolution of the thickness of the structure over time, and to intervene before the break.

A third example is the minimization of the acoustic signature of a submarine. The submarine has to chose its trajectory in order to minimize at each time step its observability by a surface ship following an unknown random trajectory.

But the spectrum of applications of the topics of the team is larger and may concern many other fields. Indeed non parametric and semi-parametric regression methods can be used in biometry, econometrics or engineering for instance. Gene selection from microarray data and text categorization are two typical application domains of dimension reduction among others. We had for instance the opportunity via the scientific program PRIMEQUAL to work on air quality data and to use dimension reduction techniques as principal component analysis (PCA) or positive matrix factorization (PMF) for pollution sources identification and quantization.

5. Software

5.1. Package "edrGraphicalTools"

This R package gives graphical tools for selecting the number of slices and the dimension of the model in SIR and SAVE approaches. It also provides the estimation of the reduction dimension subspace and the non parametric estimation of the link function using smoothing techniques. The package is available via the link http://cran.r-project.org/web/packages/edrGraphicalTools/index.html.

5.2. Package "ClustOfVar"

This R package is dedicated to cluster analysis of a set of variables. Variables can be quantitative, qualitative or a mixture of both. It provides hierarchichal and k-means clustering of a set of variables when the homogeneity criterion is based on squared correlations and correlation ratios with the variable centers. The package is available via the link http://lib.stat.cmu.edu/R/CRAN/web/packages/ClustOfVar/index.html

6. New Results

6.1. Average Continuous Control of Piecewise Deterministic Markov Processes

Participant: François Dufour.

This work deals with the long run average continuous control problem of piecewise deterministic Markov processes (PDMP's) taking values in a general Borel space and with compact action space depending on the state variable. The control variable acts on the jump rate and transition measure of the PDMP, and the running and boundary costs are assumed to be positive but not necessarily bounded. Our first main result is to obtain an optimality equation for the long run average cost in terms of a discrete-time optimality equation related to the embedded Markov chain given by the post-jump location of the PDMP. Our second main result guarantees the existence of a feedback measurable selector for the discrete-time optimality equation by establishing a connection between this equation and an integro-differential equation. Our final main result is to obtain some sufficient conditions for the existence of a solution for a discrete-time optimality inequality and an ordinary optimal feedback control for the long run average cost using the so-called vanishing discount approach. Two examples are presented illustrating the possible applications of the results developed in this work. These results have been obtained in collaboration with Oswaldo Luis Do Valle Costa from Escola Politécnica da Universidade de São Paulo, Brazil. It has been published in SIAM Journal of Control and Optimization [18].

6.2. The Policy Iteration Algorithm for Average Continuous Control of Piecewise Deterministic Markov Processes

Participant: François Dufour.

The main goal of this work is to apply the so-called policy iteration algorithm (PIA) for the long run average continuous control problem of piecewise deterministic Markov processes (PDMP's) taking values in a general Borel space and with compact action space depending on the state variable. In order to do that we first derive some important properties for a pseudo-Poisson equation associated to the problem. In the sequence it is shown that the convergence of the PIA to a solution satisfying the optimality equation holds under some classical hypotheses and that this optimal solution yields to an optimal control strategy for the average control problem for the continuous-time PDMP in a feedback form. These results have been obtained in collaboration with Oswaldo Luis Do Valle Costa from Escola Politécnica da Universidade de São Paulo, Brazil. It has been published in Applied Mathematics and Optimization [19].

6.3. Multi-objective stopping problem for discrete-time Markov processes

Participant: François Dufour.

The purpose of this work is to study an optimal stopping problem with constraints for a Markov chain with general state space by using the convex analytic approach. The costs are assumed to be non-negative. Our model is not assumed to be transient or absorbing and the stopping time does not necessarily have a finite expectation. As a consequence, the occupation measure is not necessarily finite which poses some difficulties in the analysis of the associated linear program. Under a very weak hypothesis, it is shown that the linear problem admits an optimal solution, guaranteeing the existence of an optimal stopping strategy for the optimal stopping problem with constraints. These results have been obtained in collaboration with Alexey Piunovskiy from Department. of Mathematical Sciences, The University of Liverpool, United Kingdom. It has been published in Journal of Applied Probability [21].

6.4. Singular Perturbation for the discounted continuous control of Piecewise Deterministic Markov Processes

Participant: François Dufour.

