Project-Team Micmac

Methods and Engineering of Multiscale Computing from Atom to Continuum

Rocquencourt
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1. Team

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2. Overall Objectives

The MICMAC team has been created jointly by the Ecole Nationale des Ponts et Chaussées (ENPC) and the INRIA in October 2002.

It is hosted in the CERMICS laboratory (Centre d’Enseignement et de Recherches en Mathématiques, Informatique et Calcul Scientifique) at ENPC. The scientific focus of the team is to analyze and improve the numerical schemes used in the simulations of computational chemistry at the microscopic level, and in the simulations coupling this microscopic scale with larger, meso or macroscopic, scales.

3. Scientific Foundations

Quantum Chemistry aims at understanding the properties of matter through the modeling of its behavior at a subatomic scale, where matter is described as an assembly of nuclei and electrons.

At this scale, the equation that rules the interactions between these constitutive elements is the Schrödinger equation. It can be considered (except in few special cases notably those involving relativistic phenomena or nuclear reactions) as a universal model for at least three reasons. First it contains all the physical information of the system under consideration so that any of the properties of this system can be deduced in theory from the Schrödinger equation associated to it. Second, the Schrödinger equation does not involve any empirical parameter, except some fundamental constants of Physics (the Planck constant, the mass and charge of the electron, ...); it can thus be written for any kind of molecular system provided its chemical composition, in terms of natures of nuclei and number of electrons, is known. Third, this model enjoys remarkable predictive capabilities, as confirmed by comparisons with a large amount of experimental data of various types.
On the other hand, using this high quality model requires working with space and time scales which are both very tiny: the typical size of the electronic cloud of an isolated atom is the Angström (10^{-10} meter), and the size of the nucleus embedded in it is 10^{-15} meter; the typical vibration period of a molecular bond is the femtosecond (10^{-15} second), and the characteristic relaxation time for an electron is 10^{-18} second. Consequently, Quantum Chemistry calculations concern very short time (say 10^{-12} second) behaviors of very small size (say 10^{-27} m^3) systems. The underlying question is therefore whether information on phenomena at these scales is or not of some help to understand, or better predict, macroscopic properties of matter.

It is certainly not true that all macroscopic properties can be simply upscaled from the consideration of the short time behavior of a tiny sample of matter. Many of them proceed (also) from ensemble or bulk effects, that are far from being easy to understand and to model. Striking examples are found in the solid state or biological systems. Cleavage, the ability minerals have to naturally split along crystal surfaces (e.g. mica yields to thin flakes) is an ensemble effect. Protein folding is also an ensemble effect which originates in the presence of the surrounding medium; it is responsible of peculiar properties (e.g. unexpected acidity of some reactive site enhanced by special interactions) on which rely vital processes.

However, it is undoubtedly true that on the other hand many macroscopic phenomena originate from elementary processes which take place at the atomic scale. Let us mention for instance the fact that the elastic constants of a perfect crystal or the color of a chemical compound (which is related to the wavelengths absorbed or emitted during optic transitions between electronic levels) can be evaluated by atomic scale calculations. In the same fashion, the lubrifying properties of graphite are essentially due to a phenomenon which can be entirely modelled at the atomic scale.

It is therefore founded to simulate the behavior of matter at the atomic scale in order to understand what is going on at the macroscopic one. The journey is however a long one. Starting from the basic principles of Quantum Mechanics to model the matter at the subatomic scale, one finally uses statistical mechanics to reach the macroscopic scale. It is often necessary to rely on intermediate steps to deal with phenomena which take place on various mesoscales. Possibly, one couples one approach to the others within the so-called multiscale models. In the following we shall indicate how this journey can be done, focusing rather on the first scale (the subatomic one), than on the latter ones.

It has already been mentioned that at the subatomic scale, the behavior of nuclei and electrons is governed by the Schrödinger equation, either in its time dependent form or in its time independent form. Let us only mention at this point that

- both equations involve the quantum Hamiltonian of the molecular system under consideration; from a mathematical viewpoint, it is a self-adjoint operator on some Hilbert space; both the Hilbert space and the Hamiltonian operator depend on the nature of the system
- also present into these equations is the wavefunction of the system; it completely describes its state; its $L^2$ norm is set to one.

The time dependent equation is a first order linear evolution equation, whereas the time-independent equation is a linear eigenvalue equation.

