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Project-Team galaad

Géométrie, Algèbre, Algorithmes

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Activity Report

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1. Team

Galaad is a joint project with Laboratoire J.A. Dieudonné U.M.R. C.N.R.S. n^{o} 6621, University of Nice Sophia-Antipolis.

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2. Overall Objectives

Our research program is articulated around effective algebraic geometry and its applications. The objective is to develop algorithmic methods for effective and reliable resolution of geometrical and algebraic problems, which are encountered in fields such as CAGD, robotics, computer vision, molecular biology, etc. We focus on the analysis of these methods from the point of view of complexity as well as qualitative aspects, combining symbolic and numerical computation.

Geometry is one of the key topics of our activity, which includes effective algebraic geometry, discrete and combinatorial geometry, differential geometry, computational geometry of semi-algebraic sets. More specifically, we are interested in problems of small dimensions such as intersection, singularity, topology computation, and questions related to algebraic curves and surfaces.

On one hand, we consider algebra, particularly the problems of resolution. We are involved in the design and analysis of new methods of effective algebraic geometry. Their developments and applications are central and often critical in concrete problems.

On the other hand, approximate numerical calculations, usually opposed to symbolic calculations, and the problems of certification are also at the heart of our approach. We intend to explore these bonds between geometry, algebra and analysis, which are currently making important strides. These objectives are both theoretical and practical. Recent developments enable us to control, check, and certify results when the data are known to a limited precision.

Finally our work is implemented in software developments. We pay attention to problems of genericity, modularity, effectiveness, suitable for the writing of algebraic and geometrical codes. The implementation and validation of these tools form another important component of our activity.

3. Scientific Foundations

3.1. Introduction

Our scientific activity is defined according to three broad topics: geometry, resolution of algebraic systems of equations, and symbolic-numeric links.

3.2. Geometry

3.2.1. Geometry of algebraic varieties

In order to solve an algebraic problem effectively, a preprocessing analyzing step is often mandatory. From such study, we will be able to deduce the method of resolution that is best suited to and thus produce an effective solver, dedicated to a certain class of systems. The effective algebraic geometry provides us tools for analysis and makes it possible to exploit the geometric properties of these algebraic varieties. For this purpose, we focus on new formulations of resultants allowing us to produce solvers from linear algebra routines, and adapted to the solutions one seeks to approach.

3.2.2. Discrete geometry

We are interested in the properties of solutions of polynomial equations, which result from the geometry of monomials appearing in equations, and based on Newton polytope associated to each polynomial. This *toric elimination theory (or sparse),* introduced by I. M. Gelfand's group [59] in 1970, offers better bounds on the number of common roots and degree of the resultant. It also provides more effective algorithms for sparse polynomial systems, and establishes algorithmic bridges between the geometry of points with integer coordinates (which represent the monomials) and effective algebraic geometry.

3.2.3. Geometric algorithms for curved arcs and surface patches

The above-mentioned tools of effective algebraic geometry make it possible to analyze in detail and separately the algebraic varieties. On the other hand, traditional algorithmic geometry deals with problems whose data are linear objects (points, segments, lines) but in very great numbers. Combining these two approaches, we concentrate on problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well known, and the traditional algorithmic geometry methods do not always extend to this context.

3.2.4. Geometry, groups, and invariants

The objects in geometrical problems are points, lines, planes, spheres, quadrics, ... Their properties are, by nature, independent from the reference one chooses for performing analytic computations. Which leads us to methods from invariant theory. In addition to the development of symbolic geometric computations that exploit these invariants, we are also interested in developing more synthetic representations for handling those expressions.

3.2.5. *Geometry of singularities and topology*

The analysis of singularities for a (semi)-algebraic set provides a better understanding of their structure. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying the theory of singularities to cases of curves silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

3.3. Resolution of algebraic systems

3.3.1. *Algebraic methods and quotient structure*

In order to deduce the solutions of a system of polynomials in n variables, we analyze and take advantage of the structure of the quotient ring from those polynomials. This raises questions of representing and calculating normal forms in such structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases, as well as combining symbolic and numeric computation.

3.3.2. *Duality, residues, interpolation*

We are interested in the "effective" use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems. Our focuses are effective computation of the algebraic residue, interpolation problems, and the relation between coefficients and roots in the case of multivariate polynomials.

3.3.3. *Structured linear algebra and polynomials*

The preceding work lead naturally to the theory of structured matrices. Indeed, the matrices resulting from polynomial problems, such as matrices of resultants or Bezoutians, are structured. Their rows and columns are naturally indexed by monomials, and their structures generalize the Toeplitz matrices to the multivariate case. We are interested in exploiting these properties and the implications in solving polynomial equations [64].

3.3.4. *Decomposition and factorisation*

When solving a system of equations, a first treatment is to transform it into several simpler subsystems when possible. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. For instance, the (difficult) problem of approximate factorization, the computation of perturbations of the data, which enables us to break up the problem, is studied. More generally, we are working on the problem of decompositing a variety into irreducible components.

3.3.5. *Deformation and homotopy*

The behavior of a problem in the vicinity of a data can be interpreted in terms of deformations. Accordingly, the methods of homotopy consist in introducing a new parameter and in following the evolution of the solutions between a known position and the configuration one seeks to solve. This parameter can also be introduced in a symbolic manner, as in the techniques of perturbation of non-generic situations. We are interested in these methods, in order to use them in the resolution of polynomial equations as well as for new algorithms of approximate factorization.