This work deals with the expected discounted continuous control of piecewise deterministic Markov processes (PDMP's) using a singular perturbation approach for dealing with rapidly oscillating parameters. The state space of the PDMP is written as the product of a finite set and a subset of the Euclidean space $\mathbb{R}^n$. The discrete part of the state, called the regime, characterizes the mode of operation of the physical system under consideration, and is supposed to have a fast (associated to a small parameter $\epsilon > 0$) and a slow behavior. By using a similar approach as developed in the book of Yin (98), the idea in this paper is to reduce the number of regimes by considering an averaged model in which the regimes within the same class are aggregated through the quasi-stationary distribution so that the different states in this class are replaced by a single one. The main goal is to show that the value function of the control problem for the system driven by the perturbed Markov chain converges to the value function of this limit control problem as ϵ goes to zero. This convergence is obtained by, roughly speaking, showing that the infimum and supremum limits of the value functions satisfy two optimality inequalities as ϵ goes to zero. This enables us to show the result by invoking a uniqueness argument, without needing any kind of Lipschitz continuity condition. These results have been obtained in collaboration with Oswaldo Luis Do Valle Costa from Escola Politécnica da Universidade de São Paulo, Brazil. It has been accepted for publication in Applied Mathematics and Optimization [20]. Part of this work has been presented at the 48$^{\text{th}}$ IEEE Conference on Decision and Control, Atlanta, USA, 2010, [29].

6.5. Optimal stopping for predictive maintenance

Participants: Benoîte de Saporta, François Dufour, Huilong Zhang.

In the continuation of our work on numerical approximation of the optimal stopping problem for Piecewise deterministic Markov Processes (PDMP's), we applied our procedure on an industrial example provided by EADS-Astrium, as part of our ANR grant.

More precisely, we studied the maintenance of an aluminum metallic structure embedded in a strategic ballistic missile. It is to stored in a nuclear submarine missile launcher in peacetime and inspected with a given periodicity. These structures are made to have potentially long storage durations. They stay for random periods in different environments that characterize the conditions of use. Such environments may degrade faster or slower the corrosion protection of the structure. The requirement for security on this structure is very strong. It is thus crucial to control the evolution of the thickness loss of the structure over time, and to intervene before reaching a critical threshold.

We modeled the system by a 4-dimensional PDMP an applied and adapted our numerical procedure described in [26] to obtain path-adapted stopping rules and find optimal maintenance dates. This work was presented in the workshop *Modern trends in controlled stochastic processes: theory and applications* in Liverpool in July [39], in the 34th conference on Stochastic Processes and their Applications in Osaka in september, in the conference Lamba-Mu 17 [35] and in several seminars in France and abroad. A full article on this topic is in progress.

6.6. Numerical method for impulse control of PDMP's

Participants: Benoîte de Saporta, François Dufour, Huilong Zhang.

We have also extended our results on optimal stopping to the impulse control problem. An impulse control strategy consists in a sequence of single interventions introducing a jump of the process at some controller-specified stopping time and moving the process at that time to some new point in the state space. Our impulse control problem consists in choosing a strategy (if it exists) that minimizes the expected sum of discounted running and intervention costs up to infinity, and computing the optimal cost thus achieved. Many applied problems fall into this class, such as inventory problems in which a sequence of restocking decisions is made, or optimal maintenance of complex systems with components subject to failure and repair.

Impulse control problems of PDMP's in the context of an expected discounted cost have been considered in [46], [48], [52], [53], [57]. Roughly speaking, in [46] the authors study this impulse control problem by using the value improvement approach while in [48], [52], [53], [57] the authors choose to analyze it by using the variational inequality approach. In [46], the authors also consider a numerical procedure based on the iteration of the single-jump-or-intervention operator and a uniform discretization of the state space. Our approach is also based on the iteration of the single-jump-or-intervention operator, but we want to derive a convergence rate for our approximation. Our method does not rely on a *blind* discretization of the state space, but on a discretization that depends on time and takes into account the random nature of the process. Our approach involves a quantization procedure. Roughly speaking, quantization is a technique that approximates a continuous state space random variable X by a a random variable $\widehat{X}$ taking only finitely many values and such that the difference between X and $\widehat{X}$ is minimal for the L_p norm.