For the reader more familiar with numerical analysis than with quantum mechanics, the linear nature of the problems stated above may look auspicious. What makes in fact extremely difficult the numerical simulation of these equations is essentially the huge size of the Hilbert space: indeed, this space is roughly some symmetry constrained subspace of $L^2(\mathbb{R}^d)$, with $d = 3(M + N)$, $M$ and $N$ respectively denoting the number of nuclei and the number of electrons the system is made of. The parameter $d$ is already 39 for a single water molecule and reaches rapidly $10^6$ for polymers or biological molecules. In addition, a consequence of the universality of the model is that one has to deal at the same time with several energy scales. In molecular systems indeed, the basic elementary interaction between nuclei and electrons (the two-body Coulomb interaction) shows itself in various complex physical and chemical phenomena whose characteristic energies cover several orders of magnitude: the binding energy of core electrons in heavy atoms is $10^4$ times as large as a typical covalent bond energy, which is itself around 20 times as large as the energy of a hydrogen bond. High precision or at least
controlled error cancellations are thus required to reach chemical accuracy when starting from the Schrödinger equation.

Clever approximations of the Schrödinger problems are therefore needed. The main two approximation strategies, namely the Born-Oppenheimer-Hartree-Fock and the Born-Oppenheimer-Kohn-Sham strategies, end up with large systems of coupled nonlinear partial differential equations, each of these equations being posed on $L^2(\mathbb{R}^3)$. The size of the underlying functional space is thus reduced at the cost of a dramatic increase of the mathematical complexity of the problem: nonlinearity. The mathematical and numerical analysis of the resulting models is one of the major concern of our work.

4. Application Domains

4.1. Large systems simulation

As the size of the systems one wants to study increases, more efficient numerical techniques need to be resorted to. In computational chemistry, the typical scaling law for the complexity of computations with respect to the size of the system under study is $N^3$, $N$ being for instance the number of electrons. The Grail in this respect is to reach a linear scaling, so as to make possible simulations of systems of practical interest in biology or material science. Efforts in this direction require to address a large variety of questions such as

- how to improve the nonlinear iterations that are the basis of any ab initio models for computational chemistry
- how to more efficiently solve the inner loop which most often consists in the solution procedure for the linear problem (with frozen nonlinearity)
- how to design a small enough variational space, whose dimension is kept limited while the size of the system increases.

An alternative strategy to diminishing the complexity of ab initio computations, is to try and couple different models at different scales. Such a mixed strategy can be either a sequential one or a parallel one, in the sense that

- in the former the results of the model at the lower scale is simply used to evaluate some parameters that are inserted in the model for the larger scale: one example is the parameterized classical molecular dynamics, which makes use of force fields that are fitted on calculations at the quantum level;
- while in the latter, the model at the lower scale is concurrently coupled to the model at the larger scale: an instance of such a strategy is the so called QM/MM coupling (standing for Quantum Mechanics/Molecular Mechanics coupling) where some part of the system (typically the reactive site of a protein) is modeled with quantum models, that therefore account for the change in the electronic structure, and therefore for the modification of chemical bonds, while the rest of the system (typically the inert part of a protein) is coarse grained, and more crudely modeled by classical mechanics.

The coupling of different scales can even go up to the macroscopic scale, with methods that couple a microscopic description of matter, or at least a mesoscopic one, with the equations of continuum mechanics at the macroscopic level.
4.2. Laser control

The laser control of chemical reactions is today an experimental reality. Experiments, carried out by many groups of researchers and in many different contexts and settings have demonstrated the feasibility of controlling the evolution of a quantum system using a laser field. All these experiments exploit the remarkable properties of quantum interactions (interferences) between one, or more, external interactions (e.g. lasers) and the sample of matter under study. In order to create the ad hoc interferences that will drive the system to the desired goal, one can either play with the dephasing between two beams, or conveniently choose the frequencies of the beams, or also make use of the two aspects mixed together, which amounts to allowing for “all” possible laser fields as in optimal control schemes.

Whatever the strategy, the success of these numerous experiments not only validate the idea of manipulating and controlling quantum systems with lasers, but also motivate the need for further theoretical studies in this direction, in order to even improve the results and the range of their applicability; interest in this research area has also been increasing in more applied communities. The standard modeling for the problem of the laser control of a molecular system involves the time-dependent Schrödinger equation which rules the evolution of the wavefunction describing the state of the system. On the basis of the Schrodinger equation, one then state a control problem, either in the framework of exact control or in the framework of optimal control.