3.4. Symbolic-numeric computation

3.4.1. *Certification*

The numerical problems are often approached locally. However in many problems, it is significant to give global answers, making it possible to certify calculations. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to answer these problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures [58].

3.4.2. Approximation

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate these objects while controlling the quality of approximation. Subdivision techniques based on the algebraic formulation of our problems are exploited in order to control the approximation, while locating interesting features such as singularities. This approach combines geometrical, algebraic and algorithmic aspects.

3.4.3. Degeneracies and arithmetic

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics. For example, we work on the computation of the sign of expressions, and on approaches combining modular and approximate computations, which speed up the exact answer [45].

4. Application Domains

4.1. CAGD

Key words: *geometric modeling*, *engineering computer-assisted*.

3D modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). The involved mathematical objects have often an algebraic nature, which are then discretized for easy handling. The treatment of such objects can sometimes be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, ...We propose the developments of methods for shape modeling that takes into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship to the industrial contacts of CAGD we have.

4.2. Computer vision and robotics

Key words: *engineering*, *reconstruction*, *calibration*.

Robotics and computer vision come with specific applications of the methods for solving polynomial equations. That is the case for instance, for the calibration of cameras, robots, computations of configurations and workspace. The resolution of algebraic problems with approximate coefficients is omnipresent.

4.3. Molecular biology and geometrical structures

Key words: *biology*, *health*.

The chemical properties of molecules intervening in certain drugs are related to the configurations (or conformations) which they can take. These molecules are seen as mechanisms of bars and spheres, authorizing rotations around certain connections, similar to robots series. *Distance geometry* thus plays a significant role, for example, in the reconstruction from NMR experiments, or the analysis of realizable or accessible configurations. The methods we develop are well suited for solving such a problem.

5. Software

5.1. synaps, a module for symbolic and numeric computations

Participants: Guillaume Chèze, Ioannis Emiris, Gabriel Dos Reis, Grégory Gatellier, Bernard Mourrain [correspondent], Jean-Pascal Pavone, Olivier Ruatta, Jean-Pierre Técourt, Monique Teillaud, Philippe Trébuchet.

Key words: *linear algebra*, *bezoutian*, *C++*, *FFT*, *genericity*, *effective algebraic geometry*, *links symbolic-numeric*, *geometry*, *curves and surfaces*, *sparse matrices*, *structured matrices*, *iterative methods*, *polynomials*, *solving*, *resultant*, *stability*, *Eigenvalues*.

See SYNAPS *web site: http://www-sop.inria.fr/galaad/logiciels/synaps/.*
We consider problems handling algebraic data structures such as polynomials, ideals, ring quotients, ..., as well as numerical computations on vectors, matrices, iterative processes, ...etc. Until recently, these approaches have been separated: the software for manipulating formulas is often not effective for numerical linear algebra; while the numerically stable and effective tools in linear algebra are usually not adapted to the computations with polynomials.

We design the software SYNAPS (SYmbolic Numeric APplicationS) for symbolic and numerical computations with polynomials. This powerful kernel contains univariate and multivariate solvers as well as several resultant-based methods for projection operations. Currently, we are developing a module that is related to factorization, which is relevant to the separation of irreducible components of a curve in $\mathbb{C}^3$.

In this library, a list of structures and functions makes it possible to operate on vectors, matrices, and polynomials in one or more variables. Specialized tools such as LAPACK, GMP, SUPERLU, RS, GB, ... are also connected and can be imported in a transparent way. These developments are based on C++, and attention is paid to the generic structures so that effectiveness would be maintained. Thanks to the parameterization of the code (*template*) and to the control of their instantiations (*traits*, *expression template*), they offer generic programming without losing effectiveness.

5.2. axel, a module for curves and surfaces

Participants: Ioannis Emiris, Grégory Gatellier, Bernard Mourrain, Jean-Pascal Pavone, Sylvain Pion [Geometrica team], Monique Teillaud.

Key words: *curve*, *surface*, *parameterisation*, *implicit equation*, *topology*, *singularity*, *intersection*, *algebra*, *resolution*.

See AXEL *web site: http://www-sop.inria.fr/galaad/logiciels/axel/.*
We are developing a module called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL is to provide a framework for such combination of tools, involving symbolic and numeric computations.

We rely on external libraries, such as CGAL (Geometric Algorithms Library) for classical computational geometry ingredients, and SYNAPS for algebraic tools in operating geometric objects.

We endeavour to provide data structures and functionalities related to curved objects (classes of basic objects provided with predicates and constructions). Currently, the module contains algorithms for computing intersection points of curves or surfaces, detecting self-intersection points of parameterized surfaces, implicitization, computation of topology, meshing implicit surfaces, ..., etc.

5.3. multires, a maple package for multivariate resolution problems

Participants: Laurent Busé, Ioannis Emiris, Bernard Mourrain [correspondent], Olivier Ruatta, Philippe Trébuchet.

Key words: *Polynomial algorithmic*, *resultant*, *residue*, *eigenvalues*, *interpolation*, *linear algebra*.