Although the value function of the impulse control problem can be computed by iterating implicit optimal stopping problems, see [46], [47], from a numerical point of view the impulse control is much more difficult to handle than the optimal stopping problem. Indeed, for the optimal stopping problem, the value function is computed as the limit of a sequence (v_n) constructed by iterating an operator L. This iteration procedure yields an iterative construction of a sequence of random variables $v_n(Z_n)$ (where (Z_n) is an embedded discrete-time process). This was the keystone of our approximation procedure. As regards impulse control, the iterative construction for the corresponding random variables does not hold anymore. This is mostly due to the fact that not only does the controller choose times to stop the process, but they also choose a new starting point for the process to restart from after each intervention. This makes the single-jump-or-intervention operator significantly more complicated to iterate that the single-jump-or-stop operator used for optimal stopping. We managed to overcome this extra difficulty by using two series of quantization grids instead of just the one we used for optimal stopping. This work has been submitted for publication (see http://hal.inria.fr/hal-00541413).

6.7. Exit time for PDMP's

Participants: Adrien Brandejsky, Benoîte de Saporta, François Dufour.

The aim of this work is to propose a practical numerical method to approximate the survival function and the moments of the exit time for a piecewise-deterministic Markov process thanks to the quantization of a discrete-time Markov chain naturally embedded within the continuous-time process.

Numerical computation of the moments of the exit time for a Markov process has been studied by K. Helmes, S. Röhl and R.H. Stockbridge in [54]. Starting from an assumption related to the generator of the process, they derive a system of linear equations satisfied by the moments. In addition to these equations, they include finitely many Hausdorff moment conditions that are also linear constraints. This optimization problem is a standard linear programming problem for which many efficient softwares are available. J.-B. Lasserre and T. Prieto-Rumeau introduced in [56] a similar method but they improved the efficiency of the algorithm by replacing the Hausdorff moment conditions with semidefinite positivity constraints of some moment matrices. Nevertheless, their approach cannot be applied to PDMP's because the assumption related to the generator of the process is generally not satisfied. In [47], M.H.A. Davis gives an iterative method to compute the mean exit time for a PDMP but his approach involves solving a large set of ODEs whose forms are very problem specific, depending on the behavior of the process at the boundary of the state space. Besides, in the context of applications to reliability, it seems important to study also the distribution of the exit time.

In our approach, the first step consists in expressing the j-th moment (respectively the survival function) as the last term of some sequence $(p_{k,j})_{k\leq N}$ (respectively $(p_k)_{k\leq N}$) satisfying a recursion equation $p_{k+1,j} = \psi(p_{k,j})$ (respectively $p_{k+1} = \psi(p_k)$). The natural way to deal with these problems is to follow the idea developed in [26] namely to rewrite the recursions in terms of an underlying discrete-time Markov chain and to replace it by its quantized approximation. The definitions of the recursive sequences $(p_{k,j})_k$ and $(p_k)_k$ involve some discontinuities related to indicator functions but as in [26], we show that they actually happen with small enough probability. However, an important feature that distinguishes the present work from [26] and which prevents a straightforward application of the ideas developed therein, is that the mapping ψ defining the recursions $p_{k+1,j} = \psi(p_{k,j})$ and $p_{k+1} = \psi(p_k)$ is not Lipschitz continuous. One of the main results of this paper is to overcome this difficulty by using sharp feature of the quantization algorithm. We are able to prove the convergence of the approximation scheme. Moreover, in the case of the moments, we even obtain bounds for the rate of convergence. It is important to stress that these assumptions are quite reasonable judging from the situations met in applications.

In addition, our method is highly flexible. Indeed, as pointed out in [45], a quantization based method is *obstacle free* in the sense that it provides, once and for all, a discretization of the process independently of the set we want to exit from. Consequently, the approximation schemes for both the moments and the distribution of the exit time are flexible w.r.t. to this exit set. This work has been submitted for publication (see http://hal.inria.fr/hal-00546339).

6.8. Asymptotic behavior of bifurcating autoregressive processes with missing data

Participants: Benoîte de Saporta, Anne Gégout-Petit.

