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# *Project-Team galaad*

# *Géométrie, Algèbre, Algorithmes*

*Sophia Antipolis*

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# 2. Overall Objectives

Our research program is articulated around effective algebraic geometry and its applications. The objective is to develop algorithmic methods for effective and reliable resolution of geometric and algebraic problems, which are encountered in fields such as CAGD, robotics, computer vision, molecular biology, etc. We focus on the analysis of these methods from the point of view of complexity as well as qualitative aspects, combining symbolic and numerical computation.

Geometry is one of the key topics of our activity, which includes effective algebraic geometry, differential geometry, computational geometry of semi-algebraic sets. More specifically, we are interested in problems of small dimensions such as intersection, singularity, topology computation, and questions related to algebraic curves and surfaces.

These geometric investigations lead to algebraic questions, and particularly to the resolution of polynomial equations. We are involved in the design and analysis of new methods of effective algebraic geometry. Their developments and applications are central and often critical in practical problems.

Approximate numerical calculations, usually opposed to symbolic calculations, and the problems of certification are also at the heart of our approach. We intend to explore these bonds between geometry, algebra and analysis, which are currently making important strides. These objectives are both theoretical and practical. Recent developments enable us to control, check, and certify results when the data are known to a limited precision.

Finally our work is implemented in software developments. We pay attention to problems of genericity, modularity, effectiveness, suitable for the writing of algebraic and geometrical codes. The implementation and validation of these tools form another important component of our activity.

# 3. Scientific Foundations

## 3.1. Introduction

Our scientific activity is defined according to three broad topics: geometry, resolution of algebraic systems of equations, and symbolic-numeric links.

## 3.2. Geometry

We are interested in geometric modeling problems, based on non-discrete models, mainly of semi-algebraic type. Our activities focus in particular on the following points:

### *3.2.1. Geometry of algebraic varieties*

In order to solve effectively an algebraic problem, a preprocessing analyzing step is often mandatory. From such study, we will be able to deduce the method of resolution that is best suited to and thus produce an efficient solver, dedicated to a certain class of systems. The effective algebraic geometry provides us tools for analysis and makes it possible to exploit the geometric properties of these algebraic varieties. For this purpose, we focus on new formulations of resultants allowing us to produce solvers from linear algebra routines, and adapted to the solutions we want to compute. Among these formulations, we study in particular *residual* and *toric* resultant theory. The latter approach relates the generic properties of the solutions of polynomial equations, to the geometry of the Newton polytope associated to the polynomials.

### *3.2.2. Geometric algorithms for curved arcs and surface patches*

The above-mentioned tools of effective algebraic geometry make it possible to analyze in detail and separately the algebraic varieties. On the other hand, traditional algorithmic geometry deals with problems whose data are linear objects (points, segments, lines) but in very great numbers. Combining these two points of view, we concentrate on problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well known, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations.

### *3.2.3. Geometry of singularities and topology*

The analysis of singularities for a (semi)-algebraic set provides a better understanding of their structure. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

### *3.2.4. Geometry, groups, and invariants*

The objects in geometrical problems are points, lines, planes, spheres, quadrics, .... Their properties are, by nature, independent from the reference one chooses for performing analytic computations. Which leads us to methods from invariant theory. In addition to the development of symbolic geometric computations that exploit these invariants, we are also interested in developing more synthetic representations for handling those expressions.

## 3.3. Resolution of algebraic systems

The underlying representation behind a geometric model is often of algebraic type. Computing with such models raise algebraic questions, which frequently appear as bottlenecks of the geometric problems. Here are the particular approaches that we develop to handle such questions.

### *3.3.1. Algebraic methods and quotient structure*

In order to compute the solutions of a system of polynomial equations in several variables, we analyze and take advantage of the structure of the quotient ring, defined by these polynomials. This raises questions of representing and calculating normal forms in such structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases.

### *3.3.2. Duality, residues, interpolation*

We are interested in the “effective” use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems. Our focuses are effective computation of the algebraic residue, interpolation problems, and the relation between coefficients and roots in the case of multivariate polynomials.

### *3.3.3. Structured linear algebra and polynomials*

The preceding work lead naturally to the theory of structured matrices. Indeed, the matrices resulting from polynomial problems, such as matrices of resultants or Bezoutians, are structured. Their rows and columns are naturally indexed by monomials, and their structures generalize the Toeplitz matrices to the multivariate case. We are interested in exploiting these properties and the implications in solving polynomial equations [48].

### *3.3.4. Decomposition and factorisation*

When solving a system of polynomials equations, a first treatment is to transform it into several simpler subsystems when possible. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. For instance, the (difficult) problem of approximate factorization, the computation of perturbations of the data, which enables us to break up the problem, is studied. More generally, we are working on the problem of decomposing a variety into irreducible components.

### *3.3.5. Deformation and homotopy*

The behavior of a problem in the vicinity of a data can be interpreted in terms of deformations. Accordingly, the methods of homotopy consist in introducing a new parameter and in following the evolution of the solutions between a known position and the configuration one seeks to solve. This parameter can also be introduced in a symbolic manner, as in the techniques of perturbation of non-generic situations. We are interested in these methods, in order to use them in the resolution of polynomial equations as well as for new algorithms of approximate factorization.

## 3.4. Symbolic-numeric computation

Either in geometric or algebraic problems, symbolic and numeric computation are closely intertwined. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods, as explained now.

### *3.4.1. Certification*

The numerical problems are often approached locally. However in many situations, it is significant to give global answers, making it possible to certify calculations. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures [43].

### 3.4.2. Approximation

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate these objects while controlling the quality of approximation. Subdivision techniques based on the algebraic formulation of our problems are exploited in order to control the approximation, while locating interesting features such as singularities. This approach combines geometrical, algebraic and algorithmic aspects.

### 3.4.3. Degeneracies and arithmetic

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics. For example, we work on the computation of the sign of expressions, and on approaches combining modular and approximate computations, which speed up the exact answer [37].

# 4. Application Domains

## 4.1. CAGD

**Keywords:** *engineering computer-assisted*, *geometric modeling*.

3D modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). The involved mathematical objects have often an algebraic nature, which are then discretized for easy handling. The treatment of such objects can sometimes be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, ...We propose the developments of methods for shape modeling that takes into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship to the industrial contacts of CAGD we have.

## 4.2. Computer vision and robotics

**Keywords:** *calibration*, *engineering*, *reconstruction*.

Robotics and computer vision come with specific applications of the methods for solving polynomial equations. That is the case for instance, for the calibration of cameras, robots, computations of configurations and workspace. The resolution of algebraic problems with approximate coefficients is omnipresent.