See MULTIRES *web site: http://www-sop.inria.fr/galaad/logiciels/multires/.*
The Maple package MULTIRES contains a list of functions related to the resolution of polynomial equations. The prime objective is to illustrate various algorithms on multivariate polynomials, and is not effectiveness which is achieved in a more adapted environment as SYNAPS. It provides methods to build matrices whose determinants are multiples of resultants on certain varieties, and solvers depending on these formulations, and based on eigenvalues and eigenvectors computation. It contains the computations of Bezoutians in several variables, the formulation of Macaulay [62] for projective resultant, Jouanolou [60] combining matrices of

Macaulay type, and Bezout and (sparse) resultant on a toric variety [49], [48]. Also being added are a new construction proposed for the residual resultant of a complete intersection [47], functions for computing the degree of residual resultant illustrated in [22], and the geometric algorithm for decomposing an algebraic variety [56]. The Weierstrass method generalized for several variables (presented in [66]) and a method of resolution by homotopy derived from such generalization are implemented as well. Furthermore, there are tools related to the duality of polynomials, particularly the computation of residue for a complete intersection of dimension 0.

6. New Results

6.1. Algebra

6.1.1. Computation of normal forms in a quotient algebra

Participants: Bernard Mourrain, Philippe Trébuchet [SPACES].

During the process of resolution of polynomial systems, computing normal forms in an algebra quotient is usually done by exact methods. This stage is immediately downstream from the process of modeling and precedes in most cases of numerical approximations for the solutions.

Computing a normal form indeed yields an exact representation for all solutions of the given system. Traditional algebraic methods for solving systems of polynomial equations do not perform well when polynomials have approximate coefficients. Based on this fact and the work of [63] for systems with a finite number of solutions, we use an algorithm allowing ad infinitum to compute robustly the structure of quotient algebra. Further improvement of this new approach has been carried out this year, accompanied by newer optimizations that improved substantially the implementation. This prototype is implemented in C++ inside the library SYNAPS [55] and currently freely accessible. It has been applied to the computation of Hausdorff distance in real algebraic sets, robotics and signal processing problems, simulation of centrifugal Chromatography, and intersections of curves and surfaces in CAGD, etc. A paper describing this new method has been submitted for publication.

We are now working on a criterion that allows the removal of the assumption of finite solutions in the system.

6.1.2. Univariate and multivariate Weierstrass-like methods

Participant: Olivier Ruatta.

We present a new method to solve univariate algebraic equations. This method is based on the integration of a special vector field [44]. It consists of the construction of a vector field admitting singularities only at vectors formed from any permutation of the roots of the input polynomial. In order to recover one of those singularities, roots of the input polynomial are approximated by integrating the associated vector field. Two approaches are tested for the integration: classical numerical integration (as Euler method) and effective analytic integration. We use the SYNAPS library for the experiments. Based on the approach developed in [65], we propose a generalization of the method by S. Fortune (see [57]) for solving zero dimensional algebraic systems. Which also provides a new interpretation for the method, and a natural generalization in the zero dimensional multivariate setting.

6.1.3. Resultants, residues and implicitization

Participants: Laurent Busé, Mohamed Elkadi, Bernard Mourrain.

Collaborations with Marc Chardin, university of Paris VI, and Carlos D'Andrea, university of Berkeley CA.
We continued our efforts on the implicitization problem, which we began to study a couple of years ago [46],[11][13]. It consists of computing an implicit representation for an algebraic surface that is given parametrically in the projective space.

In collaboration with Marc Chardin (University Paris VI), we detailed and improved the method based on approximation complexes we had introduced in [13]. We provided an in-depth study of the cohomology of such complexes, and presented an algorithm for implicitizing rational hypersurfaces when there are at most a finite number of base points [40] (article presented at the international conference ACA2003 and submitted to a journal). By taking into account the possible existence of base points, we significantly improved the regularity bound needed for the computations. In addition, we gave explicit algorithms which only involve linear algebra routines. They have been implemented in the software MAGMA and are available at http://www-sop.inria.fr/galaad/personnel/Laurent.Buse/.

We collaborate with Carlos D'Andrea on the irreducibility of multivariate subresultants. These subresultants were introduced by Marc Chardin in [51] where the open problem of irreducibility was stated. Let $P_1, ..., P_n$ be generic homogeneous polynomials in n variables of degrees $d_1, ..., d_n$ respectively. In [41] (article submitted to a journal) we proved that if ν is an integer satisfying $\sum_{i=1}^{n} d_i - n + 1 - \min\{d_i\} < \nu$, then all multivariate subresultants associated to family $P_1, ..., P_n$ in degree ν are irreducible. We showed that the lower bound is sharp. As a by-product, we obtained a formula for computing the residual resultant for $\binom{\rho-\nu+n-1}{n-1}$ smooth isolated points in $\mathbb{P}^{n-1}$, while assuming the appropriate number of base points and using [46], new "Macaulay-style" formulas for the implicitization problem.

The third direction we investigated is to solve the implicitization problem from the effective computation of residues by matrix formulation. In general, Bezoutian matrices give a multiple of the implicit equation. The unwanted factor in this equation can be removed (without factorization) using an algorithm for the residue computation in the multivariate setting. The advantage of this approach is to treat surfaces with base points without restrictive geometric hypotheses on the zero-locus of base points. These results are published in [15].

6.1.4. The Bezoutian conjecture

Participant: Bernard Mourrain.

The Bezoutian is a fundamental tool which, surprisingly, appears in many areas of constructive algebra. In a work presented at MEGA'03 conference [20], we described different results related to Bezoutian matrices and residue theory. We consider particularly the problem of computing the structure of quotient ring by an affine complete intersection. It was conjectured in [50] that the quotient structure can be recover from these Bezoutian matrices by simple linear operations on their rows and columns. In this work, we analyzed this algorithm in detail and proved the validity of the conjecture for a modification of the initial method. Direct applications of the results in effective algebraic geometry are given. In particular, we consider the effective membership and representation problem for an ideal which is a zero-dimensional complete intersection. An article has been submitted to a journal for publication.