Bifurcating autoregressive processes (BAR) generalize autoregressive (AR) processes, when the data have a binary tree structure. Typically, they are involved in modeling cell lineage data, since each cell in one generation gives birth to two offspring in the next one. Cell lineage data usually consist of observations of some quantitative characteristic of the cells, over several generations descended from an initial cell. Recently, experiments made by biologists on aging of *Escherichia coli*, see [60], motivated mathematical and statistical studies of the asymmetric BAR process, that is when the quantitative characteristics of the even and odd sisters are allowed to depend from their mother's through different sets of parameters. The originality of this work is that we take into account possibly missing data in the estimation procedure of the parameters of the asymmetric BAR process. This is a problem of practical interest, as experimental data are often incomplete, either because some cells died, or because the measurement of the characteristic under study was impossible or faulty. For instance, among the 94 colonies studied in [60], only two data sets are complete, with respectively 2 and 6 generations. In average over the 94 colonies dividing up to 9 times, there are about 23% of missing data. It is important to take this phenomenon into account.

The *naive* approach to handle missing data would be to replace the sums over all data in the estimators by sums over the observed data only. Our approach is slightly more subtle. We propose a structure for the observed data based on a two-type Galton-Watson process consistent with the possibly asymmetric structure of the BAR process. Basically, the probability to observe a cell depends on the type of both this cell and its mother. We propose an estimation procedure of the parameters of the asymmetric BAR process in this context, and prove sharp results of convergence and asymptotic normality for our estimators.

This work is in collaboration with Laurence Marsalle of Lille 1 university. It has been presented at the *Journées de Statistique* in Marseille in may [33] and in the seminar of the team probability and statistics of Bordeaux and is submitted for publication (see http://hal.inria.fr/inria-00494793/en/).

6.9. Modeling crack propagation by PDMP's

Participants: Romain Azaïs, Anne Gégout-Petit.

We propose a method to validate a model of PDMP for the propagation of a crack in alloy. The model is given in [44]. We have developed a method of actualization in order to predict the behavior of a crack knowing the beginning of the history of its propagation. The method is the following: we propose a model which parameters are estimated without a crack and we use it for the prediction of the crack. We define a criteria to measure the quality of the prediction. We compare the model of PDMP to a stochastic model without jump of the parameter using this criteria and the dispersion of the simulated curves. We carried out this work in collaboration with Marie Touzet of the LMP "Laboratoire de Mécanique Physique" from Bordeaux. The results are very satisfying. This work is published in [34].

6.10. Optimal quantization applied to Sliced Inverse Regression

Participants: Romain Azaïs, François Dufour, Anne Gégout-Petit, Jérôme Saracco.

We tackle the well known Slice Inverse Regression (SIR) method for a semiparametric regression model involving a quantitative variable X and including a dimension reduction of X via a parameter β. The response variable Y is real. Our goal is to estimate β and to predict the response variable conditionally to X. We adapt SIR method using optimal quantization [59] in the first time only for the independent variable X for the estimation of β. In a second time, we quantize the variable $(\widehat{\beta}_n, Y)$ in order to propose a discrete conditional law of Y given $X = x$. We show the convergence of the estimator of β and of the conditional law. Simulation studies show the numerical qualities of our estimates. This work will submitted for publication very soon.

6.11. A semiparametric approach to estimate reference curves for biophysical properties of the skin

Participant: Jérôme Saracco.

Reference curves which take one covariable into account such as the age, are often required in medicine, but simple systematic and efficient statistical methods for constructing them are lacking. Classical methods are based on parametric fitting (polynomial curves). In this work, we describe a methodology for the estimation of reference curves for data sets, based on nonparametric estimation of conditional quantiles. The derived method should be applicable to all clinical or more generally biological variables that are measured on a continuous quantitative scale. To avoid the curse of dimensionality when the covariate is multidimensional, a semiparametric approach is proposed. This procedure combines a dimension-reduction step (based on sliced inverse regression) and kernel estimation of conditional quantiles step. The usefulness of this semiparametric estimation procedure is illustrated on a sim- ulated data set and on a real data set collected in order to establish reference curves for biophysical properties of the skin of healthy French women. This work is published [38] and was presented in [32].

6.12. Recursive and non recursive versions for SIR and SIRoneslice (a new one slice-based SIR approach)

Participants: Bernard Bercu, Thi Mong Ngoc Nguyen, Jérôme Saracco.