The first fact to underline as a crucial feature of the problem of laser control is the orders of magnitude in time and space that are typically encountered here. The space scale is indeed that of an atom, say $10^{-10}$ m, but more important than that the time scale is of the order of $10^{-15}$ s (the femtosecond) and can even go down to the attosecond ($10^{-18}$ s). As surprising as it may seem, the laser fields can literally be “tailored” on these tiny timescales. They can involve huge intensities $10^{12}$ W/cm$^2$ and above, and their shots can be cycled at 1 KHz. Apart from being very impressive, these orders of magnitude mean one thing for whom is not an expert: one can do several thousands of experiments in a minute. This ability changes the whole landscape of the control problem, for making an experiment is here far cheaper than running a numerical simulation. This has motivated the paradigm of closed-loop optimization when the criterion to be optimized is evaluated on-the-fly on an experimental device. One of the current challenging issue for the mathematicians taking part into the field is to understand how to take advantage of a combined experimental/numerical strategy. In this respect, it is to be noted that the experimental side can come from on-the-fly experiments (how to decide what to do ?), but may also come from the tremendous amount of data that can be (and actually is) stored from the billions of experiments done to this day (how to dig into this data base?).

A second point is to remark the way in which the control enters the problem: the control multiplies the state. Theoretically and numerically, this bilinear nature causes difficulties. Finally, we deal here with open-loop control, at least for two reasons: first, the timescale on which the phenomenon goes is too short for the current capabilities of electronic devices, which prevents closing the loop within one experiment; but secondly, feedback control means measuring something, which in a quantum framework means interacting with and thus perturbing the system itself. These two bottlenecks might be overcome in the future, but this will undoubtedly require a lot of theoretical and technical work.

A third peculiarity regards the choice of admissible laser fields as control: what types of $E(t)$ should we allow when setting up the control problem ? This question leads to a dichotomy: one can choose either to restrict oneself to the experimentally feasible fields, or to basically let the field free, therefore allowing for very general laser fields, even those out of reach for the contemporary technology. The two tracks may be followed. In particular, the second track, the most “imaginative” one (rather unusual in comparison to other contexts), can serve as a useful technical guide for building the lasers for tomorrow’s technology.

A final key issue is robustness. It is of course a standard fact in every control problem that the control obtained needs to be robust, for obvious practical reasons. The somewhat unusual feature in the present setting is that the experiments show that they are surprisingly robust with respect to all kinds of perturbations (noise, uncertainties in the measures, ...). Clearly, there is here something to be understood on the theoretical level, e.g. by envisioning new modeling strategies that incorporate undesirable perturbations.
5. New Results

5.1. Electronic structure calculations

Participants: Maxime Barrault, Eric Cancès, Hervé Galicher, Claude Le Bris, François Lodier, Mohamed El Makrini, Gabriel Stoltz, Gabriel Turinici.

We have continued our studies on the algorithms used for electronic structure calculations. The different tracks followed are

- domain decomposition techniques for the linear subproblem
- reduced basis approaches: work in collaboration with the group of Antony Patera at MIT [5].

Apart from this main stream we have continued the development of techniques [11] for locating electrons in molecules and thereby bridging the classical chemical description in terms of covalent, ionic, ... bonds and the quantum description of the electronic structure in terms of wavefunctions. This is a joint effort with Andreas Savin, from the theoretical chemistry laboratory at University Paris 6.

All of these new developments and methods are meant to be inserted in the long term inside the software platform that we are currently developing (in collaboration with Yves Achdou, University Paris 7). The current status of this software allows us to perform electronic structure calculations on simple systems, within a C++ environment, and relying on rigorous numerical analysis tools.

In parallel with these numerical works, we have pursued in our enterprise to put the models and the techniques on a sound mathematical basis. Instances of such a program are:

- the study by Gabriel Stoltz of a possible multiscale modelling of detonation waves in materials,
- the study by Gabriel Stoltz and Eric Cancès of models coupling different levels of description of molecular systems such as QM/MM models (models coupling quantum mechanics with classical mechanics), QM/QM models (models coupling quantum models of different nature).
- the study by Eric Cancès, Benjamin Jourdain and Tony Lelièvre of the basis of the diffusion Monte Carlo method.
- the study by Eric Cancès and Hervé Galicher in collaboration with Mathieu Lewin (Univ. Paris Dauphine) of the definition and the existence of excited states together with the possible numerical strategies to evaluate the first one [33].