## 4.3. Molecular biology and geometrical structures

**Keywords:** *biology*, *health*.

The chemical properties of molecules intervening in certain drugs are related to the configurations (or conformations) which they can take. These molecules are seen as mechanisms of bars and spheres, authorizing rotations around certain connections, similar to robots series. *Distance geometry* thus plays a significant role, for example, in the reconstruction from NMR experiments, or the analysis of realizable or accessible configurations. The methods we develop are well suited for solving such a problem.

# 5. Software

## 5.1. synaps, a module for symbolic and numeric computations

**Keywords:** *C++*, *Eigenvalues*, *FFT*, *bezoutian*, *curves and surfaces*, *effective algebraic geometry*, *genericity*, *geometry*, *iterative methods*, *linear algebra*, *links symbolic-numeric*, *polynomials*, *resultant*, *solving*, *sparse matrices*, *stability*, *structured matrices*.

**Participants:** Guillaume Chèze, Ioannis Emiris, Grégory Gatellier, Bernard Mourrain [contact person], Jean-Pascal Pavone, Olivier Ruatta, Jean-Pierre Técourt, Philippe Trébuchet, Elias Tsigaridas.

*See* SYNAPS *web site: http://www-sop.inria.fr/galaad/logiciels/synaps/.*

We consider problems handling algebraic data structures such as polynomials, ideals, ring quotients, ..., as well as numerical computations on vectors, matrices, iterative processes, ...etc. Until recently, these domains were separated: software for manipulating formulas is often not effective for numerical linear algebra; while the numerically stable and efficient tools in linear algebra are usually not adapted to the computations with polynomials.

We design the software SYNAPS (SYmbolic Numeric APplicationS) for symbolic and numerical computations with polynomials. This powerful kernel contains univariate and multivariate solvers as well as several resultant-based methods for projection operations. Currently, we are developing a module that is related to factorization, which is relevant to the separation of irreducible components of a curve in $\mathbb{C}^3$.

In this library, a list of structures and functions makes it possible to operate on vectors, matrices, and polynomials in one or more variables. Specialized tools such as LAPACK, GMP, SUPERLU, RS, GB, ... are also connected and can be imported in a transparent way. These developments are based on C++, and attention is paid to the generic structures so that effectiveness would be maintained. Thanks to the parameterization of the code (*template*) and to the control of their instantiations (*traits*, *template expression*), they offer generic programming without losing effectiveness.

## 5.2. axel, a module for curves and surfaces

**Keywords:** *algebra*, *curve*, *implicit equation*, *intersection*, *parameterisation*, *resolution*, *singularity*, *surface*, *topology*.

**Participants:** Grégory Gatellier, Bernard Mourrain, Jean-Pascal Pavone, Olivier Ruatta, Jean-Pierre Técourt.

*See* AXEL *web site: http://www-sop.inria.fr/galaad/logiciels/axel/.*

We are developing a module called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

The library contains data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterisation, B-Spline, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for for computing intersection points of curves or surfaces, detecting and computing self-intersection points of parameterized surfaces, implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

This library is connected to external libraries such as SYNAPS for algebraic tools such as polynomial solvers or CGAL (Geometric Algorithms Library) for classical computational geometry ingredients. Some components of the library are connected to industrial CAGD software. Many functionalities of the library are also available through the computer algebra system MATHEMAGIX, as dynamic binary libraries.

## 5.3. multires, a maple package for multivariate resolution problems

**Keywords:** *Polynomial algorithmic*, *eigenvalues*, *interpolation*, *linear algebra*, *residue*, *resultant*.

**Participants:** Laurent Busé, Ioannis Emiris, Bernard Mourrain [contact person], Olivier Ruatta, Philippe Trébuchet.

*See* MULTIRES *web site: http://www-sop.inria.fr/galaad/logiciels/multires/.*

The Maple package MULTIRES contains a set of routines related to the resolution of polynomial equations. The prime objective is to illustrate various algorithms on multivariate polynomials, and is not effectiveness, which is achieved in a more adapted environment as SYNAPS. It provides methods to build matrices whose determinants are multiples of resultants on certain varieties, and solvers depending on these formulations, and based on eigenvalues and eigenvectors computation. It contains the computations of Bezoutians in several variables, the formulation of Macaulay [47] for projective resultant, Jouanolou [46] combining matrices of Macaulay type, and Bezout and (sparse) resultant on a toric variety [41], [40]. Also being added are a new construction proposed for the residual resultant of a complete intersection [38], functions for computing the degree of residual resultant illustrated in [39], and the geometric algorithm for decomposing an algebraic variety [44]. The Weierstrass method generalized for several variables (presented in [50]) and a method of resolution by homotopy derived from such generalization are implemented as well. Furthermore, there are tools related to the duality of polynomials, particularly the computation of residue for a complete intersection of dimension 0.

# 6. New Results

## 6.1. Algebra

### 6.1.1. Computation of normal forms in a quotient algebra

**Participants:** Bernard Mourrain, Philippe Trébuchet [SALSA].

We develop a new method for computing the normal form of a polynomial modulo a zero-dimensional ideal $I$. We give a detailed description of the algorithm, a proof of its correctness, and finally experimentations on classical benchmark polynomial systems. The method we propose can be thought as an extension of both the Gröbner basis method and the Macaulay construction. As such it establishes a natural link between these two methods. We have weaken the monomial ordering requirement for Gröbner bases computations, which allows us to construct new type of representations for the associated quotient algebra. This approach yields more freedom in the linear algebra steps involved, which allows us to take into account numerical criteria while performing the symbolic steps. This is a new feature for a symbolic algorithm, which has an important impact on the practical efficiency, as it is illustrated by the experiments. This work is submitted for publication.

### 6.1.2. Univariate polynomial solvers in Bernstein basis

**Participants:** Bernard Mourrain, Fabrice Rouillier [SALSA], Marie-Francoise Roy [IRMAR, Rennes].

In this expository work accepted for publication (see also [33]), we explain how the Bernstein's basis, widely used in Computer Aided Geometric Design, provides an efficient method for real root isolation, using De Casteljau's algorithm. We explain the link between this approach and more classical methods for real root isolation such as Uspensky's method. We also present a new improved method for isolating real roots in the Bernstein's basis.