We consider a semiparametric single index regression model involving a p-dimensional quantitative covariable x and a real dependent variable y. A dimension reduction is included in this model via an index $x'\beta$. Sliced inverse regression (SIR) is a well-known method to estimate the direction of the euclidean parameter β which is based on a "slicing step" of y in the population and sample versions. The goal of this work is twofold. We first propose an estimator of the direction of β based on the use of only one "optimal" slice chosen among the H slices. We call this new method SIRoneslice. Then we provide the recursive versions of the SIR and SIRoneslice estimators. We give an asymptotic result for SIRoneslice approach. Simulation study shows good numerical performances of SIRoneslice method and clearly exhibits the main advantage of using recursive versions of the SIR and SIRoneslice methods from a computational times point of view. Some extensions are also discussed. The proposed methods and criterion have been implemented in R and the corresponding codes are available from the authors. This work is in revision for publication [12].

6.13. A graphical tool for selecting the number of slices and the dimension of the model in SIR and SAVE approaches

Participant: Jérôme Saracco.

Sliced inverse regression (SIR) and related methods were introduced in order to reduce the dimensionality of regression problems. In general semiparametric regression framework, these methods determine linear combinations of a set of explanatory variables $\mathbf{X}$ related to the response variable Y, without losing information on the conditional distribution of Y given X. They are based on a "slicing step" in the population and sample versions. They are sensitive to the choice of the number H of slices, and this is particularly true for SIR-II and SAVE methods. At the moment there are no theoretical results nor practical techniques which allow the user to choose an appropriate number of slices. In this paper, we propose an approach based on the quality of the estimation of the effective dimension reduction (EDR) space: the square trace correlation between the true EDR space and its estimate can be used as goodness of estimation. We introduce a naïve bootstrap estimation of the square trace correlation criterion to allow selection of an “optimal" number of slices. Moreover, this criterion can also simultaneously select the corresponding suitable dimension K (number of the linear combination of **X**). From a practical point of view, the choice of these two parameters H and K is essential. We propose a 3D-graphical tool, implemented in R, which can be useful to select the suitable couple (H, K). An R package named “edrGraphicalTools” has been developed. In this work, we focus on the SIR-I, SIR-II and SAVE methods. Moreover the proposed criterion can be used to determine which method seems to be efficient to recover the EDR space, that is the structure between Y and **X**. We indicate how the proposed criterion can be used in practice. A simulation study is performed to illustrate the behavior of this approach and the need for selecting properly the number H of slices and the dimension K. This work is in collaboration with Benoît Liquet from the Biostatistic team of the Public Health Unit INSERM and is in revision for publication [25].

6.14. A sliced inverse regression approach for a stratified population

Participants: Marie Chavent, Jérôme Saracco.

We consider a semiparametric single index regression model involving a real dependent variable Y, a p-dimensional quantitative covariable X and a categorical predictor Z which defines a stratification of the population. This model includes a dimension reduction of X via an index $X'\beta$. We propose an approach based on sliced inverse regression in order to estimate the space spanned by the common dimension reduction direction β. We establish $\sqrt{n}$-consistency of the proposed estimator and its asymptotic normality. Simulation study shows good numerical performance of the proposed estimator in homoscedastic and heteroscedastic cases. Extensions to multiple indices models, q-dimensional response variable and/or SIR_α-based methods are also discussed. The case of unbalanced subpopulations is treated. Finally a practical method to investigate if there is or not a common direction β is proposed. This work is in collaboration with Benoît Liquet (Biostatistic tea of INSERM) and Vanessa Kuentz (CEMAGREF Bordeaux) and is accepted for publication [16].

6.15. Handling Missing Values with Regularized Iterative Multiple Correspondence Analysis

Participant: Marie Chavent.

A common approach to deal with missing values in Exploratory Data Analysis consists in minimizing the loss function over all non-missing elements. This can be achieved by EM-type algorithms where an iterative imputation of the missing values is performed during the estimation of the axes and components. This work proposes such an algorithm, named iterative MCA, to handle missing values in Multiple Correspondence Analysis (MCA). This algorithm, based on an iterative PCA algorithm, is described and its properties are studied. We point out the overfitting problem and propose a regularized version of the algorithm to overcome this major issue. Finally, performances of the *regularized iterative MCA* algorithm are assessed from both simulations and a real dataset. Results are promising for MAR and MCAR values with respect to other methods such as *missing-data passive modified margin*, an adaptation of *missing passive* method used in Gifi’s Homogeneity analysis framework. This work is submitted [55]

7. Contracts and Grants with Industry

7.1. EADS Astrium

Participants: Romain Azaïs, Adrien Brandejsky, Benoîte de Saporta, François Dufour, Anne Gégout-Petit, Huilong Zhang.