6.2. Geometry

6.2.1. Autointersection

Participants: André Galligo, Bernard Mourrain, Jean-Pascal Pavone.

Self-intersections often occur as an unexpected result of CAGD operations such as offset, draft, fillet or sweep, and detecting them is a real challenge for many modeling tools. We developed a new sampling algorithm for finding self-intersections of parameterized surfaces relying on the definition of region in the parameter domain and proceeding in three steps: region/region intersections, neighbor regions intersections and regions self-intersections. This work is supported by the European project GAIA II, whose main target is the study of self-intersection locus. Depending on the kind of information wearied by the regions, several versions of the algorithm can be used, and each provides specific ways for handling self-intersection and intersections of regions. Each version shares a common way to encode the regions, the neighborhood information, and a bounding volume hierarchy upon. The algorithm is often faster than the generation of sample points, so that it is interesting when handling surfaces defined procedurally. An article has been submitted to the ACM Symposium on Solid Modeling and Applications 2004.

6.2.2. Arrangements

Participants: Bernard Mourrain, Jean-Pierre Técourt, Monique Teillaud.

We proposed a sweeping plane algorithm for computing an arrangement of quadrics in 3D [21][32]. The algorithm computes the so-called *vertical decomposition* of the arrangement, where cells are of constant size, and it is output-sensitive in terms of the complexity of this decomposition.

Though this vertical decomposition has a much lower complexity than the classical Collins' *cylindrical decomposition*, it is still quite high. We are working on *partial decompositions* that would have a better complexity.

We focused our study on geometric predicates needed by the algorithm (see Section 6.3.3). In fact, the algorithm requires the manipulation of algebraic numbers of degree up to 256, whereas the description of the non-decomposed arrangement involves only numbers of degree 64. It is not clear whether an algorithm can fill this gap, but it seems that a partial decomposition may ameliorate the problem. Currently, we are working in this direction.

6.2.3. Classification

Participants: Bernard Mourrain, Jean-Pierre Técourt.

We consider the effective classification of Steiner surfaces, that are parameterizations of degree 2 from $\mathbb{P}^2$ to $\mathbb{P}^3$ (with base field $\mathbb{R}$ or $\mathbb{C}$) without base point. We introduced algebraic invariants using Veronese variety and its Chordal. These invariants permit to characterize different classes of equivalence up to homography and they are easy to compute. We carried on our collaboration with INRA (Avignon) concerning the use of Steiner patches in the modeling of canopy. This work was presented at the SIAM Conference on Geometric Design and Computing [38] and submitted for publication.

6.2.4. Semi-implicit representation of algebraic surfaces

Participants: Laurent Busé, André Galligo.

In a recent work [17] presented at the international conference MEGA 2003, we introduced an intermediate representation of surface that we called semi-implicit. We gave a general definition in the language of projective complex algebraic geometry, and began its systematic study with an effective view point. We applied this representation to the investigation of intersection of two bi-cubic surfaces, surfaces widely used in Computer Aided Geometric Design, and used a new kind of resultants developed by our team, the determinantal resultants [39]. A paper has been submitted to a journal and we are still working on a larger use of semi-implicit representations in shape modeling through interpolation, self-intersection, intersection, and classification problems.

6.2.5. Differential geometry

Participant: Gabriel Dos Reis.

We are interested in applying techniques from differential geometry in the study of geometric objects, such as curves and surfaces used in Computer Aided Geometric Design, and performing robust algorithms for dealing with singularities. In [42], we presented a method to numerically construct a very important class of surfaces, the Constant Mean Curvature (CMC for short) surfaces. Their geometry was investigated with radial metric, putting an emphasis on the link between Hopf differential and extrinsic geometry. New asymptotic estimations of the metrics are given at the umbilical point and at infinity. We also deduced an efficient algorithm for numerical constructions. This work was presented at the international MEGA conference [19].

6.3. Symbolic numeric computation

6.3.1. Exact and approximate factorization

Participants: Guillaume Chèze, André Galligo.

We study a major step in factorization algorithms that proceed via approximations (see for example [52], [67], [68]). We propose two algorithms for computing an exact absolute factorization of a bivariate polynomial from an approximate one. These algorithms are based on certain properties of algebraic integers over $\mathbb{Z}$. The first method relies on an efficient heuristic which uses only gcd computations for determining the denominator of an algebraic integer in $\mathbb{Q}[\alpha]$, where α is an integer over $\mathbb{Z}$. The second method is a certified one, and it is based on a study of perturbation in a Vandermonde system.

6.3.2. Symbolic-numeric sparse interpolation of multivariate polynomials

Participant: Wen-shin Lee.

Collaborations with Mark Giesbrecht and George Labahn.

We develop effective solutions for the problem of sparse interpolation of a multivariate black box polynomial in floating point arithmetic. We also inquire into some techniques designed for exact computations in the setting of floating point, such as early termination and post check. Based on our observation of the similarity between the exact Ben-Or/Tiwari interpolation and the classical Prony's method for interpolating a sum of exponential functions, we exploit the generalized eigenvalue reformulation of Prony's method. Our methods are implemented in Maple, and we now investigate the sensitivity of our solutions in collaboration with Mark Giesbrecht and George Labahn.

As an application, through black box polynomial interpolations, we may dramatically increase the efficiency of computing the determinant of the non-singular maximal principal minor in a Bezoutian matrix that is for a set of multivariate polynomials. We further investigate the exploitation of the sparsity in the decomposition of a Bezoutian.