### 6.1.3. Multivariate polynomial solvers in Bernstein basis

**Participants:** Bernard Mourrain, Jean-Pascal Pavone.

We develop a new algorithm for solving a system of polynomials in Bernstein form. It can be seen as an improvement of the *Interval Projected Polyhedron* algorithm proposed by Sherbrooke and Patrikalakis. It uses a powerful reduction strategy thanks to a univariate root finder based on *Bezier clipping* and *Descarte's rule*. The improvement of this reduction compared to the classical *IPP* method, is illustrated on systems with tangent solutions, which are the worst case situation of all iterative algorithms. We analyse the behavior and complexity of the method, proving a generalisation of Vincent's theorem for multivariate polynomials. We also show an application to the computation of self-intersection of rational Bezier curves, with examples taken up to degree 20 and some experiments on classical polynomial benchmark problems.

### 6.1.4. Univariate and multivariate Weierstrass-like methods

**Participant:** Olivier Ruatta.

The Weierstrass iteration is an iterative algorithm allowing to compute simultaneously all the roots of an algebraic system. This method was generalized to overconstrained systems in [51]. More recently in [34][28], we used this method in order to address the problem of the approximate GCD and we shown the links of the iteration with the distance to the nearest consistent system. This method allows us to compute the common roots of a system near an inexact algebraic system. This work was presented as a poster to the conference ISSAC 04 at Santander and received the prize of "distinguished poster". In another way, in the univariate setting, we proposed a method derived from the Weierstrass iteration allowing to approximate simultaneously all the roots of an algebraic equation by integration of the proposed vector field.

### 6.1.5. Multivariate interpolation

**Participant:** Olivier Ruatta.

We proposed different algorithms to solve the multivariate Lagrange interpolation problem. Based on a previous work [49] where we introduced a generalization of the Lagrange basis, we proposed a generalization of the Newton basis in [27]. This work has been presented at the EACA conference and has been published as an extended abstract in the proceedings of this conference. We obtained the better complexity bound for the multivariate Lagrange interpolation problem. The complexity of our algorithm is basically cubic in the number of interpolation points and quadratic in the complexity of the evaluation of some multivariate polynomials.

### 6.1.6. Irreducibility of multivariate subresultants

**Participants:** Laurent Busé, Carlos D'Andrea.

Classical subresultants of two univariate polynomials have been studied by Sylvester. Multivariate subresultants, introduced by Chardin [42], provide a criterion for over-constrained polynomial systems to have Hilbert function of prescribed value, generalizing the classical case. They have been used in computational algebra for polynomial system solving as well as for providing explicit formulas for the representation of rational functions. The study of their properties is an active research area, in particular it is important to know which of them are irreducible. In this work, published in [14], we proved the following result: let $P_1, ..., P_n$ be generic homogeneous polynomials in $n$ variables of degrees $d_1, ..., d_n$ respectively. We prove that if $\nu$ is an integer satisfying $\sum_{i=1}^{n} d_i - n + 1 - \min\{d_i\} < \nu$, then all multivariate subresultants associated to the family $P_1, ..., P_n$ in degree $\nu$ are irreducible. We show that the lower bound is sharp. As a byproduct, we get a formula for computing the residual resultant of $\binom{\rho-\nu+n-1}{n-1}$ smooth isolated points in $\mathbb{P}^{n-1}$.

### 6.1.7. Properness and inversion problems of parameterized surfaces

**Participants:** Laurent Busé, Carlos D'Andrea.

Rational surfaces play an important role in the frame of practical applications, especially in Computer Aided Geometric Design. Such surfaces can be parameterized, i.e. can be seen as the image of a generically finite rational map. In this collaboration, we addressed the following questions: decide if the parameterization map is invertible, and if it is the case find an inverse. Both questions have been already solved theoretically and algorithmically by means of Gröbner bases. Our approach is based on the matrix formulation of some projection operators allowing to eliminate variables. We give a general method, generalizing preliminar results obtained and presented at the international conferences EACA and ISSAC 2004 [19], for solving both problems by means of matrices. In particular we introduce the notion of *implicitization matrices*; these matrices can be used to solve simultaneously both problems by extracting information from a square matrix whose determinant is an implicit equation (that we do not need to compute!) of the surface. This work as been submitted for publication.

## 6.2. Geometry

### 6.2.1. Geometric modeling and Steiner surfaces

**Participants:** Franck Aries [INRA Avignon], Jean-Pierre Técourt, Bernard Mourrain.

We study quadratic parameterization patches (generically Steiner patches) for modeling purpose. We give an overview of the properties and algorithmic of these surfaces, including classification, implicitization, and inversion. Our new approach for the classification is based on rational and certified computation, using effective algebraic geometry tools. We use resultant-based constructions to deduce the implicit equations of such patches. Exploiting the properties of these resultant matrices, we propose a new approach to solve the inversion problem. We show how it reduces to a linear algebra problem, for which we propose a stable and efficient algorithm including the treatment of singularities. The approach is illustrated by experiments on canopy models. This work [18] is going to be published.

### 6.2.2. Geometry of parametrized bicubic surfaces

**Participants:** André Galligo, Mike Stillman.

We start the study of the problem of describing the double point locus of a bicubic surface. Our motivation is to determine, whether a real bicubic patch over the unit square will have self-intersections. And if so, to identify useful points and curves in order to determine basic feature and to help analysis the surface accurately. The work was presented at the conference in honor of D. Lazard in Paris.

### 6.2.3. Classification of parameterised surfaces

**Participants:** Mohamed Elkadi, André Galligo, Thi-Ha Le.

Parametrized surfaces of low degrees are very useful in applications, specially in Computer Aided Geometric Design and Geometric Modeling. The precise description of their geometry is not easy in general. Here we study surfaces of bidegree $(1, 2)$. We show that, generically up to linear changes of coordinates, they are classified by two continuous parameters (modulus). We present an elegant combinatorial description where these modulus appear as cross ratios. We provide compact implicit equations for these surfaces and for their singular locus together with a geometric interpretation (see [22]).

### 6.2.4. Semi-implicit representation of algebraic surfaces

**Participants:** Laurent Busé, André Galligo.

We continued our work on this new representation of algebraic surfaces which is an interesting intermediate representation between parameterized and implicit representations. In a work accepted for publication, we first develop further the theoretical side by giving a general definition in the language of projective complex algebraic geometry. Then we applied it to investigate the intersection of two bi-cubic surfaces, these surfaces are widely used in Computer Aided Geometric Design. In [20] we mainly addressed the topic of performing the usual CAD operations with semi-implicit representation of surfaces. We derived formulae for computing the normal and the curvatures at a regular point. We provided exact algorithms for computing self-intersections of a surface and more generally its singular locus. We also presented some surface/surface intersection algorithms relying on generalized resultant calculations.