The goal of this project is to propose models for fatigue of structure and to study an approach to evaluate the probability of occurrence of events defined by the crossing of a threshold. In this context, EADS-Astrium funds the PhD Thesis of Adrien Brandejsky since september 2009.

7.2. Thales Optronique

Participants: Camille Baysse, Benoîte de Saporta, François Dufour, Anne Gégout-Petit, Jérôme Saracco.

The goal of the project is the optimization of the maintenance of a on board system with a HUMS (Health Unit Monitoring Systems). The collaboration begins with the PhD of Camille Baysse (CIFRE) on this subject.

7.3. DCNS

Participants: Benoîte de Saporta, François Dufour, Huilong Zhang.

In september 2010, an industrial collaboration started with DCNS on the application of Markov Decision Processes to optimal stochastic control of a submarine to minimize its acoustic signature. This work gave rise to the technical report *Contrôle optimal stochastique, application à l'optimisation de trajectoire* [51].

7.4. EDF

Participants: Benoîte de Saporta, François Dufour, Huilong Zhang.

The objective of this project is develop new methodologies for studying the dynamic reliability of controlled systems used in the critical area of power generation and process industries.

8. Other Grants and Activities

8.1. Regional Initiatives

Marie Chavent participates to a project financed by the Région Aquitaine for three years (2010-2013), named *PSI : Etude des interactions états psychophysiologiques et musique* including the PHD-grant of Laurent Vezard. The subject of this PHD, co-directed by M. Chavent, F. Faita and P. Legrand from Project-Team ALEA, is *Dimension reduction in the context of supervised learning. Applications to the electrical brain activity study*.

8.2. National Initiatives

The goal of the project "FAUTOCOES" (number ANR-09-SEGI-004) of the ARPEGE program of the French National Agency of Research (ANR) can be described as follows. Today, complex technological processes must maintain an acceptable behavior in the event of random structural perturbations, such as failures or component degradation. Aerospace engineering provides numerous examples of such situations: an aircraft has to pursue its mission even if some gyroscopes are out of order, a space shuttle has to succeed in its re-entry trip with a failed on-board computer. Failed or degraded operating modes are parts of an embedded system history and should therefore be accounted for during the control synthesis.

These few basic examples show that complex systems like embedded systems are inherently vulnerable to failure of components and their reliability has to be improved through fault-tolerant control. Embedded systems require mathematical representations which are in essence dynamic, multi-model and stochastic. This increasing complexity poses a genuine scientific challenge:

- to model explicitly and realistically the dynamical interactions existing between the physical state variables defining the system: pressure, temperature, flow rate, intensity, etc, and the functional and dysfunctional behavior of its components;
- to estimate the performance of the system through the evaluation of reliability indexes such as availability, quality, and safety;
- to optimize the control to prevent system failures, as well as to maintain the system function when a failure has occurred.

Our aim is to meet the previously mentioned challenge by using the framework of piecewise deterministic Markov processes (PDMP's in short) with an emphasis on probabilistic and deterministic numerical methods. More precisely, our objectives are

- to use the framework of piecewise deterministic Markov processes to model complex physical systems and phenomena;
- to compute expectations of functionals of the process in order to evaluate the performance of the system;
- to develop theoretical and numerical control tools for PDMP's to optimize the performance and/or to maintain system function when a failure has occurred.

More details are available at http://fautocoes.bordeaux.inria.fr/.

9. Dissemination

9.1. Editorial activities

M. Chavent is member of the scientific committee of SFC'10 and EGC'10.

F. Dufour is associate editor of the journal: SIAM Journal of Control and Optimization since 2009.

A. Gégout-Petit is member of the Review Committee of COMPSTAT 2010.

J. Saracco is an associate editor of the journal Case Studies in Business, Industry and Government Statistics (CSBIGS) since 2006.

J. Saracco is a member of the scientific committee of the conference "Rencontres des Jeunes Statisticiens" in Aussois in 2011.