Two state of the art review articles have been written in order to provide the community with a reliable account of all recent developments in the field both from a mathematical and a numerical viewpoint [18][20].
5.2. Molecular dynamics

**Participants:** Eric Cancès, Claude Le Bris, Frédéric Legoll, Antonin Orriols, Gabriel Turinici.

Molecular dynamics is often used in statistical physics as an alternative to Monte Carlo methods for computing ensemble averages. Based on a theoretical result concerning the convergence of the numerical averages toward the exact result, we were able to propose a new method to accelerate this convergence. An application oriented work has been published in [8] and the numerical analysis is submitted [9].

This scientific program has been performed in collaboration with Francois Castella, Philippe Chartier and Erwan Faou from INRIA Rennes (work supported by ARC PRESTISSIMO 2003-2004). It will be continued with a different funding (ACI “Nouvelles interfaces des mathématiques” 2004-2005).

5.3. Laser control

**Participants:** Fabrice Cohen, Claude Le Bris, Mazyar Mirrahimi, Gabriel Turinici.

Following a series of works devoted to the optimal control of the alignment of a molecule by laser field and of high harmonic generation [7][1], our interest now focuses on the very practical issues of the laboratory implementations of closed loop optimal control. This is done in close collaboration with the group of H. Rabitz (Princeton University). We have introduced direct search algorithms into this field and assessed their efficiency versus more commonly used algorithms such as Genetic-like algorithms [28]. A practical implementation of these ad-hoc developed algorithm is under progress in an experimental setting at Princeton University. This is made possible by a PICS CNRS-NSF grant.

Different from the experimental algorithms, the so-called monotonically convergent search procedures were also studied in collaboration with Y. Maday and J. Salomon (Paris 6), H. Rabitz (Princeton) and Y.Ohtsuki (Tohoku Univ. Japan) [27].

We are currently addressing in collaboration with Pierre Rouchon (Ecole des Mines, Paris) some questions related to the inversion paradigm: use the laser field as a tool to obtain additional information on the molecular system. Lyapounov methods for the numerical resolution of the evolution equations have also been introduced in joint works with Mazyar Mirrahimi and Pierre Rouchon [26][25].

Finally, studies concerning intuitive controllability criterions [30] and the controllability of ensembles of molecules and the impact of the form of the laser profile have been addressed in a collaboration with H. Rabitz [29].

5.4. Multiscale simulation of solids

**Participants:** Antoine Gloria, Claude Le Bris, Frédéric Legoll.

This research program divides into a theoretical part and a more numerical one.

On the theoretical side, following a series of works, in collaboration with Xavier Blanc (Laboratoire Jacques-Louis Lions, Paris), Isabelle Catto (University Paris 9), and Pierre-Louis Lions Collège de France, Paris), we have continued to address the question of how to define ground state energies for some systems composed of an infinite number of particles. The framework is that of quantum chemistry, where the state of matter is modelled though variational problems that couple a classical description of the nuclei with a quantum description of the electrons. Starting from a model for the molecule (finite number \(N\) of nuclei-say of unit charge- and an equal finite number \(N\) of electrons), a prototypical example being the Thomas-Fermi-von Weizsäcker model, we pass to the limit when \(N\) goes to infinity. When the nuclei are enforced to sit on the sites of a periodic lattice, asymptotically filling the entire lattice \(\mathbb{Z}^3\), then the problem is solved by the so-called bulk limit problem for periodic crystals (series of work by I. Catto, C. Le Bris and P.-L. Lions).

We are currently addressing the same type of questions, but when the sites where the nuclei are located are random variables. We thus have at the microscopic level a notion of stochastic lattices, for which we define the ground state energy, the latter of course depending on the model chosen for the modelling of interactions. In addition, we have continued our program consisting in passing from the microscale to the macroscale on the
basis of quantum models at the microscale. The writing of two articles describing the results and techniques is currently under progress.

On a more numerical level, Claude Le Bris and Frédéric Legoll, in collaboration with Xavier Blanc, have completed the mathematical and numerical analysis of a prototypical model for simulations in materials science. The model under study couples an atomistic description of the sample is coupled with a macroscopic continuum description [32].

In addition to this, Antoine Gloria is beginning his Phd work on questions related to multiscale methods for the simulation of materials.