6.3.3. Certified geometric predicates

Participants: Ioannis Emiris, Bernard Mourrain, Monique Teillaud, Jean-Pierre Técourt, Elias Tsigaridas.

Our work on certified predicates started a few years ago. We first studied the case of predicates for arrangements of circular arcs [53][54]; see also [61] for a related work. More work was done this year on predicates for arrangements of conic arcs [28][27]. In fact most of these predicates reduce to comparison of roots of polynomials of degrees 2 to 4. We propose efficient methods, based on Sturm sequences and root isolation techniques, to achieve such comparisons. Some harder predicates, that require comparisons of roots of polynomials whose coefficients belong to an algebraic extension of the rational field, are also currently studied.

As mentioned in Section 6.2.2, we are also working on 3D arrangements of quadric surfaces. In our algorithm, the predicates boil down to comparisons of roots of algebraic systems in three variables and up to four equations of degree two. Algebraic tools like Rational univariate representations, Sturm sequences and Descartes' rule can be used to achieve the comparisons in an exact manner.

6.3.4. Topology of curves and surfaces

Participants: Lionel Deschamps, Gregory Gatellier, Abder Labrouzy, Bernard Mourrain, Jean-Pierre Técourt.

Deducing the geometry or topology of a curve or surface from its algebraic definition is an important task we have to perform efficiently when dealing with geometric problems such as arrangement computations, structure analysis, ..., etc. In order to avoid incoherent computation, this task often requires certificated methods.

In relation with the ECG and GAIA project, we consider first the problem of computing the topology of a curve defined as the intersection of two surfaces. The method that we developed computes the critical values for the projection along a plane direction, regular points in-between these critical points and connect the branches according to the regularity of the end points. The approach depends on univariate and multivariate polynomial solvers. A first prototype has been implemented based on the SYNAPS library. See [33] for more details.

The problem of meshing a surface given by an implicit equation has also been studied. We develop a new method, which allows us to guarantee the topology in the smooth part and give a model of singularity elsewhere, while producing a number of linear pieces related to the geometry of the object. We use Bernstein bases to represent the function in a box and subdivide this representation according to a generalization of Descartes rule, until the problem in each box boils down to the case where either the implicit object is proven to be homotopic to its linear approximation in the cell or the size of the cell is smaller than ϵ. Experiments on examples from the classification of singularities show the efficiency of the approach. A paper related to this approach is submitted to a conference presentation [23].

6.4. Software aspects of scientific computations

6.4.1. For a better support to scientific computation

Participant: Gabriel Dos Reis.

ISO/CEI JTC1/SC22/WG21
The work on the normalization of the language C++ has been pursued. Two contributions in the form of technical reports to the ISO/CEI JTC1/SC22/WG21 committee, have been proposed. The first one is on template aliasing in C++ [25] to simplify the use of multiple template parameters in implementations devised for a high degree of control and flexibility. The second report [31] is on the control of implicit template instantiation. These propositions have been presented at the Association of C and C++ Users (ACCU) Conference 2003 in Oxford.

The visit of B. Stroustrup in summer was the occasion of further work on the parameterisation of code in C++, leading in particular to the definition of the “concept” mechanism and to a proposition on template argument checking [35], [36]. The previous work on template aliasing was extended and detailled in [37]. This proposition received a strong acceptation and encouragment from the Evolution Working Group of C++.

6.4.2. Predicates for curve arrangements

Participants: Ioannis Emiris, Athanasios Kakargias, Sylvain Pion [Geometrica team], Monique Teillaud, Elias Tsigaridas.

See the CGAL-LIKE CURVED KERNEL *web site http://www-sop.inria.fr/galaad/teillaud/kernel.html and the* AXEL *web site http://www-sop.inria.fr/galaad/logiciels/axel/index.html.*
The theoretical work on certified predicates (see Section 6.3.3) allowed us to work on an efficient implementation and to improve last year’s first preliminary code [69]. We now provide:

- as [34]: a kernel with circular arcs and all the predicates needed to interface with the CGAL arrangement algorithms; a concept RootOf_2 for algebraic numbers of degree 2 and some models of this concept; a demo showing the computation of an arrangement of circular arcs, using the CGAL arrangements and this kernel.
- in A. Kakargias’ internship report [43]: some experimental studies, comparing several number types to support the predicates.
- as [27]: a prototype implementation of a complete set of comparisons between algebraic numbers of degree from 1 to 4; experimental results and applications to geometric predicates.

6.4.3. Synaps

Participant: Nicolas Chleq.

See the SYNAPS *web site http://www-sop.inria.fr/galaad/logiciels/synaps/index.html.*
The SYNAPS library benefits for the work of N. Chleq (DREAM engineer) for the configuration and documentation process. The tools and techniques used now to install the package follow more standard rules, relying on the autoconf, automake tools. An automatic test suite as been settled down to reinforce the robustness of the implementation. Documentation is now generated automatically, also by classical tools. Besides, the existing implementation has been extended and improved for applications in geometric problems.

6.4.4. Triangulation of points

Participant: Monique Teillaud.

See the CGAL *web site: http://www.cgal.org.*

The package "3D Triangulation" of CGAL is maintained in collaboration with Sylvain Pion (Geometrica team). It was released in CGAL 3.0 in November, 2003.