All the member of the team are regular reviewers for the most important journals in applied probability and statistics.

9.2. Scientific responsibilities

B. de Saporta is in charge of the "Tache 3" of the ANR project FAUTOCOES.

F. Dufour is the leader of the ANR project FAUTOCOES.

F. Dufour is member of the IFAC Technical Committee TC 1.4 Stochastic Systems, term Period 2008-2011.

9.3. Organization of workshops and conferences

The team CQFD has been very involved in the organization of the "Journées MAS 2010". This biennial meeting is the most important event for French probabilists and statisticians. It took place in Bordeaux University in september 2010. The topic of this meeting where stochastic algorithms and combinatorics. http://www.math.u-bordeaux1.fr/MAS10/

The team CQFD has organized the meeting of the GTR 22, group "Methodology" of the IMdR (French Institute of Risk Control).

9.4. Administration of the universities and research institutes

M. Chavent is co-director of the cursus *Statistique et Fiabilité* of the master MIMSE *Ingénierie Mathématique, Statistique et Economique* of the University of Bordeaux.

B. de Saporta is member of the "Congress and Colloquium" commission of the INRIA Bordeaux Sud-Ouest.

B. de Saporta is correspondant of the cursus *Ingénierie Economique* of the master MIMSE *Ingénierie Mathématique, Statistique et Economique* of the University of Bordeaux.

B. de Saporta is in charge of the seminar of the team ”Statistics and Probability” of the Institute of Mathematics of Bordeaux (IMB).

F. Dufour is member of the scientific council of the engineering school ENSEIRB-MATMECA.

F. Dufour is member of the commission INRIA ”Jeunes Chercheurs”.

A. Gégout-Petit is a member of the CEVU (Conseil des Etudes et de la Vie Universitaire) of the Bordeaux 2 University.

J. Saracco was a member of the administration council of the University of Bordeaux 4.

J. Saracco is the leader of the team ”Statistics and Probability” of the Institute of Mathematics of Bordeaux (IMB).

H. Zhang is director of the cursus *Ingénierie Mathématique* of the Licence de Mathématiques of the University of Bordeaux.

9.5. Administration of the learned societies

M. Chavent is an elected member of the administration council of the SFdS. She was vice-secretary of the SFDS (Société Française de Statistique) until june 2010.

B. de Saporta belongs to the board of SMAI-MAS group. She is webmaster of this website.

A. Gégout-Petit is an elected member of the administration council of the SFdS (Société Française de Statistique); she is vice-secretary of the SFDS since june 2010

A. Gégout-Petit is in the board team of the web-domain *emath.fr*. In this function, she manages the project "carte des masters" a web site which gathers together the informations on all the french masters in mathematics.

9.6. Promotion, dissemination of the science

B de Saporta is a member of the "Cellule Grand Public" of the SMAI.

A. Gégout-Petit is in charge for the promotion of "Licence MASS" (Applied mathematics degree) of the University of Bordeaux 2 to the secondary school pupils.

9.7. Teaching

The actual members of the team are professor or senior lecturer at University of Bordeaux and each of them gives around 200 hours of teaching every year.

Master of mathematical engineering M. Chavent, B. de Saporta, F. Dufour, A. Gégout-Petit, J. Saracco, and H. Zhang teach graduate probability and statistics in the cursus "Statistique et Fiabilité" of the Master "Ingénierie Mathématique Statistique et Economique" at the Universities of Bordeaux 1, 2, 4, and Institut Polytechnique de Bordeaux. All of them are academic tutors for students work experiences in companies or research organisms.

Licence of applied mathematics M. Chavent and A. Gégout-Petit teach statistics in licence MASS of University of Bordeaux 2. H. Zhang teach probability and statistics in licence ”Ingénierie Mathématique” of University Bordeaux 1.

Engineering schools F. Dufour teaches probability and its applications to the students of the engineering schools MATMECA and ENSEIRB. J. Saracco teaches statistics to the students of the engineering school ENSC.

Finance and economics B. de Saporta teaches undergraduate mathematics and postgraduate probability and finance at the university of Economics Montesquieu Bordeaux 4. J. Saracco taught statistics in ”Magistère d'Economie et de Finance Internationale” (MAGEFI), University Bordeaux 4 and linear algebra and statistics in Licence of Economics, University Bordeaux 4.

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