5.5. Multiscale simulation of complex fluids

**Participants:** Eric Cancès, Yousra Gati, Claude Le Bris, Tony Lelièvre.

The subject of this activity covers two different applications and settings.

The first one is that of the modeling of polymeric fluid flows. We have continued our endeavor to mathematically study the models commonly used in numerical simulations by experts at nonnewtonian fluid dynamics. Various issues have been addressed: variance reduction techniques in [17], theoretical studies of the stochastic partial differential equations that may arise in this context [23][16][21]. We have also begun the study of the long time limit of such systems. The work is done in collaboration with Felix Otto (Univ. of Bonn).

The second topic of interest deals with concentrated suspensions, and is done in collaboration with Isabelle Catto (University Paris 9). So far, the study is mainly of theoretical nature (see however [14]). We have established the well-posedness of the Cauchy problem for the equations modeling the suspension [10] and also studied the long time limit of those.

5.6. Magnetohydrodynamics flows

**Participants:** Claude Le Bris, Tony Lelièvre, Antonin Orriols.

In collaboration with Jean-Frédéric Gerbeau (Inria, BANG), and in association with Aluminium Péchiney, we have pursued our efforts for the numerical simulation of electrolytic cells for the industrial production of Aluminium [15]. The major work completed this year has been the parallelization of the current version of the code. This is performed in the context of the PhD thesis of Antonin Orriols. The purpose was of course to speed up the calculation time, thereby making it possible to envision a control and optimization strategy. The next step indeed consists in addressing the following question: how can we design the electromagnetic boundary conditions on the cell, or some geometrical features of the cell, in order to make the cell as efficient as possible and keep it as stable and robust as possible with respect to external perturbations.

In addition to this, a state of the art review article has been written [19], and a book on the subject is in preparation.

5.7. Mathematical models for models issued from biology

**Participants:** Frédéric Legoll, Eric Cancès, Gabriel Turinici.

The acceleration of simulations of the stochastic evolutions equations that appear in cell and molecular biology was the topic of a collaboration with A.Alfonsi (CERMICS / ENPC), W.Huisingsa(Frei U. Berlin) and B. Di Ventura (EMBL). A sound mathematical foundation for the simulations in hybrid joint stochastic -deterministic frameworks has been introduced and tested on a system of biological interest.

6. Contracts and Grants with Industry

Many research activities of the team are indeed conducted in close collaboration with private or public companies: Pechiney for the modeling of electrolytic cells, Electricité de France and Commissariat à l'Energie
Atomique for computational chemistry, molecular dynamics and multiscale simulation of solids, and companies from the elastomer industry for the multiscale simulation of rubber-like materials.

7. Other Grants and Activities

7.1. Regional activities
The team is shared between INRIA and Ecole Nationale des Ponts et Chaussées.

7.2. National activities
The team is part of the research action GDR Density Functional Theory whose scientific leader is H. Dreyssé, devoted to the development of DFT methods for the simulation of materials and complex systems. It is also a part of the research action GDR Interaction de particules, whose scientific leader is Th. Goudon, on questions related to the modeling of many particles systems.
Claude Le Bris is the scientific leader of a new program initiative ACI “Nouvelles interfaces des mathématiques” devoted to various questions related to computational chemistry, molecular simulations and multiscale problems.

7.3. European Community financed activities
Some members of the team participate into the european project IHP “HYKE” (Hyperbolic and kinetic equations: Asymptotics, numerics, Applications) whose Scientist in charge is Benoît Perthame (BANG), on theoretical aspects related to the resolution of the Schrödinger equation for large systems and long times.
Some members of the team participate into the european project (Marie Curie Research Training Networks) “MULTIMAT” devoted to the multiscale modelling of materials, scientist in charge Nick Schryvers.

7.4. Bilateral international relations

7.4.1. Americas
Continuous and permanent cooperations have been established with the group of Gustavo Scuseria at Rice University on questions related to electronic structure calculations for large systems, that of Herschel Rabitz at Princeton University and that of André Bandrauk at Unversity of Sherbrooke (Canada), respectively on questions related to laser control and to the solution of the Schrödinger equation for a large number of degrees of freedom. The collaboration with Herschel Rabitz is part of a PICS CNRS-NSF collaboration between Princeton University and the Laboratoire J.L.Lions (Paris 6).