### 6.2.5. Autointersection

**Participants:** André Galligo, Bernard Mourrain, Jean-Pascal Pavone.

In CAGD it is important not only to detect but also to describe and compute the self-intersection curves of surfaces bounding a solid constructed via usual CAGD operators. Such surfaces are called procedural: they are parametric but not necessarily rational. J.P. Pavone developed an efficient algorithm, based on sampling, to compute the self-intersection locus of such a parametric surface. It relies on the segmentation of the parameter domain into subdomains on which the parameterization map is injective, then applies adapted multiple surface/surface intersections. As the approach is sampling-based, there is no guarantee that all self-intersection points are located, also there is not guarantee that singular cases can be handled. Sampling based methods or lattice evaluation is used to a great extent within CAGD systems for surface-surface intersection calculation. For detecting self-intersections this approach is novel. The implementation of this algorithm has been connected to an industrial CAGD tool. Experimentations in this context shows the efficiency of the approach.

### *6.2.6. Arrangements of quadrics*

**Participants:** Bernard Mourrain, Jean-Pierre Técourt, Monique Teillaud.

In the paper [17], we study a sweeping algorithm for computing the arrangement of a set of quadrics in $\mathbb{R}^3$. We define a “trapezoidal” decomposition in the sweeping plane, and we study the evolution of this subdivision during the sweep. A key point of this algorithm is the manipulation of algebraic numbers. In this perspective, we put a large emphasis on the use of algebraic tools, needed to compute the arrangement, including Sturm sequences and Rational Univariate Representation of the roots of a multivariate polynomial system.

### *6.2.7. Algebraic identities for the configurations of pairs of conics and quadrics*

**Participant:** Emmanuel Briand.

Emmanuel Briand continued his work started at the *University of Cantabria* in Santander on algebraic identities – equations, inequations, inequalities – characterizing the configurations of pairs of projective conics, and studied applications for arrangements of conics and generalizations to quadrics. A publication is in preparation answering a question about the characterization of the configurations of pairs of conics by means of the signature function, raised in [55]. The similar question about pairs of quadrics is also under study.

### *6.2.8. Topology of curves and surfaces*

**Participants:** Grégory Gatellier, Georges Comte [UNSA], Bernard Mourrain, Jean-Pierre Técourt.

In this work, we can distinguish two parts, one concerning the topology of implicit curves (defined by two implicit equations) [24] to be published and another on algebraic surfaces (defined by one equation) which is still in progress. In both cases, starting from one or two implicit equations, the algorithm outputs an isotopic meshing of the curve or surface. Both algorithms are based on the strategy of sweeping algorithms: we choose a direction of sweeping, compute when the topology of the sections (with respect to the sweeping) change. We compute the topology of the sections and connect them. The main ingredients are projection tools, based on resultants and 0-dimensional solvers and from a more theoretical point of view, singularity theory with Whitney stratification.

### *6.2.9. Meshing real algebraic surfaces*

**Participants:** Lionel Alberti, George Comte [UNSA], Bernard Mourrain.

We develop a new method to mesh surfaces defined by an algebraic equation, which is able to isolate the singular points of the surface, to guaranty the topology in the smooth part, and to give a topological model of singularity elsewhere, while producing a number of linear pieces, related to the Vitushkin variations of the surface. It applies to surfaces defined by a polynomial equation or a B-spline equation. We use Bernstein bases to represent the function in a box and subdivide this representation according to a generalization of Descartes rule, until the problem in each box boils down to the case where either the implicit object is proved to be homeomorph to its linear approximation in the cell or the size of the cell is smaller than $\varepsilon$. This ensures that the topology of the smooth part of an implicit surface is cached within a precision $\varepsilon$, where $\varepsilon$ is a tunable parameter. For the singular cases, we use a combination of tools coming from singularity theory, real geometry and algebraic geometry. In particular, using the notion of Whitney stratification and Milnor balls, the method allows us to compute a finite partition of the space in cubes so that the zero set of the polynomial in each cube has the same topology as a cone. Once one has such a decomposition, it is easy to build a mesh for the zero set. The actual computation of the Whitney stratification is done using projective resultants. Part of this work is described in [29] and submitted to the proceedings of the conference “Mathematical methods for curves and surfaces” 2004, Tromsoe, Norway.

#### 6.2.10. Applications to Computer Vision

**Participants:** Jean Ponce [University of Illinois at Urbana-Champaign], Theo Papadopoulo [ODYSSÉE], Monique Teillaud, Bill Triggs [LEAR].

In a work in progress, we introduce the *absolute quadratic complex* formed by all lines that intersect the absolute conic. A simple relation between a camera's intrinsic parameters, its projection matrix expressed in a projective coordinate frame, and the metric upgrade separating this frame from a metric one, provides a new framework for autocalibration, particularly well suited to typical digital cameras with rectangular or square pixels since the skew and aspect ratio are decoupled from the other intrinsic parameters.

### 6.3. Symbolic numeric computation

#### 6.3.1. A symbolic numeric algorithm for the absolute factorization

**Participant:** Guillaume Chèze.

A recent algorithmic procedure for computing the absolute factorization of a polynomial $P(X,Y)$, after a linear change of coordinates, is via a factorization modulo $X^3$. This was proposed by A. Galligo and D. Rupprecht in [45], [52]. Then absolute factorization is reduced to finding the minimal zero sum relations between a set of approximated numbers $b_i$, $i = 1$ to $n$ such that $\sum_{i=1}^{n} b_i = 0$, (see also [53]). Here this problem with an a priori exponential complexity bound, is efficiently solved for large degrees ($n > 100$). We rely on LLL algorithm, used with a strategy of computation inspired by van Hoeij's treatment [56]. For that purpose we prove a theorem on bounded integer relations between the numbers $b_i$, also called linear traces in [54]. This work has been presented at the "International Symposium on Symbolic and Algebraic Computation" [21].

#### 6.3.2. A symbolic algorithm for the absolute factorization

**Participants:** Guillaume Chèze, Grégoire Lecerf.

In the vein of recent algorithmic results on polynomial factorization based on lifting and recombination techniques, we propose a new faster method for computing the absolute factorization of a bivariate polynomial. The complexity of our probabilistic algorithm is sub-quadratic in the dense size of the input polynomial, with respect to its total degree. In addition, we present a deterministic version with only soft quadratic worst case complexity. This work has been accepted at the"International Conference on Polynomial System Solving".