6.5. Applications

6.5.1. Applications to CAGD

In [14] we are interested in computing effectively cylinders through 5 points, and in other problems involved in metrology and CAGD. In particular, we consider the cylinders through 4 points with a fix radius and with extremal radius. For these different problems, we gave bounds on the number of solutions and examples showing that these bounds are optimal. Finally, we described two algebraic methods which can be used to efficiently solve these problems and some experimental results. Since we also investigated another technique based on residual resultants for solving these problems described in [22], we are currently in contact with Thomas Chaperon (working at MENSI) to incorporate this method in extracting cylinders from a noisy 3D point cloud.

We also investigated other applications to CAGD through the European project ECG. In [26] and [23] we studied different representations of algebraic surfaces as parameterized, implicit, semi-implicit and meshed representations, and developed tools for converting one representation into another. Through the GAIA II European project we gave a survey of resultant theory and its applications to CAGD [12].

The self-intersection algorithm developed by Jean-Pascal Pavone has been integrated in the ThinkDesign software of the Think3 company as a COM component.

6.5.2. Shape optimisation

Participants: Mehmet Celikbas, Bernard Mourrain.

Collaboration with Opale.

In the context of the COLORS Forme, for simulation and optimization purposes, we consider the question of compact encoding of wing shapes. Thanks to the basic properties of control, degree elevation, ..., the representation of parametric curves in the Bernstein basis offers very powerful tools for geometric optimization. Based on B-spline representation, we extended this approach and obtained more flexibility and a smoother behavior in numerical simulation. This has been experimented during the internship of M. Celikbas, on an academic case, which is a test step in the global project on wing shape optimization. See [24].

6.5.3. Bioinformatic

Participants: Antoine Marin, Bernard Mourrain.

In molecular biology, biochemistry and structural biology, knowing the structures of proteins is the key to learn their functions. Understanding the functions of proteins is one of the main goals in numerous ongoing genomic projects. In these projects, more and more protein structures are elucidated through two major experiments: X-ray crystallography and Nuclear Magnetic Resonance (NMR). Even if the proportion of solved structures using NMR is only 15% today, this technique has made a lot of improvements during the past few years.

The usage of NMR is mainly hampered by the difficulty in analyzing the data collected from these experiments. NMR gives two types of information: correlations between atom nuclei through chemical bounds and correlations between atom nuclei through space. The second type of information is critical in solving the 3D structure of proteins. As described by Crippen and Havel in 1988, distance geometry is the mathematical basis for a geometric theory of molecular conformation. As the natural approach to solve this problem, distance geometry gives a clear representation of what a protein structure is and what are the relations between atom nuclei distances.

In the GALAAD project, a set of programs have been developed to generate and analyze these data automatically in testing and developing various distance geometry tools for this problem. Based on preliminary results, although with basic distance geometry tools we are not able to solve the problem with such noisy and missing data provided by NMR experiments, we are not that far. Further research is required for improvements and to correctly tackle this problem.

8. Other Grants and Activities

8.1. National actions

8.1.1. Bioinformatic action

Participants: Ioannis Emiris, Bernard Mourrain [correspondent], Antoine Marin.

The project "Géométrie de Distances et Génomique Structurale à haut débit par RMN" aims at deducing the conformation of molecules in Euclidean space, from partial information on the relative distances between atoms or the radicals (group of atoms). This information is obtained by experiments called NMR or Nuclear Magnetic Resonance. During his post-doctorate, partly in our team, Antoine Marin has been working on typing techniques for the amino acids starting from their chemical shifts, regrouping of the residues observed on the spectra in the sequence of protein, and rebuilding of the 3D-structure starting by matrix methods from information of distance.

8.2. International actions

8.2.1. ecg : Effective Computational Geometry for Curves and Surfaces

Participants: Laurent Busé, Ioannis Emiris, André Galligo, Grégory Gatellier, Bernard Mourrain, Olivier Ruatta, Jean-Pierre Técourt, Monique Teillaud [correspondent].

See ECG *project web site http://www-sop.inria.fr/prisme/ECG/*.
INRIA (GEOMETRICA and GALAAD) is coordinating the European project:
- Acronym: ECG, number IST-2000-26473
- Title: Effective Computational Geometry for Curves and Surfaces.
- Specific Programme: IST
- RTD (FET Open)
- Start date: may 1st 2001 - Duration: 3 years
- Participation of Inria as coordinating site
- Other participants:
    ETH Zürich (Switzerland),
    Freie Universität Berlin (Germany),
    Rijksuniversiteit Groningen (Netherlands),
    MPI Saarbrücken (Germany),
    Tel Aviv University (Israel)
- Abstract: Efficient handling of curved objects in Computational Geometry. Geometric algorithms for curves and surfaces, algebraic issues, robustness issues, approximation.

Monique Teillaud is the technical coordinator of the project within the Board and the members, as well as the communication with the *Project Officer* in Brussels.

She is also maintaining the public web site of the project, together with internal web sites with restricted access, and the mailing lists for internal communication (the members mailing list contains 66 addresses).

Steve Oudot (GEOMETRICA team) is maintaining the `php` scripts for submission of technical reports, originally written by Menelaos Karavelas (PRISME team). He also wrote new scripts for deliverables submission.

Olivier Ruatta settled a *MySQL* server that is used for the database of technical reports.

8.2.2. gaia

Participants: Laurent Busé, Mohamed Elkadi, Ioannis Emiris, André Galligo [correspondant], Michel Merle, Bernard Mourrain, Jean-Pascal Pavone, Olivier Ruatta.