8. Dissemination

8.1. Animation of the scientific community
Claude Le Bris is member of the board of directors of the SMAI (french SIAM), more particularly in charge of relation with the industrial companies.
Eric Cancès has organized a mini-symposium at CANUM (with Jean-Frédéric Gerbeau), Obermay June 2004 on the topic “Modeling and numerical simulations in biology and medicine”. He has also organized, again with JF Gerbeau, the summer school CEMRACS 2004 “Mathematics and applications in biology and medicine”, CIRM, Marseille July 26 - Sept. 3.
As a part of the ARC Prestissimo, the team has co-organized the international conference on “ Molecular dynamics: mathematical and numerical aspects”, Institut Henri Poincaré, Dec. 2004

8.2. Teaching activities
- Simulation moléculaire: aspects théoriques et numériques, cours de DEA, université Paris 6 (C. Le Bris).
- Simulation moléculaire, cours de DEA, université Paris 9 (E. Cancès).
- Systèmes multïchêles, cours de la majeure SEISM, Ecole Polytechnique (C. Le Bris).
- Calcul scientifique et Analyse, cours à l’Ecole Nationale des Ponts et Chaussées, (E. Cancès).
- Analyse en fréquences, cours à l’Ecole Nationale des Ponts et Chaussées, (E. Cancès).
- Modéliser, Programmer, Simuler, cours à l’Ecole Nationale des Ponts et Chaussées, (C. Le Bris).
- DEUG, Université Evry (Y. Gati)
- Cours Probabilités et Applications, (janv 2004) (T. Lelièvre)
- Cours de Méthodes Mathématiques pour la Finance, ENPC. (mars 2004 - juin 2004) (T. Lelièvre)
- Cours de Calcul scientifique, ENPC. (janvier 2004 - mars 2004) (T. Lelièvre)

8.3. Conference participation

Members of the team have delivered lectures in the following seminars, workshops and international conferences:

- Seminar of the Mathematics department (Grenoble, toulouse, Nancy, Bonn, Montreal, McGill, Sherbrooke) (Claude Le Bris)
- Workshop Mathematical aspects of material science: discrete and continuum descriptions of matter, Castle Ringberg, February 01-07, 2004. (Claude Le Bris)
- SIAM Conference on Mathematical Aspects of Materials Science, Los Angeles, May 23-26 2004. (Tony Lelièvre, Frederic Legoll, Claude Le Bris)
- 7ème Colloque Franco-Roumain de Mathématiques Appliquées, Craiova (Roumanie), 30 août- 3 septembre 2004. (Claude Le Bris)
- International workshop on rheological models for fluids, Montréal, November 2004 (Eric Cancès, Claude Le Bris, Tony Lelièvre)
- International workshop on the mathematical aspects of quantum chemistry, Warwick, December 2004, series of three lectures (Claude Le Bris)
- ICCMSE conference, Athens, November 2004 (E. Cancès)
- ICFD, Oxford 2004 (Y. Gati)
- "Mathematical theory of systems and networks", June 2004, Louvain-Belgium (M. Mirrahimi)
- NOLCOS (IFAC symposium on nonlinear control systems), September 2004, Stuttgart- Germany (M. Mirrahimi)
- IPM Workshop on nonlinear PDE’s, Dec. 2004, Teheran-Iran (M.Mirrahimi)
- Computation and Numerical Analysis for Multiscale and Multiphysics Modelling, the University of Warwick, April 2004. (T. Lelièvre)
- MC2QMC 2004 conference, Juan-les-Pins, June 2004. (T. Lelièvre)
- ICTAM 2004, Varsovie, Aug. 2004 (A. Gloria)
- 36ième Congrès National d’Analyse Numérique, Obernay, juin 2004. (E.Cancès)
- Workshop Cecam "Accelerating Dynamical Simulations", Lyon, mar. 2004 (F. Legoll)
- CEMRACS 2004: Mathematics and Applications in Biology and Medecine, Marseille, July-Aug 2004 (E. Cancès, H. Galicher, A. Gloria, F. Legoll, T. Lelièvre, G. Stoltz, G. Turinici)
- 21st International Congress on Theoretical and Applied Mechanics, Varsovie, août 2004 (F. Legoll)
- Workshop on Molecular simulation: Algorithmic and Mathematical Aspects, Paris, décembre 2004 (F. Legoll)
9. Bibliography

Books and Monographs


Doctoral dissertations and Habilitation theses


Articles in referred journals and book chapters


**Internal Reports**