#### 6.3.3. Hybrid symbolic-numeric sparse interpolation

**Participant:** Wen-shin Lee.

*Joint work with Mark Giesbrecht and George Labahn (University of Waterloo)].*

In the floating-point arithmetic, we developed effective solutions for the problem of sparse interpolation for a black box polynomial in different bases. Our methods are implemented in Maple, and we are still working on a more formal analysis and related experiments.

Based on the polynomial relations between trigonometric functions, we extend progress in floating-point sparse polynomial interpolation to trigonometric interpolation. This work has been presented in two conferences [25][26]. Full paper versions are in preparation.

#### 6.3.4. Interpolating the determinant of a polynomial matrix

**Participants:** Wen-shin Lee, Mohamed Elkadi, Bernard Mourrain.

When computing the determinant of a polynomial matrix (or specifically a Bezoutian matrix), whose entries are multivariate polynomials, the size of intermediate expressions can easily become impractical. However, such determinant can be regarded as a black box, and the determinant polynomial can be computed via a black box polynomial interpolation method.

We explore and compare available black box interpolation algorithms. To further improve the efficiencies, we develop strategies for different interpolation methods in different arithmetics.

Currently, our developments are being implemented in the SYNAPS library. We intend to compare and test different interpolation methods, as well as the approaches of computing residues with respect to a polynomial map.

#### *6.3.5. Geometric predicates*

**Participants:** Ioannis Emiris, Athanasios Kakargias, Sylvain Pion [Geometrica team], Monique Teillaud, Elias Tsigaridas.

*See the Curved Kernel web site http://www-sop.inria.fr/galaad/teillaud/kernel.html.*

The work on geometric predicates was pursued further this year as part of the ECG project. The design of classes and concepts of a CGAL kernel for curved objects led to a publication [23]. The implementation of a CGAL kernel for simple curves is in progress. It will provide the user with the elementary operations that are necessary for running arrangements. This work will be submitted to the CGAL Editorial Board for inclusion in the CGAL library in the coming months.

Preliminary benchmarks were performed in collaboration with ECG partners (Max-Planck-Institut für Informatik and Tel Aviv University) [32].

#### *6.3.6. Triangulation of points*

**Participant:** Monique Teillaud.

*See the* CGAL *web site:http://www.cgal.org.*

*The package "3D Triangulation" of CGAL is maintained in collaboration with Sylvain Pion (Geometrica team).* CGAL 3.1 was released in December, 2004.

#### *6.3.7. Shape optimisation*

**Participants:** Nikos Pavlidis, Bernard Mourrain, Michael Vrahatis [University of Patras].

During the internship of N. Pavlidis, we investigate the application of optimization methods to geometric problems, and in particular Covariance Matrix Adaptation Evolution Strategies, Differential Evolution Algorithms and Particle Swarm Optimization methods. All three methods exploit a set (population or swarm) of potential solutions (individuals or particles) to probe the search space for possible best solutions. At each iteration, they employ a number of operators to refine the potential solutions so as to evolve the population towards more promising regions of the search space (in the case of minimization such regions are characterized by lower function values). They are designed to handle non-linear, non-differentiable and discontinuous objective functions, as well as constraint optimization problems. They are also robust to the presence of noise and imprecise information, and can cope with the existence of multiple local minima.

In collaboration with OPALE team, these techniques have been applied to shape optimization problems. For the wing and arch problems, we use Bezier and B-spline to represent the profile with few parameters, in order to optimise more efficiently the required performance criteria. These optimisation tools have also been applied on distance matrices in molecular conformation problems, in order to find valid matrices in the search space. A technical report on this work is in preparation.

# 7. Other Grants and Activities

## 7.1. European actions

#### *7.1.1. ecg : Effective Computational Geometry for Curves and Surfaces*

**Participants:** Laurent Busé, Ioannis Emiris, André Galligo, Grégory Gatellier, Athanasios Kakargis, Bernard Mourrain, Olivier Ruatta, Jean-Pierre Técourt, Monique Teillaud [contact person], Elias Tsigaridas.

*See the* ECG *project web site.*

INRIA (GEOMETRICA and GALAAD) is coordinating the European project:

- Acronym: ECG, number IST-2000-26473

- Title: Effective Computational Geometry for Curves and Surfaces.
- Specific Programme: IST
- RTD (FET Open)
- Start date: may 1st 2001 - Duration: 3 years
- Participation of Inria as coordinating site
- Other participants:
  ETH Zürich (Switzerland),
  Freie Universität Berlin (Germany),
  Rijksuniversiteit Groningen (Netherlands),
  MPI Saarbrücken (Germany),
  Tel Aviv University (Israel)
- Abstract: Efficient handling of curved objects in Computational Geometry. Geometric algorithms for curves and surfaces, algebraic issues, robustness issues, approximation.

ECG ended on April 30, 2004. Jean-Daniel Boissonnat was the project manager, and Monique Teillaud was the technical project manager.

She was in charge of the communication within the Board and the members, and with the Project Officer in Brussels. She was also maintaining the public web site of the project, together with internal web sites with restricted access, and the mailing lists for internal communication (the members mailing list contains 66 addresses).

Steve Oudot (GEOMETRICA team) was maintaining the `php` scripts for submission of technical reports, originally written by Menelaos Karavelas (PRISME team). Olivier Ruatta settled a *MySQL* server that is used for the database of technical reports.

### 7.1.2. *gaia*

**Participants:** Laurent Busé, Stéphane Chau, Mohamed Elkadi, Ioannis Emiris, André Galligo [contact person], Bernard Mourrain, Jean-Pascal Pavone, Olivier Ruatta.

*See the* GAIA II *project web site*

In collaboration with the university of Nice UNSA, the GALAAD team is involved in the European project GAIA:
- Acronyme : GAIA II, number IST-2001-35512
- Title : Intersection algorithms for geometry based IT-applications using approximate algebraic methods
- Specific program of the project : IST
- Type of project: FET-Open
- Beginning date : 1st of july 2002 - During : 3 years
- Participation mode of INRIA: participant via the UNSA
- Partners list :
  SINTEF Applied Mathematics, Norvegia,
  Johannes Kepler University, Austria,
  UNSA, France,
  Université de Cantabria, Spain,
  Think3 SPA, Italy and France,
  University of Oslo, Norvegia.
- Abstract of the project : Detection and treatment of intersections and self-intersections, singularity analysis, classification, approximate algebraic geometry and applications to CAG.

### 7.1.3. *aim@ shape*

**Participants:** Laurent Busé, Emmanuel Briand, Stéphane Chau, Mohamed Elkadi, Ioannis Emiris, André Galligo, Thi Ha Le, Bernard Mourrain [contact person], Olivier Ruatta, Monique Teillaud.