See the GAIA II *project web site http://www.math.sintef.no/gaiatwo/*
In collaboration with the university of Nice UNSA, the GALAAD team is involved in the European project GAIA:

- Acronyme : GAIA II, number IST-2001-35512
- Title : Intersection algorithms for geometry based IT-applications using approximate algebraic methods
- Specific program of the project : IST
- Type of project : FET-Open
- Beginning date : 1st of july 2002 - During : 3 years
- Participation mode of INRIA : participant via the UNSA
- Partners list :

    SINTEF Applied Mathematics, Norvegia
    Johannes Kepler University, Austria
    UNSA, France
    Université de Cantabria, Spain
    Think3 SPA, Italy and France
    University of Oslo, Norvegia

- Abstract of the project : Detection and treatment of intersections and self-intersections, singularity analysis, classification, approximate algebraic geometry and applications to CAG.

8.2.3. Bilateral action

Participants: Laurent Busé, Guillaume Cheze, Mohamed Elkadi, Ioannis Emiris [correspondent], André Galligo, Bernard Mourrain, Olivier Ruatta.

Collaboration with A. Dickenstein and C. D'Andrea (Univ. of Buenos Aires).
Collaboration with the Mathematics Department of the University of Buenos Aires in Argentina, in the framework of an ECOS-Sud project for 3 years (2001-03). This project supports 2-week visits of researchers (this year M. Elkadi visited Buenos Aires) and longer stays of PhD students (this year, G. Cheze spent one month in Buenos Aires) or Postdocs. It is titled "Robust solution of algebraic systems and applications in computer-aided design" and concerns elimination theory, resultant matrices, but also newer research subjects related to applications in modeling and CAGD.

The year 2003, last year of the collaboration was marked by the organization of a Cimpa Graduate School at Buenos Aires, in July 2003, on Systems of Polynomial Equations. The organizers were A. Dickenstein (U. Buenos Aires) and I. Emiris (U. Athens, Greece). Among the lecturers, there were A. Galligo and B. Mourrain. The School was followed by the 1st latin-american workshop on "Systems of polynomial equations", in which spoke M. Elkadi and G. Cheze.

8.2.4. Bilateral action

Participants: Ioannis Emiris, Athanasios Kakargis, Bernard Mourrain [correspondent], Jean-Pierre Técourt, Monique Teillaud, Elias Tsigaridas.

The Team of Geometric and Algebraic Algorithms at the National University of Athens, Greece, has been associated with Galaad for a period of 3 years (2003-05). See its web site http://www-sop.inria.fr/galaad/collab/grece/ea.html.
This bilateral collaboration is titled CALAMATA (CALculs Algebriques, MATriciels et Applications). The Greek team (http://www.di.uoa.gr/~erga/) is headed by Ioannis Emiris.

The focus of this project is the solution of polynomial systems by matrix methods. Our approach leads naturally to problems in structured and sparse matrices. Real root isolation, either of one univariate polynomial or of a polynomial system, is of special interest, especially in applications in geometric modeling, CAGD or computational geometry. We are interested in computational geometry, actually, in what concerns curves and surfaces. The framework of this work is the European project ECG.

In 2003, 6 members of the Greek team visited Inria, either for week-long visits or for longer visits (from one to 3 months). Three Inria researchers visited Athens for one week. We also mention the participation of members of both teams in international or national conferences : the meeting of Computational geometry at Dagstuhl (Germany), the Cimpa Graduate School in Argentina, and the French conference on Computational geometry.

8.2.5. Bilateral action

Participant: Olivier Ruatta [correspondent].

Agnes Szanto works at the departement of mathematics of the North Carolina State University.
This is a project on overdetermined algebraic systems funded by a U.S. grant of one year. A visit of O. Ruatta to the North Carolina State University is planned.

The objective of this investigation is to develop and implement highly efficient algorithms for the solution of over-constrained polynomial systems with finitely many, possibly multiple roots over the complex numbers, when the input is given with inexact coefficients. We refer to such problems as inexact degenerate systems. Both “resultant based” and analytic iterative methods are considered to tackle this problem using the large number of already existing works. The researchers will address the problem of the definition of “nearly consistent” systems, with computational methods generalizing the S.V.D. of the linear case. The complexity is one of the central issues of this research since we want efficient methods. Implementations will be available via Java applets on the internet.

9. Dissemination

9.1. Animation of the scientific community

9.1.1. Seminar organization

We continued to organize a (almost) weekly seminar called “Table Ronde”. The list of talks is available at http://www-sop.inria.fr/galaad/seminaires/TABLERONDE/seminaire.php.

9.1.2. Comittee participations

- Special issue of “Theoretical Computer Science” on Algebraic Numeric algorithms. Guest editors: I. Emiris, B. Mourrain et V.Y. Pan.

9.1.3. Editorial committees

- Monique Teillaud is a member of the CGAL Editorial Committee.

9.1.4. Other committees

- B. Mourrain is in charge, with Thierry Vieville, of the “Formation par la recherche” at INRIA Sophia-Antipolis,
- M. Teillaud and J.P. Técourt are member of the “Comité de centre” at INRIA Sophia-Antipolis.

9.1.5. WWW server

More information on our activities can be found at http://www-sop.inria.fr/galaad/.

9.2. Participation to conferences and invitations

- L. Busé attended the international conference ACA 2003 hosted in North Carolina State University at Raleigh, USA. He gave a talk based on both works [13][40].
- G. Chèze attended the CIMPA Graduate School at Buenos Aires in July 2003 on “Systems of Polynomial Equations.” He gave a talk at the workshop on “Systems of Polynomial Equations.” In September, he went to Strasbourg and Nancy for one week in order to work with M. Mignotte, P. Zimmermann and G. Hanrot.