*See the* AIMSHAPE *project web site*

- Acronym : aim@shape, number NoE 50766
- Title : AIM@SHAPE, Advanced and Innovative Models And Tools for the development of Semantic-based systems for Handling, Acquiring, and Processing knowledge Embedded in multidimensional digital objects.
- Type of project: network of excellence
- Beginning date : 1st of january 2004 - During : 4 years
- Partners list :

    CNR - Consiglio Nazionale delle Ricerche,
    DISI - Universita di Genova,
    EPFL - Swiss Federal Institute of Technology,
    IGD - Fraunhofer,
    INPG - Institut National Polytechnique de Grenoble,
    INRIA
    CERTH - Center for Research and Technology Hellas,
    UNIGE - Université de Genève,
    MPII - Max-Planck-Institut für Informatik,
    SINTEF,
    Technion CGGC,
    TUD - Darmstadt University of Technology,
    UU - Utrecht University,
    WIS - Weizmann Institute of Science.

- Abstract of the project : it is aimed at coordinating research on representing, modeling and processing knowledge related to digital shapes, where by shape it is meant any individual object having a visual appearance which exists in some (two-, three- or higher- dimensional) space (e.g., pictures, sketches, images, 3D objects, videos, 4D animations, etc.).

## 7.2. Bilateral actions

### *7.2.1. Associated team CALAMATA*

**Participants:** Ioannis Emiris, Athanasios Kakargias, Bernard Mourrain [contact person], Nikos Pavlidis, Jean-Pierre Técourt, Monique Teillaud, Elias Tsigaridas, Michael Vrahatis.

*The Team of Geometric and Algebraic Algorithms at the National University of Athens, Greece, has been associated with GALAAD since 2003. See its web site.*

This bilateral collaboration is entitled CALAMATA (CALculs Algebriques, MATriciels et Applications). The Greek team (http://www.di.uoa.gr/~erga/) is headed by Ioannis Emiris.

The focus of this project is the solution of polynomial systems by matrix methods. Our approach leads naturally to problems in structured and sparse matrices. Real root isolation, either of one univariate polynomial or of a polynomial system, is of special interest, especially in applications in geometric modeling, CAGD or nonlinear computational geometry. We are interested in computational geometry, actually, in what concerns curves and surfaces. The framework of this work has been the European project ECG.

In 2004, 4 members of the Greek team visited INRIA, either for week-long visits or for longer visits (from one to 3 months). Two INRIA researchers visited Athens for one week. We also mention the participation of members of both teams in international or national conferences: the final workshop of ECG in Paris, the conference on Algebraic geometry and geometric modeling in Nice, and the conference in honor of Daniel Lazard in Paris.

### *7.2.2. NSF-INRIA collaboration*

**Participants:** Laurent Busé, André Galligo, Mohamed Elkadi, Bernard Mourrain [contact person], Jean-Pierre Técourt.

The objective of this collaboration between GALAAD and the Geometric Modeling group at Rice University in Houston, Texas (USA) is to investigate techniques from Effective Algebraic Geometry in order to solve some of the key problems in Geometric Modeling and Computational Biology. The two groups have similar interests and complementary strengths. Effective Algebraic Geometry is the branch of Algebraic Geometry that pursues concrete algorithms rather than abstract proofs. It deals mainly with practical methods for representing polynomial curves and surfaces along with robust techniques for solving systems of polynomial equations. Many applications in Geometric Modeling and Computational Biology require fast robust methods for solving systems of polynomial equations. Here we concentrate our collective efforts on solving standard problems such as implicitization, inversion, intersection, and detection of singularities for rational curves and surfaces. To aid in modeling, we shall also investigate some novel approaches to representing shape. In contemporary Computational Biology, many problems can be reduced to solving large systems of low degree polynomial equations. We plan to apply our polynomials solvers together with new tools for analyzing complex shapes to help study these currently computationally intractable problems.

#### 7.2.3. NCSU-GALAAD collaboration

**Participant:** Olivier Ruatta.

*Agnes Szanto works at the department of mathematics of the North Carolina State University.*

This is a project on overdetermined algebraic systems funded by a U.S. grant of one year. Olivier Ruatta went to Raleigh in February 2004 for 18 days and Agnes Szanto came at Sophia Antipolis in May 2004 for 13 days. This collaboration is prolongated and is partially supported by a NFS grant obtained by A. Szanto which include founds for this work.

The objective of this investigation is to develop and implement highly efficient algorithms for the solution of over-constrained polynomial systems with finitely many, possibly multiple roots over the complex numbers, when the input is given with inexact coefficients. We refer to such problems as inexact degenerate systems. Both “resultant based” and analytic iterative methods are considered to tackle this problem using the large number of already existing works. The researchers addresses the problem of the definition of “nearly consistent” systems, with computational methods generalizing the S.V.D. of the linear case. The complexity is one of the central issues of this research since we want efficient methods.The first results obtained are related to overdetermined Weierstrass iteration in [28][34] which had been presented in different conferences.

# 8. Dissemination

## 8.1. Animation of the scientific community

### 8.1.1. Seminar organization

We continued to organize a bi-weekly seminar called “Table Ronde”. The list of talks is available at http://www-sop.inria.fr/galaad/seminaires/TABLERONDE/seminaire.php.

### 8.1.2. Comittee participations

- B. Mourrain was a member of the programm comittee of the conference ISSAC 2004, july 4-7 2004, (International Symposium on Symbolic and Algebraic Computation 2004, University of Cantabria, Santander, Spain.)

### 8.1.3. Editorial committees

- Monique Teillaud is a member of the CGAL Editorial Board.

### *8.1.4. Organisation of conferences and schools*

- Jean-Daniel Boissonnat (GEOMETRICA) and Monique Teillaud organized the final workhop of the European Project ECG, which took place in the École Normale Supérieure in Paris, March 31 - April 2, 2004. The local organizer was Michel Pocchiola and the administrative organization was done by Monique Simonetti.
  There were 48 participants. The full program with the abstracts and the slides of the talks, and the list of participants, are available from the web site of the workshop http://www-sop.inria.fr/prisme/ECG/workshop/final which is also accessible from the ECG public web site http://www-sop.inria.fr/prisme/ECG/.
- M. Elkadi, B. Mourrain and R. Piene organised the conference AGGM'04 (Algebraic geometry and Geometric Modeling), 27-29 September 2004, Nice . There were 45 participants. The publication of proceedings is planned. For more details, see http://www-sop.inria.fr/galaad/conf/04aggm/index.html.
- Monique Teillaud and Sylvain Pion (GEOMETRICA) organized a CGAL course at LORIA (November 30 - December 12). The local organizer was Sylvain Lazard (ISA-VEGAS). There were 18 participants: 16 researchers or students from five different INRIA project-teams, and two from Eindhoven University. See http://www.loria.fr/~lazard/CGAL/2004/.

### *8.1.5. PHD thesis commitees*

- B. Mourrain was a referee of the PHD. thesis of L. Dupont, University of Nancy.
- B. Mourrain was a referee of the PhD. thesis of P. Giorgi, University of Lyon I.

### *8.1.6. Other comittees*

- Wen-Shin Lee was member of the Poster committee at the conference ISSAC 2004, july 4-7 2004, (International Symposium on Symbolic and Algebraic Computation 2004, University of Cantabria, Santander, Spain.).
- B. Mourrain is in charge, with Thierry Vieville, of the "Formation par la recherche" at INRIA Sophia-Antipolis.
- M. Teillaud (until fall 2004) and J.P. Técourt are members of the "Comité de centre" at INRIA Sophia-Antipolis.
- M Teillaud is a member of the "Comité Local Hygiène et Sécurité" of INRIA Sophia-Antipolis.

### *8.1.7. WWW server*

http://www-sop.inria.fr/galaad/.

## 8.2. Participation to conferences and invitations

- L. Busé : Visit to Marc Chardin, University Paris VI, 3-4 may; visit to J.P. Jouanolou, University of Strasbourg, 5-7 may; EACA conference, Santander, Spain, 30 june to 3 july; ISSAC conference, Santander, Spain, 4-8 july; visit to Ron Goldman, Rice University, Houston, Texas, 21-29 october.
- S. Chau: Toulouse, 1-3 december 2004: Journées Liens Calcul Numérique - Calcul Formel.
- G. Chèze: Toulouse, 1-3 december 2004: Journées Liens Calcul Numérique - Calcul Formel; Paris, 24-26 november 2004: International Conference on Polynomial System Solving; Paris, 4-8 october 2004: Algebraic Geometry and Number Theory with Magma; Santander, 4-7 july 2004: International Symposium on Symbolic and Algebraic Computation; Santander, 1-3 july 2004: Encuentro de Algebra Computacional y Aplicaciones; Grenoble, 29 march-2 april 2004: École Jeunes Chercheurs en Calcul Formel et Algorithmique;
- M. Elkadi: Santander, 4-7 july 2004: International Symposium on Symbolic and Algebraic Computation; Santander, 1-3 july 2004: Encuentro de Algebra Computacional y Aplicaciones;
- A. Galligo: presentation at the conference in honor of D. Lazard (International Conference on Polynomial System Solving), 24-26 november, Paris;
- Wen-Shin Lee: presentations at the conference RWCA 2004, march 25-26, (Ninth Rhine Workshop on Computer Algebra, University of Nijmegen, Nijmegen, The Netherlands) and CASC 2004, july 12-19, (The 7th International Workshop on Computer Algebra in Scientific Computing, Euler International Mathematical Institute, Saint Petersburg, Russia).
- B. Mourrain: participation to “Calcul formel, algorithmes certifiés, preuves constructives”, Lumniny, 12-16 january, 2004; presentation at the Aim@Shape workshop on applications of shape modeling, 17-19 march; Darmstadt; at the ECG workshop; 31 march-02 April, Paris; at the conference on Mathematical methods for curves and surfaces, Tromsoe, 29 june-3 july, 2004; participation to Issac’2004, International Symposium on Symbolic and Algebraic Computation, 4-7 july, Santander; organisation of the conference on Algebraic Geometry and Geometric Modeling, 27-29 september, Nice; visit to Ron Goldman, Rice University, Houston, Texas, 21-29 october. presentation at the International Conference on Polynomial System Solving, 24-26 november, Paris; visit to Athens University, Greece, 6-10 december.
- O. Ruatta: Marseille, 12 - 16 january 2004, Calcul formel, algorithmes certifiés preuves constructives; Marseille, 19 - 20 january 2004: journées ALEA. Limoges, 20-24 january 2004: visit to LACO and presentation (Approximations de toutes les racines d’un polynôme par intégration d’un champ de vecteurs); Raleigh, 3 - 20 february 2004: visit to North Carolina State University for collaboration with A. Szanto and presentation (Symbolic/Numeric and Algebraic/Differential interfaces). Rennes, 2-4 april 2004: visit to IRMAR and presentation (Approximer toutes les racines d’une équatation algébrique par intégration de champs de vecteurs); Santander, Spain, 30 june to 3 july: EACA conference presentation of two works (one is a joined work with A. Szanto and M. Sciabica); Santander, 4-7 july 2004: International Symposium on Symbolic and Algebraic Computation, presentation of a poster; Nice, 27-29 september: conference on Algebraic Geometry and Geometric Modeling; Limoges 11-14 october 2004: visit to LACO and presentation (Schéma d’interpolation de Newton multivarié)); Lyon, 8-9 november 2004: visit to MAPLY and presentation (Approximer toutes les racines d’une équation algébrique par intégration de champs de vecteurs); Toulouse, 1-3 december 2004: Journées Liens Calcul Numérique - Calcul Formel; Haifa (Israel), 6-17 december, visit to Technion (Aim@shape) and presentation (Solving polynomial problems in geometrical issues);

- Monique Teillaud: Dagstuhl, Germany, january 5-9, CGAL developers meeting;
  Paris, march 31 - april 2, ECG final workshop, *Towards a* CGAL *curved kernel*;
  New York, june 14-18, meeting of the GENEPI associated team (GEOMETRICA);
  University of Illinois at Urbana Champaign, september 14-29, collaboration with Jean Ponce and Theo Papadopoulo (ODYSSÉE), discussions with Sariel Hal-Peled and other researchers, talk on CGAL;
  LORIA, ISA-VEGAS project-team, november 29 - december 3;
- Jean-Pierre Técourt: Grenoble, 29 march-2 april 2004: École Jeunes Chercheurs en Calcul Formel et Algorithmique; Genova 14-18 june 2004 International Summer School on Computational Methods for Shape Modelling and Analysis Nice, 27-29 september: Conference on Algebraic Geometry and Geometric Modeling; Visit to Athens University, Greece, 6-10 december.