- M. Elkadi attended CIMPA Graduate School at Buenos Aires in July 2003 on “Systems of Polynomial Equations.” He gave a talk at the workshop on “Systems of Polynomial Equations.” He also attended the meeting “Effectivity Problems V” in Diamante (Italy) and gave a talk.
- G. Dos Reis gave a talk at the Journée de Calcul Formel (Luminy, january) and presented a paper at the MEGA conference (june, Kaiserslautern).
- A. Galligo gave a talk at the MEGA Conference (Kaiserslautern, Germany, June). He also gave a talk at the Journées de Calcul Formel (Marrakech, Maroc, April). He taught a course at the CIMPA Graduate School at Buenos Aires in July on “Absolute polynomial factorization.”
- B. Mourrain was in charge of a session at the Journées Nationales de Calcul Formel (Luminy, January), and gave talks at the Dagsthul workshop on Verification and Constructive Algebra (Dagsthul, Germany, January), the Journées de Calcul Formel (Marrakech, Maroc, April), ECG workshop (Berlin, Germany, June), MEGA Conference (Kaiserslautern, Germany, June), the Rencontres Mondiales des Logiciels Libres (Metz, July), at the Surfaces Symposium (Treilles, July), COMPASS conference (Kefermarkt, Austria, September), SIAM conference on Geometric Design (Seattle, USA, November). He also taught a course at the CIMPA Graduate School in Buenos Aires in July on “Systems of Polynomial Equations,” and was invited to give a talk at the 5th Real Number and Computer conference (Lyon, September) and the workshop of the AS Constraint (Strasbourg, December).
- J.P. Pavone attended the COMPASS conference (Kefermarkt, Austria, September).
- O. Ruatta was an invited lecturer for Real Algebraic and Analytic Geometry summer school (June 2003 in Rennes, France) and attended the R.A.A.G. meeting for young researchers (Septembre 2003 in Coma-Ruga, Spain).
- Monique Teillaud gave the following talks in conferences and workshops: 14th ACM-SIAM Sympos. Discrete Algorithms (SODA), Baltimore (January), *Perturbations and Vertex Removal in a 3D Delaunay Triangulation"*; Journées Nationales de Calcul Formel, Luminy (January), *Étude de prédicats géométriques*; Dagstuhl Seminar 03121, Computational Geometry, invited talk (March), *Arrangement of Quadrics in 3D*; ECG Workshop on Applications Involving Geometric Algorithms with Curved Objects, Saarbruücken (September), *Sweeping an arrangement of quadrics*. She also attended: the ECG General Workshop, Berlin (June), the Journées de Géométrie Algorithmique, Giens (September), Workshop on Geometric Compression, Sophia Antipolis (November).
- Jean-Pierre Técourt attended the 19th European Workshop on Computational Geometry, March 24-26, University of Bonn, Germany.

9.3. Formation

9.3.1. Teaching at universities

- G. Chèze: Practical sessions, Introductive course to C language, Algorithmic and Computational Sciences, UNSA, (78h). Mathematics Applied to Social Sciences, first year of the DEUG.
- M. Elkadi: Courses in Algebra (Deug MI2) and differential calculus (Licence MASS).
- A. Galligo : Course of mathematics and computer algebra in DEUG MASS1 at UNSA (192h).
- B. Mourrain : DEA Algorithmique, Paris VI (10h). Maîtrise Math-Info, UNSA (40h).
- J.P. Pavone : Computer Sciences at the IUT of Nice (96h).
- O. Ruatta : Pratical sessions and exercices Algorithmic et programmation for the first year of the DEUG at UNSA (tronc commun 50h then Math-Info 20h). Exercices of Symbolic Computation for the DEUG MASS at UNSA (40h). Introduction to Unix-Linux for MST Geologie-Geophysique at UNSA (15h). Languages and tools for the web for MST Geologie-Geophysique at UNSA (18 h).

- Jean-Pierre Técourt : Computer Sciences (Java) in Deug Tronc Commun, UNSA (64h).
- Monique Teillaud : Geometric Computation, ESSI (École Supérieure en Sciences Informatiques) (20h). Computational Geometry, UNSA (Université de Nice Sophia Antipolis), Maîtrise d'informatique (8h). Geometric Computation, ISIA (Institut Supérieur d'Informatique et d'Automatique) (11h). Computational Geometry, École doctorale I2S (Module Imagerie), Montpellier (3h).

9.3.2. PhD theses in progress

- Guillaume Chèze, Factorisation de polynômes à plusieurs variables, UNSA.
- Thi Ha LE, Classification and intersections of some parametrized surfaces and applications to CAGD (UNSA).
- Jean-Pascal Pavone, Étude de la géométrie des surfaces paramétrées utilisées en CAO, UNSA.
- Jean-Pierre Técourt, Algorithmique des courbes et surfaces implicites, UNSA.

9.3.3. Internships

See the web page of our interships.

- Mehmet Celikbas (INSA Toulouse), *Surface de Bezier et B-spline pour l'optimisation géométrique*, from June 20th to September 20th.
- Lionel Deschamps, (DEA SIC Image Vision -ESSI), *Maillage de surfaces implicites*, from April 1st to September 30th.
- Athanasios Kakargias (Athens National University), *Experimenting with the Curved Kernel*, from June 29th to September 28th.
- Abder Labrouzy (ISIMA Clermont-Ferrand) *Intersection de courbes et surfaces algébriques*, from April 7th to September 5th.
- Nicolas Martin (ESSI, 2nd year), *Arrangements of evolving circles in the plane*, from June 30th to September 22th.

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