## 8.3. Formation

### *8.3.1. Teaching at universities*

- L. Busé : Course and exercices of "differential geometry and applications" at the university of Nice, maîtrise MIM (52 hours). Course in Master, 2nd year, on "algebraic curves and surfaces for CAGD" at the university of Nice, with A. Galligo and M. Elkadi (9 hours).
- G. Chèze : TD MI2 in algebra (course by M. Elkadi).
- M. Elkadi : Course and exercises of "Arithmetic" in DEUG Mathématiques et informatique. Course and exercices of "Effective Algebraic Geometry", maîtrise MIM. Cours in Master 2 of Mathemathics.
- A. Galligo : Course in Master 2. TD de Mathemathematics in Mass section, first year.
- B. Mourrain : Algorithmique, Maîtrise Mathématiques-Informatique, UNSA (40h).
- O. Ruatta : Course and exercices of "Web languages" at Nice university, Master of Earth Sciences; Exercices of "Algorithm and implementation" at university of Nice;
- Jean-Pierre Técourt: TD and TP "Algorithmic and implementation" in Deug at University of Nice. (64h)
- Monique Teillaud :
  - Computational Geometry, Arrangements (6h). Master MIGS (Mathématiques pour l'Informatique Graphique et les Statistiques), université de Bourgogne, Dijon.
  - Geometric Computation, CGAL (11h). Institut Supérieur d'Informatique et d'Automatique de l'École des Mines de Paris, Sophia Antipolis.

### *8.3.2. PhD theses in progress*

- Stéphane Chau, *Study of singularities used in CAGD*, UNSA.
- Thi Ha Lê, *Classification and intersections of some parametrized surfaces and applications to CAGD*, UNSA.
- Jean-Pierre Técourt, *Algorithmique des courbes et surfaces implicites*, UNSA.

### *8.3.3. Defended PhD thesis*

- Jean-Pascal Pavone, *Auto-intersection de surface paremétrées réelles*, 1st december 2004, UNSA.
- Guillaume Chèze, *Des méthodes symboliques-numériques et exactes pour la factorisation absolue des polynômes en deux variables*, 16 december 2004, UNSA.

### 8.3.4. Internships

*See the web page of our interships.*

- Lionel Alberti (ENS Cachan), *Maillage de surfaces algébriques réelles singulières*, from june 1st to august 31th [35].
- Houssam Khalil, (University of Lyon I), *Résolution des systèmes polynomiaux par valeurs propres et vecteurs propres*, from april 1st to june 31th [36].
- Van Thoi Tran (IFI), *Approximate implicitization*, from may 1st to july 3th.
- Nikos Pavlidis (University of Patras), *Application of optimization methods in geometric problems*, from september 15th to december 15th.
- Elias Tsigaridas (National University of Athens), *Algebraic numbers and polynomial system solving*, 4 weeks in june and 2 weeks in november.

# 9. Bibliography

## Major publications by the team in recent years

[1] L. BUSÉ, M. ELKADI, B. MOURRAIN. *Resultant over the residual of a complete intersection*, in "J. of Pure and Applied Algebra", vol. 164, 2001, p. 35-57.

[2] L. BUSÉ, M. ELKADI, B. MOURRAIN. *Using projection operators in computer aided geometric design*, in "Topics in algebraic geometry and geometric modeling, Providence, RI", Contemp. Math., vol. 334, Amer. Math. Soc., 2003, p. 321–342.

[3] M. ELKADI, B. MOURRAIN. *Algorithms for residues and Lojasiewicz exponents*, in "J. of Pure and Applied Algebra", vol. 153, 2000, p. 27-44.

[4] I. EMIRIS. *Algorithmes Algébriques et Géométriques*, Ph. D. Thesis, Université de Nice – Sophia-Antipolis, École Doctorale des Sciences pour l'Ingénieur, January 2000.

[5] I.Z. EMIRIS, A. GALLIGO, H. LOMBARDI. *Certified Approximate Univariate GCDs*, in "J. Pure & Applied Algebra, Special Issue on Algorithms for Algebra", vol. 117 & 118, May 1997, p. 229–251.

[6] I.Z. EMIRIS, B. MOURRAIN. *Matrices in Elimination Theory*, in "J. Symbolic Computation, Special Issue on Elimination", vol. 28, 1999, p. 3–44.

[7] A. GALLIGO. *Théorème de division et stabilité en géométrie analytique locale*, in "Ann. Inst. Fourier", vol. 29, 1979, p. 107–184.

[8] A. GALLIGO, S. WATT. *A Numerical Absolute Primality Test for Bivariate Polynomials*, in "Proc. Annual ACM Intern. Symp. on Symbolic and Algebraic Computation", 1997, p. 217–224.

[9] B. MOURRAIN. *Algorithmes et Applications en Géométrie Algébrique*, Ph. D. Thesis, Univ. de Nice, September 1997.

[10] B. MOURRAIN, V. Y. PAN. *Multivariate Polynomials, Duality and Structured Matrices*, in "J. of Complexity", vol. 16, n$^o$ 1, 2000, p. 110-180.

## Doctoral dissertations and Habilitation theses

[11] G. CHÈZE. *Des méthodes symboliques-numériques et exactes pour la factorisation absolue des polynômes en deux variables*, Ph. D. Thesis, Université de Nice Sophia-Antipolis, 2004.

[12] J. PAVONE. *Auto-intersection de surfaces pamatrées réelles*, Ph. D. Thesis, Université de Nice Sophia-Antipolis, 2004.

## Articles in referred journals and book chapters

[13] L. BUSÉ. *Resultants of determinantal varieties*, in "J. Pure Appl. Algebra", vol. 193, n$^o$ 1-3, 2004, p. 71–97.

[14] L. BUSÉ, C. D'ANDREA. *On the irreducibility of multivariate subresultants*, in "C. R. Math. Acad. Sci. Paris", vol. 338, n$^o$ 4, 2004, p. 287–290.

[15] M. ELKADI, B. MOURRAIN. *Residue and implicitization problem for rational surfaces*, in "Appl. Algebra Eng. Comm. Comput.", vol. 14, n$^o$ 5, 2004.

[16] I. Z. EMIRIS, B. MOURRAIN, V. Y. PAN. *Guest editors of the special Issue Algebraic and numerical algorithms*, in "Theoret. Comput. Sci.", vol. 315, n$^o$ 2-3, 2004, p. 307–308.

[17] B. MOURRAIN, J.-P. TÉCOURT, M. TEILLAUD. *On the computation of an arrangement of quadrics in 3D*, in "Comput. Geom. Theory Appl.", Special issue, 19th European Workshop on Computational Geometry, Bonn, 2004.

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